Analyses of Field Measured Data With Rheology and Hydraulics Models

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Abstract

In this paper, the rheology & the hydraulic predictive power of several non-Newtonian models has been evaluated. These are Bingham, Power law, Herschel-Buckley, Robertson & Stiff, and Unified models. Field data were considered for the investigation. The analysis results show that the Robertson & Stiff and Power law models exhibit good prediction. However, the main conclusion from this work is to indicate that hydraulics models always need calibration since the models don’t capture the physics of the entire physical phenomenon during operations. It should also be noted that the predictive nature of the models varies as the mud system changes.

1. INTRODUCTION

Drilling fluid is an essential part of drilling operation. Drilling fluid is pumped through a drill string and return back through an annulus. The main function of drilling fluid among others is to cool and lubricate the drill bit, to bring drill cutting to the surface and maintains wellbore pressure.

Poorly designed drilling fluid properties along with wrongly used operational parameters may result undesired cutting accumulation in a wellbore. This leads to several drilling related problems including: excessive over pull on trips; increasing torque; stuck pipe; hole pack-off; increasing ECD; formation fracturing; slow rates of penetration; and difficulty running casing and logs (API, 13D).

Wrongly designed wellbore pressure may result borehole fracturing or collapse, which leads to huge mud loss and pack-off respectively. The wellbore instability problems in
average increase the non-productive time up to 14-20% and the overall drilling budget by about 10% (Aadnøy, 2003). Properly designed equivalent circulation density (ECD) can handle these problems. The ECD is determined from the sum of the static mud weight, and the frictional pressure loss. Therefore, the accurate prediction of frictional pressure loss is very important.

During planning and operation phase, optimization of drilling hydraulics requires calculation of frictional pressure losses in the system and the minimum fluid velocity to carry the cuttings in the annulus. For wellbore instability and hole cleaning problems, determination of a safe operational window is underpinned by good wellplan, best drilling practices, and proper selection of the rheological characteristics of the drilling fluid, which is still a challenging task for engineers (API, 13D).

The theoretical calculation of pressure losses in a wellbore require the knowledge of the correct fluid properties as the fluid flows through each interval of a borehole. In literature, there exists several rheology and hydraulics models. However, it is important to analyze the predictive power of the models. Some hydraulic prediction methods use an iterative procedure in order to match model with experimental data. Since hydraulics models don’t capture all physics, (Lohne et al, 2010) have generated a model calibration factor, which could contain some hidden physical parameters.

The main objectives of this paper is to analyze the predictive power of different hydraulic models. Literature documented field data was considered (Roberto et al. 1996) for this analysis, where the authors analyzed field data with Bingham, Power law, and Herschel-Buckley models. However, in this paper Unified, and Robert Stiff model were also used for the analysis, in addition to Bingham, Power law, and Herschel-Buckley used previously.

2 RHEOLOGY MODELS
This section presents the summary of non-Newtonian rheology models. Their corresponding hydraulics models are not presented in the paper, except the results of the analysis, however More details on this can be found in the papers referenced in introduction section.

Bingham Plastic model
According to the model, fluid behavior exhibits a linear shear stress vs. shear rate relationship. Bingham plastic fluid is two parameters model, which has a yield point (intercept) and constant slope (Plastic viscosity). The intercept of the line is part of the fluid viscosity, which is caused by forces of attraction between charged ions in the drilling fluid. In other word, according to the model, certain minimum pressure is required to overcome the shear yield stress in order to start the fluid to flow. Plastic viscosity is part of the fluid resistance, which is due to the fluid-fluid or fluid–solid or solid-solid interaction in the drilling fluid. The model reads:
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\[ \tau = \mu_p \gamma + \tau_y \]

Where, yield point \((\tau_y)\) and plastic viscosity \((\mu_p)\)

For hydraulic calculation, the yield stress and the plastic viscosity are determined directly from the Fann data as:

\[ \mu_p \text{ (cP)} = Q_{600} - Q_{300} \]

\[ \tau_y \text{ (lbf/100sqft)} = 2Q_{300} - Q_{600} \]

**Power law model**

Drilling fluid reduces its viscosity as the shear rate increases. Power law model describes polymer based drilling fluid behavior better than the Bingham plastic model. The power law model is described by two parameters as:

\[ \tau = k \gamma^n \]

Where, \(k\) is the consistence index and \(n\) is flow behavior index. Even though this model provides reasonable predictions of fluid behavior at higher shear rates, it also fails to simulate the shear behavior of most drilling fluids in lower shear rates.

