Independent Attributes for $m$-Concepts in a Soft Context Induced by a Soft Set

Young Key Kim$^1$ and Won Keun Min$^2$ *

$^1$Department of Mathematics, MyongJi University, Youngin 17058, Korea.

$^2$Department of Mathematics, Kangwon National University, Chuncheon 24341, Korea.

Abstract

For the purpose of studying more effective ways of finding the reduction in a formal context, we have combined the formal contexts with the soft sets to form so-called soft contexts, and proposed the notion of soft concepts. And to study the structure of soft contexts, we introduced a new type of soft concept (called $m$-concept or object oriented soft concept) based on soft sets and the set of all $m$-concepts. In this paper, we introduce and study the notion of $m$-dependent and $m$-independent attributes in a given soft context. And, we show that every $m$-dependent attribute is generated by some $m$-independent attributes and the family of all $m$-independent attributes generates all $m$-concepts in a given soft context. Finally, we show that a reduction of a soft concept lattice is obtained by the family of all $m$-independent attributes.

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*Corresponding author: wkmin@kangwon.ac.k
1. Introduction

Wille introduced the formal concept analysis in [18], which is an important theory for the research of information structures induced by a binary relation between the set of attributes and objects attributes. The basic notions of formal concept analysis are formal context, formal concept, and concept lattice. A formal context is a kind of information system, which is a tabular form of an object-attribute value relationship [3, 4, 6, 7]. A formal concept is a pair of a set of objects as called the extent and a set of attributes as called the intent. The set of all formal concepts together with the order relation forms a complete lattice called the concept lattice [6, 17]. Formal concept lattice is the core data structure and a kind of a formal knowledge representation.

Molodtsov introduced the notion of soft set in 1999 [15], which is to deal complicated problems and uncertainties. Maji et al. introduced the operations for soft set theory in [12]. In [1], Ali et al. proposed new operations modified some concepts introduced by Maji. Until recently, researches combining soft sets with other mathematical concepts have been extensively studied. [2, 4, 5, 11, 13, 16]

In [14], we have formed a soft context by combining the concepts of the formal context and the soft set defined by the set-valued mapping. And we introduced and studied the new concepts named soft concepts and soft concepts lattices. Furthermore, in [8], we introduced some operations on a parameter set of a soft set, and studied some properties of such notions. In [9], for a soft set over a universe set, we investigated a special operation induced by two operations defined in [8], and studied some related properties and several characterizations. And also, by using the two operation, we investigated the new concept of $m$-concepts related closely the object oriented concept in formal context, and showed that the family of all the $m$-concepts in a soft context is a supra topology but not a topology. Moreover, we studied the notion of independent and dependent $m$-concept. In particular, we showed that the set of all independent $m$-concepts completely determines every $m$-concept in a soft context and the smallest base for the set of all soft concepts as a supratopological structure.

In this paper, we introduce and study the notion of $m$-dependent and $m$-independent attributes in a given soft context (Definition 3.1). And, we show that every $m$-dependent attribute is generated by some $m$-independent attributes (Theorem 3.9) and the family of all $m$-independent attributes generates all $m$-concepts in a given soft context (Theorem 3.13). Finally, we show that a reduction of a soft concept lattice is obtained by the family of all $m$-independent attributes (Theorem 3.16).

2. Preliminaries

A formal context is a triplet $(U, V, I)$, where $U$ is a non-empty finite set of objects, $V$ is a nonempty finite set of attributes, and $I$ is a relation between $U$ and $V$. Let $(U, V, I)$ be a formal context. For a pair of elements $x \in U$ and $y \in V$, if $(x, y) \in I$, then it
means that object $x$ has attribute $y$ and we write $xIy$. The set of all attributes with a given object $x \in U$ and the set of all objects with a a given attribute $y \in V$ are denoted as the following [17,18]:

$$x^* = \{y \in V | xIy\}; \ y^* = \{x \in U | xIy\}.$$ 

And, the operations for the subsets $X \subseteq U$ and $Y \subseteq V$ are defined as:

$$X^* = \{y \in V | \text{for all } x \in X, xIy\}; \ Y^* = \{x \in U | \text{for all } y \in Y, xIy\}.$$ 

In a formal context $(U, V, I)$, a pair $(X, Y)$ of two sets $X \subseteq U$ and $Y \subseteq V$ is called a formal concept of $(U, V, I)$ if $X = Y^*$ and $X = Y^*$, where $X$ and $Y$ are called the extent and the intent of the formal concept, respectively.

