Some More Results on Super Heronian Mean Labeling

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Abstract

Here we look into some more results on Super Heronian Mean Labeling for some standard graphs. In this paper we prove that \( P_n \circ K_{1,2}, P_n \circ K_{1,3}, P_n \circ K_3, (P_n \circ K_1) \circ K_{1,2}, (P_n \circ K_1) \circ K_{1,3} \) are Super Heronian Mean graphs.

Keywords: Graph, Super Heronian mean graph, \( P_n \circ K_{1,2}, P_n \circ K_{1,3} \), \( P_n \circ K_3, (P_n \circ K_1) \circ K_{1,2}, (P_n \circ K_1) \circ K_{1,3} \).

1. INTRODUCTION

We start with simple, finite and undirected graph and have \( p \) vertices and \( q \) edges. For a detailed survey of graph labeling, we refer to J.A Gallian [1]. For standard terminology and notation we follow Harary [2]. The concept of Super Heronian Mean Labeling was introduced by S.S. Sandhya, E. Ebin Raja Merly and G.D. Jemi in [7]. In this paper, we discuss some more results on Super Heronian Mean Labeling for some special graphs.

Definition: 1.1

Let \( f : V(G) \rightarrow \{1,2,\ldots,p+q\} \) be an injective function. For a vertex labeling “\( f \)” the induced edge labeling \( f^*(e=uv) \) is defined by,

\[
f^*(e) = \left\lfloor \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rfloor \quad \text{OR} \quad \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil
\]

Then “\( f \)” is called a Super Heronian Mean Labeling if
\{f(V(G)) \cup \{f(e) : e \in E(G) = \{1,2,\ldots,p+q\}\}\). A graph which admits Super Heronian Mean Labeling is called **Super Heronian Mean Graph**.

**Theorem 1.2** Any Path \(P_n\) is a Super Heronian Mean Graph.

**Theorem 1.3** Any Cycle \(C_n\) is a Super Heronian Mean Graph.

**Theorem 1.4** Any Comb \((P_n \odot K_1)\) is a Super Heronian Mean Graph.

### 2. MAIN RESULTS

**Theorem 2.1**

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a Comb graph. Then G is a Super Heronian mean graph.

**Proof:**

Comb \((P_n \odot K_1)\) is a graph obtained from a path \(P_n = v_1v_2\ldots v_n\) by joining a vertex \(v_i\) to \(u_i, 1 \leq i \leq n\). Let G be a graph obtained by joining pendant vertices \(w\) and \(z\) respectively.

Define a function \(f: V(G) \rightarrow \{1,2,\ldots,p\}\) by,

\[
\begin{align*}
f(w) &= 1, \\
f(v_1) &= 3, \\
f(v_i) &= 4i+1; 2 \leq i \leq n \\
f(z) &= 4n+3 \\
f(u_1) &= 5, \\
f(u_i) &= 4i-1; 2 \leq i \leq n
\end{align*}
\]

Edges are labeled with,

\[
\begin{align*}
f(wv_1) &= 2 \\
f(v_1v_{i+1}) &= 4i+2; 1 \leq i \leq n-1 \\
f(v_nz) &= 4n+2 \\
f(v_iu_i) &= 4i; 1 \leq i \leq n
\end{align*}
\]

\[f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1,2,\ldots,p\}\]

Thus \(f\) provides Super Heronian mean labeling of G.

Hence G is a Super Heronian mean Graph.
**Example: 2.2** A Super Heronian mean labeling of G when n=5 is given below

\[
\begin{align*}
&1 & 2 & 3 & 6 & 9 & 10 & 13 & 14 & 17 & 18 & 21 & 22 & 23 \\
&4 & 7 & 8 & 12 & 16 & 20 & & & & & & &
\end{align*}
\]

**Figure 1**

**Theorem: 2.3**

Let G be a graph obtained by attaching each vertex of \( P_n \) to the central vertex of \( K_{1,2} \). Then G is a Super Heronian mean graph.

**Proof:**

Let \( P_n \) be the path \( u_1u_2 \ldots u_n \) and \( v_i, w_i \) be the vertices of \( K_{1,2} \) which are attached to vertex \( u_i \) of \( P_n \). The graph contain \( 3n \) vertices and \( 3n-1 \) edges.

Define a function \( f: V(G) \rightarrow \{1, 2, \ldots, p+q\} \) by,

\[
\begin{align*}
f(u_i) &= 6i-3 ; 1 \leq i \leq n \\
f(v_i) &= 6i-5 ; 1 \leq i \leq n \\
f(w_i) &= 6i-1 ; 1 \leq i \leq n
\end{align*}
\]

Edges are labeled with,

\[
\begin{align*}
f(u_iu_{i+1}) &= 6i ; 1 \leq i \leq n-1 \\
f(u_iv_i) &= 6i-4 ; 1 \leq i \leq n \\
f(u_iw_i) &= 6i-2 ; 1 \leq i \leq n
\end{align*}
\]

This gives a Super Heronian mean labeling of G.

**Example: 2.4** A Super Heronian mean labeling of \( P_4 \bigcirc K_{1,2} \) is given below.

\[
\begin{align*}
&3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 & 30 & 33 & 36 \\
&2 & 4 & 8 & 10 & 14 & 16 & 20 & 22 & & & &
\end{align*}
\]

**Figure 2**
Theorem: 2.5
Let $G$ be a graph obtained by attaching each vertex of $P_n$ to the central vertex of $K_{1,3}$. Then $G$ is a Super Heronian mean graph.

Proof:
Let $P_n$ be the path $u_1, u_2, \ldots, u_n$ and $v_i, w_i, z_i$ be the vertices of $K_{1,2}$ which are attached to the vertex $u_i$ of $P_n$.