**Herschel Buckley**

The Herschel-Buckley is yielded power law model. Unlike the power law model, the fluid can be set into flow when the applied minimum pressure overcomes the yield stress, which is defined as \(\tau_0\). This value is different from the Bingham yield stress. The model defines a fluid by three-parameters and can be described as:

\[ \tau = \tau_0 + k \gamma^n \]

It is documented in literature that the Herschel-Buckley model is preferred to Power-law or Bingham relationships since it better describes rheological behavior of drilling fluids.

The flow and consistency indexes \(n\) and \(k\) values can be determined graphically. The value \(\tau_0\) is determined from:

\[ \tau_0 = \frac{\tau_* - \tau_{max} \times \tau_{min}}{2 \tau_* - \tau_{max} - \tau_{min}} \]

Where \(\tau_*\) is the shear stress value corresponding to the geometric mean of the shear rate, \(\gamma_*\).
Unified model\textsuperscript{1,10}  
Unified model\textsuperscript{2} is another version of yielded power law model, which is described by three parameters as:
\[ \tau = \tau_{yd} + k \gamma^* \]  

The \( n \) and \( k \) parameters calculation use the Bingham plastic viscosity (\( \mu_p \)), yield point (\( \tau_y \)). The yield stress, \( \tau_{yl} \) for the Unified model is calculated from the 3 and 6 RPM Fann data as: (\( \tau_{yl}=2\theta_3-\theta_6 \)).

Robertson and stiff\textsuperscript{9}  
Robertson and Stiff developed a more general model to describe the rheological behavior of drilling fluids and cement slurries. The model reads:
\[ \tau = A (\gamma + C)^B \]  

where \( A, B, \) and \( C \) are model parameters. \( A \) and \( B \) can be considered similar to the parameters \( k \) and \( n \) of the Power-law model. The third parameter \( C \) is a correction factor to the shear rate, and the term (\( \gamma + C \)) is considered effective shear rate. The parameter \( C \) is determined from:
\[ C = \frac{\gamma_{max} \gamma_{min} - \gamma^2}{2\gamma^* - \gamma_{min} - \gamma_{max}} \]  

Where \( \gamma^* \) is the shear rate value corresponding to the geometric mean of the shear stress, \( \tau^* \). The geometric mean of the shear stress (\( \tau^* \)) is then calculated from:
\[ \tau^* = (\tau_{min} \times \tau_{max})^{\frac{1}{2}}. \]

Bit hydraulics model  
Pressure drop across a bit occurs as drilling mud flowing through bit nozzles. This pressure drop is part of the entire pressure loss in the system and is important for drilling hydraulic optimization such as for designing maximum hydraulic horse power and hole cleaning and ECD determination. The pressure drop at the nozzle area is a function of total flow area of the bit, flow rate and mud density. The pressure drop across the bit nozzles also can be calculated by the following equation\textsuperscript{13}
\[ P_{bit} = \frac{156WQ^2}{(D_{bit}^2 + D_{noz}^2 + D_{ns}^2)^2} \]  

Where, \( P_{bit} \), psi, \( W \)-mud weight, ppg, \( Q \)-Flow rate, gpm, \( D_n \) bit nozzles diameters, inch
3. ANALYSIS OF FIELD DATA
The Roberto et al.'s (1996)\textsuperscript{4} experimental data were used for models evaluation. Table 1 shows the rheology of the mud system and Table 2 shows the drill string sizes. The bit has 4 nozzles having a size of 3x15/32” + 1x14x32”. Based on the dataset, the well structure is sketched and shown as Figure 1.