Let $U$ be a universe set and $E$ be a collection of properties of objects in $U$. We will call $E$ the set of parameters with respect to $U$.

A pair $(F, E)$ is called a soft set [15] over $U$ if $F$ is a set-valued mapping of $E$ into the set $P(U)$ of all subsets of the set $U$, i.e.,

$$F : E \rightarrow P(U).$$ 

In other words, for $a \in E$, every set $F(a)$ may be considered as the set of $a$-elements of the soft set $(F, E)$.

Let $U = \{z_1, z_2, \ldots, z_m\}$ be a non-empty finite set of objects, $E = \{e_1, e_2, \ldots, e_n\}$ a non-empty finite set of attributes, and $F : E \rightarrow P(U)$ a soft set. Then the triple $(U, E, F)$ is called a soft context [14].

And, in a soft context $(U, E, F)$, we introduced the following mappings:

For each $Z \in P(U)$ and $Y \in P(E)$,

- (1) $F^+ : P(E) \rightarrow P(U)$ is a mapping defined as $F^+(Y) = \bigcap_{y \in Y} F(y)$;
- (2) $F^- : P(U) \rightarrow P(E)$ is a mapping defined as $F^-(Z) = \{a \in E : Z \subseteq F(a)\}$;
- (3) $\Psi : P(U) \rightarrow P(U)$ is an operation defined as $\Psi(Z) = F^+ F^-(Z)$.

Then $Z$ is called a soft concept [14] in $(U, E, F)$ if $\Psi(Z) = F^+ F^-(Z) = Z$. The set of all soft concepts is denoted by $sC(U, E, F)$.

In [10], we introduced the notion of $m$-concepts which is independent of the notion of soft concepts to each other as the following: For each $X \in P(U)$,

$$\mathcal{F} : P(U) \rightarrow P(U) \text{ is an operation defined by } \mathcal{F}(X) = \mathcal{F}^\mathcal{F} (X),$$

where two operators $\mathcal{F} : P(A) \rightarrow P(U)$ and $\mathcal{F}^\mathcal{F} : P(U) \rightarrow P(A)$ are defined by:

$$\mathcal{F}(C) = \cup_{c \in C} F(c); \quad \mathcal{F}^\mathcal{F} (X) = \{c \in A : F(c) \subseteq X\}.$$
Then for $X \in P(U)$, $X$ is called an $m$-concept (or object oriented soft concept) in $(U, A, F)$ if $\mathcal{F}(X) = \bigcup_{a \in M_d} F(a) = X$.

The set of all $m$-concepts is denoted by $m(U, A, F)$.

**Theorem 2.1 ([10])** Let $(U, A, F)$ be a soft context. Then we have:

1. $\mathcal{F}(\emptyset) = \emptyset$.
2. $\mathcal{F}(X)$ is an $m$-concept.
3. For $B \subseteq A$, $\mathcal{F}(B)$ is an $m$-concept.
4. For $a \in A$, $\mathcal{F}(a)$ is an $m$-concept.
5. $X$ is an $m$-concept if and only if there is some $B \subseteq A$ such that $X = \bigcup_{a \in M_d} F(a)$.

In [10], we introduced the notion of independent and dependent soft concepts: Let $(U, A, F)$ be a soft context. Then for $Z \in m(U, A, F)$,

1. $Z$ is said to be dependent on $m(U, A, F)$ if there exist $Z_1, \cdots, Z_n \in m(U, A, F)$ satisfying $Z_1 \subseteq Z$ and $Z = \bigcup_{i=1}^{n} Z_i$.
2. $Z$ is said to be independent of $m(U, A, F)$ if $Z$ is not dependent.

We will denote:

$mD = \{Z \in m(U, A, F) \mid X \text{ is dependent on } m(U, A, F)\}$;

$mI = \{Z \in m(U, A, F) \mid X \text{ is independent of } m(U, A, F)\}$.

**Theorem 2.2 ([10])** Let $(U, A, F)$ be a soft context. Then

1. $mD \cap mI = \emptyset$; $mD \cup mI = m(U, A, F)$.
2. For each $X \in mD$, there is a family $B \subseteq mI$ satisfying $X = \bigcup B$.
3. For $Z \in mI$, there is $c \in A$ satisfying $F(c) = Z$.