Define a function $f: V(G) \to \{1, 2, \ldots, p+q\}$ by,

$$
\begin{align*}
  f(u_i) &= 8i-3 ; 1 \leq i \leq n \\
  f(v_i) &= 8i-7 ; 1 \leq i \leq n \\
  f(w_i) &= 8i-5 ; 1 \leq i \leq n \\
  f(z_i) &= 8i-1 ; 1 \leq i \leq n
\end{align*}
$$

Edges are labeled with,

$$
\begin{align*}
  f(u_iu_{i+1}) &= 8i ; 1 \leq i \leq n-1 \\
  f(u_iv_i) &= 8i-6 ; 1 \leq i \leq n \\
  f(u_iw_i) &= 8i-4 ; 1 \leq i \leq n \\
  f(u_iz_i) &= 8i-2 ; 1 \leq i \leq n
\end{align*}
$$

This gives a Super Heronian mean labeling of $G$.

Example: 2.6 A Super Heronian mean labeling of $P_4 \odot K_{1,3}$ is given below.

\[\text{Figure: 3}\]

Theorem: 2.7
Let $G=P_n \odot C_3$ be a graph obtained by attaching $C_3$ to each vertex of a path $P_n$. Then $G$ is a Super Heronian mean graphs.
Some More Results on Super Heronian Mean Labeling

**Proof:**
Consider a graph $G$ is obtained by attaching $C_3$ to each vertex of a Path $P_n$. Let $P_n$ be a path $u_1, u_2 \ldots u_n$. Let $u_i, v_i, w_i, 1 \leq i \leq n$ be the vertices of $C_3$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, p+q\}$ by,

$f(u_i)=7i-3 ; 1 \leq i \leq n$

$f(v_i)=7i-6 ; 1 \leq i \leq n$

$f(w_i)=7i-1 ; 1 \leq i \leq n$

Edges are labeled with,

$f(u_iu_{i+1})=7i ; 1 \leq i \leq n-1$

$f(u_iv_i)=7i-5 ; 1 \leq i \leq n$

$f(v_iw_i)=7i-4 ; 1 \leq i \leq n$

$f(u_iw_i)=7i-2 ; 1 \leq i \leq n$

$f(V(G)) \cup \{f(e) \in E(G)\} = \{1, 2, \ldots, p+q\}$

Hence $G$ is a Super Heronian mean graph.

**Example:** A Super Heronian mean labeling of $P_4 \bigodot C_3$ is displayed below.

![Figure 4](image)

**Theorem:** 2.9

A graph obtained by attaching $K_{1,2}$ at each pendant vertex of a Comb is a Super Heronian mean graph.

**Proof:**
Let $G_1$ be a comb and $G$ be the graph obtained by attaching $K_{1,2}$ at each pendant vertex of $G_1$. Let its vertices be $u_i, v_i, w_i, x_i, 1 \leq i \leq n$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, p+q\}$ by,

$f(u_i)=8i-3 ; 1 \leq i \leq n$
\[ f(v_i) = 8i - 1; \ 1 \leq i \leq n \]
\[ f(w_i) = 1 \]
\[ f(w_i) = 8i - 8; \ 2 \leq i \leq n \]
\[ f(x_i) = 8i - 6; \ 1 \leq i \leq n \]

Edges are labeled with,
\[ f(u_i u_{i+1}) = 8i + 1; \ 1 \leq i \leq n - 1 \]
\[ f(u_i v_i) = 8i - 2; \ 1 \leq i \leq n \]
\[ f(v_i w_i) = 8i - 5; \ 1 \leq i \leq n \]
\[ f(v_i x_i) = 8i - 4; \ 1 \leq i \leq n \]

Thus both vertices and edges together get distinct labels from \{1, 2, \ldots, p+q\}.

Hence \( G \) is a Super Heronian mean graphs.

**Example:** 2.10 A Super Heronian mean labelling of \( G = (P_5 \odot K_{1,2}) \) is given below.

**Theorem:** 2.11

A graph obtained by attaching a triangle at each pendant vertex of a Comb is a Super Heronian mean graph.

**Proof:**

Let \( G_1 \) be a comb and \( G \) be the graph obtained by attaching a triangle at each pendant vertex of \( G_1 \). Let its vertices be \( u_i, v_i, w_i, x_i, 1 \leq i \leq n \).
Define a function \( f: V(G) \rightarrow \{1, 2, \ldots, p+q\} \) by,
\[
\begin{align*}
    f(u_i) &= 9i - 1 ; 1 \leq i \leq n \\
    f(v_i) &= 9i - 3 ; 1 \leq i \leq n \\
    f(w_1) &= 1 \\
    f(w_i) &= 9i - 9 ; 2 \leq i \leq n \\
    f(x_i) &= 9i - 5 ; 1 \leq i \leq n
\end{align*}
\]

Edges are labeled with,
\[
\begin{align*}
    f(u_iu_{i+1}) &= 9i + 3 ; 1 \leq i \leq n - 1 \\
    f(u_iv_i) &= 9i - 2 ; 1 \leq i \leq n \\
    f(v_1w_1) &= 3, f(v_1w_i) = 9i - 7 ; 2 \leq i \leq n \\
    f(v_ix_i) &= 9i - 4 ; 1 \leq i \leq n \\
    f(w_1x_1) &= 2, f(w_ix_i) = 9i - 8 ; 2 \leq i \leq n
\end{align*}
\]

Thus \( f \) provides a Super Heronian mean graph.

**Example:** 2.1 A Super Heronian mean labeling of \( G=(P_4 \odot K_1) \odot K_{1,2} \) is given below,
REFERENCES