Table 1: Mud Rheology Measurement (Roberto et al (1996)\textsuperscript{4})

<table>
<thead>
<tr>
<th>RPM</th>
<th>Dial reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>38</td>
</tr>
<tr>
<td>300</td>
<td>26</td>
</tr>
<tr>
<td>200</td>
<td>22</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 2: Drilling well consist of a 513,6m drill string of 5x4.28” size. The drill string is connected with 137m long heavy weight having a size of 5x3in. In addition, two different ODs, but same ID size of drilling collar with a total length of 77m.

Table 2: Drill string data (Roerto et al, (1996)\textsuperscript{4})

<table>
<thead>
<tr>
<th>Type</th>
<th>Length (m)</th>
<th>OD(in)</th>
<th>ID (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standpipe</td>
<td>20</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>Rotary hose</td>
<td>20</td>
<td>-</td>
<td>3,5</td>
</tr>
<tr>
<td>Swivel</td>
<td>3.5</td>
<td>-</td>
<td>3,5</td>
</tr>
<tr>
<td>Kelly</td>
<td>12</td>
<td>-</td>
<td>3,5</td>
</tr>
<tr>
<td>Drillpipe</td>
<td>513,6</td>
<td>5</td>
<td>4,28</td>
</tr>
<tr>
<td>Heavy weight</td>
<td>137</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Drill collar</td>
<td>70</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Drill collar</td>
<td>77</td>
<td>11 ¼</td>
<td>3</td>
</tr>
<tr>
<td>Bit</td>
<td>0,40</td>
<td>3x15/32” + 1x14x32”</td>
<td></td>
</tr>
</tbody>
</table>
3.1 Rheology property prediction and analysis
The measured drilling fluid rheology shown Table 1 was analyzed by the reviewed rheology models. The calculated average % error between the models and the measured data are presented in Table 3 along with the model parameters. Figure 2 shows the comparisons between measured and the predicted rheologies of the drilling fluid.

Figure 1: Well structure /not to scale
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**Figure 2**: Rheology prediction of field scale data

**Table 3**: Error analysis of rheology prediction

<table>
<thead>
<tr>
<th>Rheology Models</th>
<th>Parameters</th>
<th>Average sum % error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bingham</td>
<td>$\mu_p=0.0327$ $\tau_y=7.1974$</td>
<td>26.79</td>
</tr>
<tr>
<td>Power law</td>
<td>$n=0.4043$ $k=2.1$</td>
<td>6.55</td>
</tr>
<tr>
<td>Herschel Buckley</td>
<td>$\tau_o=3.73$ $n=0.7304$ $k=0.2399$</td>
<td>4.25</td>
</tr>
<tr>
<td>Unified</td>
<td>$\tau_m=4.0$ $n=0.8073$ $k=0.148$</td>
<td>6.35</td>
</tr>
<tr>
<td>Robertson &amp; Stiff</td>
<td>$A=0.7342$ $B=0.5714$ $C=19.04$</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Based on the computed sum of the absolute average errors between the model and the measured data, the results show that the Robertson-Stiff model exhibit the least error.
rate, whereas Bingham model results with higher error rate. The best curve fitting for this drilling fluid can be compared in the order of:

**Robertson and stiff > Herschel Buckley > Unified > Power law > Bingham**

### 3.2 Hydraulic prediction analysis

Figure 3 shows the computed standpipe pressure (SPP) obtained the hydraulic models with the measured data. For better visualization, the analysis results are provided in Table 4. For each flow rate, the % error between the measurement and the model prediction were calculated. As can be seen, the error rate at different flow rates show different values. For all flow rates, the % average of the errors were calculated for comparison purpose. The result shows that Robertson and Stiff model exhibits a lower % error rate. In the order of good prediction performance, we can observe:

**Robertson and stiff > Power law > unified > Bingham > Herschel Buckley**

However, the original researchers have reported the opposite and it is difficult to tell how they did the calculations. However, in this paper, the calculation is very straightforward and the bit models used is different than those used in previous researches.