### 3. Main Results

First, we study the notion of $m$-dependent and $m$-independent attributes in a given soft context. And, we show that the family of all $m$-independent attributes is a base for the set of all $m$-concepts in a given soft context. Finally, we show that a reduction of a soft concept lattice $mL(U, A, F)$ is obtained by the family of all $m$-independent attributes.

**Definition 3.1** Let $(U, A, F)$ be a soft context. Put $M_d = \{g \in A \mid F(a) \supseteq F(g)\}$. Then for $d \in A$, $d$ is said to be $m$-dependent on $A$ if there exists $M_d \neq \emptyset$ satisfying $F(d) = \mathcal{F}(M_d) = \bigcup_{a \in M_d} F(a)$. 
Otherwise, \(d\) is said to be \(m\)-independent on \(A\).

We denote: 
\[ M_D = \{a \in A \mid a \text{ is } m\text{-dependent on } A\}; \]
\[ M_I = \{a \in A \mid a \text{ is } m\text{-independent on } A\}. \]

**Example 3.2** Let \(U = \{1, 2, 3, 4, 5\}\) and \(A = \{a, b, c, d, e, f, g\}\). Consider a soft context \((U, A, F)\) as Table 1.

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Then, the set-valued mapping \(F : A \rightarrow P(U)\) is defined as follows:
\[ F(a) = \{1, 2, 4\}; \quad F(b) = F(f) = \{1, 3, 5\}; \quad F(c) = \{2, 4\}; \quad F(d) = \{1, 3\}; \]
\[ F(e) = \{1, 5\}; \quad F(g) = \{1\}. \]

So,
\[ M_a(A) = \{c, g\}; \quad M_b(A) = M_f(A) = \{d, e, g\}; \quad M_c(A) = \emptyset; \]
\[ M_d(A) = M_e(A) = \{g\}; \quad M_g(A) = \emptyset. \]

For \(a, b, f \in A\),
\[ F(a) = \mathbb{F}(M_a) = F(c) \cup F(g); \]
\[ F(b) = F(f) = \mathbb{F}(M_b) = \mathbb{F}(M_f) = F(d) \cup F(e) \cup F(f). \]

So, \(a, b\) and \(f\) are \(m\)-dependent. But since \(F(d) \neq \mathbb{F}(M_d) = F(g)\) and \(F(e) \neq \mathbb{F}(M_e) = F(g)\), \(d\) and \(e\) are not \(m\)-dependent.

Then, we have:
\[ M_D = \{a, b, f\}; \quad M_I = \{c, d, e, g\}. \]

**Theorem 3.3** Let \((U, A, F)\) be a soft context. Then
(1) \(M_D \cap M_I = \emptyset; \quad M_D \cup M_I = A\).
(2) \(a\) is \(m\)-independent if and only if either \(M_a = \emptyset\) or if \(M_a \neq \emptyset\), then \(\mathbb{F}(M_a) = \bigcup_{g \in M_a} F(g) \neq F(a)\).
(3) For \( a \in A \), \( a \in M_D \) if and only if \( F(a) \in mD \).

(4) For \( a \in A \), \( a \in M_I \) if and only if \( F(a) \in mI \).

Proof.

(1) and (2) Obvious.

(3) Let \( a \in M_D \). Then \( M_a(A) = \{ g \in A \mid F(a) \supseteq F(g) \} \neq \emptyset \) and \( \mathbb{F}(M_a) = \bigcup_{g \in M_a} F(g) = F(a) \). Hence, by definition of dependency of soft concepts, \( F(a) \in mD \).

For the converse, let \( F(a) \in mD \) for \( a \in A \). Then, by (5) of Theorem 2.1, there exists \( B \in P(A) \) such that \( \mathbb{F}(B) = F(a) \). It implies that \( B \subseteq M_a = \{ g \in A : F(a) \supseteq F(g) \} \). And from \( \mathbb{F}(B) \subseteq \mathbb{F}(M_a) \), it follows \( F(a) \supseteq \mathbb{F}(M_a) \supseteq \mathbb{F}(B) = F(a) \). Consequently, there is nonempty set \( M_a \) satisfying \( \mathbb{F}(M_a) = F(a) \). So, \( a \in M_D \).

(4) For \( a \in M_I \), suppose \( F(a) \notin mI \). Then from \( mD \cap mI = \emptyset \) and \( mD \cup mI = m(U, A, F) \), \( F(a) \in mD \). Then by (1), \( a \in M_D \) and \( a \notin M_I \), which is a contradiction. Hence, \( F(a) \in mI \).