![Figure 3: Comparison of hydraulics model & measured SPP](image)
Table 4: Absolute and average % error analysis

| Pump rate lpm | Measured SPP [bar] (Roberto et al) | Models | | | | | |
|--------------|-----------------------------------|--------|--------|--------|--------|--------|
| 1640         | 1640                              | 46      | 55.5    | 45.2    | 58.0    | 36.8    | 46.2    |
| 2460         | 2460                              | 103     | 114.8   | 95.0    | 86.7    | 96.9    | 96.7    |
| 3270         | 3270                              | 176     | 193.0   | 160.8   | 129.7   | 163.1   | 163.3   |

Absolute and average % error between the measurement and the models prediction

<table>
<thead>
<tr>
<th>Pump rate lpm</th>
<th>% Error</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1640</td>
<td>20.6</td>
<td>1.7</td>
<td>26.0</td>
<td>20.0</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>2460</td>
<td>11.4</td>
<td>7.7</td>
<td>15.8</td>
<td>6.0</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>3270</td>
<td>9.64</td>
<td>8.64</td>
<td>26.30</td>
<td>7.32</td>
<td>7.23</td>
<td></td>
</tr>
</tbody>
</table>

Average error %

| Pump rate lpm | Average error % | | | | |
|--------------|-----------------|--------|--------|--------|
| 1640         | 13.9            | 6.0    | 22.7   | 11.1   | 4.6    |

3.3 Calibration of models

As shown on Table 4, none of the models used in this analysis were predict the measured data. This suggests that models always require calibration in order to match real measured data.

Lohne at al. (2008) also analyzed the hydraulics of field-measured data with the commonly used friction loss model, where their analysis also shows a discrepancy between the measurement and the model. The frictional pressure loss for both through drill-string and annulus flow as a function of friction factor can be calculated with Eq. 8. In addition to the friction factor, there is uncertainty in the temperature of the well, dynamic characteristics of the pump, pressure loss through BHA and bit, and the density and rheology of the drilling fluid as functions of temperature and pressure. Since the accurate information about these parameters is difficult to determine, the authors use a correlation factor, c, to match the model to the measurements. This factor will account for the hidden physical parameters that the model doesn’t take into account. The modified pressure loss written as [5]

\[
\frac{dP}{dt} = c \times \frac{2f}{D_{hy}} \rho_{mix} v_{mix}^2
\]

Where, \(D_{hy}\) is the hydraulic diameter, \(D_{hy} = D\) for drillstring and \(D_{hy} = D_w - D_c\) for annulus and \(v_{mix}\) is the fluid mixture velocity.
Figure 4 displays the correlation factors generated for the five models, which are a function of flow rate. The best correlation is 1 or closer to 1. Power law and Roberson & Stiff models show nearly the same correlation for all flow rates. Unified model also shows similar correlation factors to these models at higher flowrates.

**Figure 4: Correlation factor**

4 SUMMARY
The paper presents the rheology and hydraulics analysis of literature documented field data using different models. Based on the considered drilling fluid and experimental well, the result of the analysis can be summarized as the following.

- Robertson & Stiff model shows lower error rate and Bingham shows a higher error rate as compared with the measured data. However, this is not a general conclusion for all types of drilling fluid. As separate study showed that changing different drilling fluid, other hydraulic models might predict better than the one mentioned earlier.
- In terms of hydraulics, the Roberson & Stiff and Power law models show good for all flow rates, but Unified also shows similar to these models at a very high flow rates.
- The overall result indicates that it is difficult to generalize which model describe best. It is always important to calibrate models with a measured real time data in order to capture the entire physical phenomenon during operations.
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NOMENCLATURE

$k$ consistency index, lbf-s°/100sqft

$n$ flow behavior index

$P$ bit pressure drop, psi

$W$ mud weight, pound per gallon

$Q$ flow rate, gal per min

$D_n$ bit nozzles diameters, inch

$\tau_y$ yield point, lbf/100sqft

$\mu_p$ plastic viscosity, cP

$lpm$ liter per minute

SPP standpipe pressure

REFERENCES