In the same way, the converse is obviously showed.

\[ \blacksquare \]

Theorem 3.4 Let \((U, A, F)\) be a soft context. If \( \varphi : M_I \to mI \) is a mapping as defined by \( \varphi(a) = F(a) \) for \( a \in M_I \), then \( \varphi \) is surjective.

Proof. Let \( a \in M_I \). Then \( F(a) \in mI \) and \( \varphi(a) = F(a) \in mI \). Thus, the mapping \( \varphi \) is well-defined. For the surjection, let \( X \in mI \). Then by (3) of Theorem 2.2, there exists an element \( a \in A \) such that \( F(a) = X \). From (4) of Theorem 3.3, \( a \in M_I \) and \( X = F(a) \). Thus, \( \varphi \) is surjective.

\[ \blacksquare \]

Definition 3.5 Let \((U, A, F)\) be a soft context. For \( a \in A \), we say that an element \( a \) is generated by finitely many elements if \( F(a) = \bigcup_{b \in B} F(b) \) for \( B = \{ b_1, b_2, \ldots, b_n \} \subseteq A \), and \( b \in B \) is called generator for \( a \).

Lemma 3.6 Let \((U, A, F)\) be a soft context. For \( d \in A_D \), \( M_d = \{ g \in A \mid F(d) \supseteq F(g) \} \) is a set of generators for \( d \).

Proof. Obvious.

\[ \blacksquare \]

Example 3.7 In Example 3.2, for \( b \in A \), \( b \) is generated by \( \{ d, e \} \) and \( M_b(A) = \{ d, e, g \} \), respectively. \( d, e, \) and \( g \) are generators of \( b \).
Theorem 3.8 ([10]) Let \((U, A, F)\) be a soft context. Then for each \(X \in mD\), there is a family \(B \subseteq mI\) such that \(X = \cup B\).

Theorem 3.9 Let \((U, A, F)\) be a soft context. For each \(d \in M_D\), there exists \(B \subseteq M_I\) such that \(F(B) = \cup_{b \in B} F(b) = F(d)\).

Proof. Let \(d \in M_D\). Then \(F(d) \in mD\) and since \(F(d)\) is a dependent soft concept, there exist \(Z_1, \ldots, Z_n \in m(U, A, F)\) such that \(F(d) \supseteq Z_i\) and \(F(d) = \cup Z_i, i = 1, \ldots, n\). And, since \(mI\) is a base for \(m(U, A, F)\), for each \(Z_i\), there exists \(T_i \subseteq mI\) such that \(\cup T_i = Z_i\) for \(i = 1, \ldots, n\).

And, for each \(T_i, T_j \subseteq mI\) \((j = 1, \ldots, l)\), by (3) of Theorem 2.2, there is an \(m_{ij} \in A\) such that \(F(m_{ij}) = T_i\). Then for each \(F(m_{ij}) = T_i\), from \(F(m_{ij}) = T_i \subseteq mI\) and (4) of Theorem 3.4, \(m_{ij} \in M_I\). Put 
\[
B_i = \{m_{ij} \in M_I \mid F(m_{ij}) = T_i \text{ for } T_i \subseteq mI\}
\]
\((i = 1, \ldots, n)\).

Then for \(i = 1, \ldots, n\), \(B = \cup B_i \subseteq M_I\) and \(F(B) = \cup_{b \in B} F(b) = \cup (\cup_{m_{ij} \in B} F(m_{ij})) = \cup (\cup T_i) = \cup Z_i = F(d)\). So, the proof is completed.

Let \((U, A, F)\) be a soft context. Then a family \(S\) of subsets of \(m(U, A, F)\) is called a base for \((U, A, F)\) if it satisfies the following two conditions:

1. \(S \subseteq m(U, A, F)\).
2. For each \(X \in m(U, A, F)\), there exists \(S' \subseteq S\) such that \(X = \cup S'\).

In [10], we obtained the properties of base for \((U, A, F)\) as the following:

Theorem 3.10 ([10]) Let \((U, A, F)\) be a soft context. Then:

1. The family \(F_A = \{F(a) \mid a \in A\}\) is a base:
2. \(mI\) is the smallest base for \(m(U, A, F)\):
3. For \(B \subseteq A\), if a set-valued mapping \(\varphi : B \to mI\) defined by \(\varphi(b) = F(b)\) for \(b \in B\) is surjective, then \(\varphi(B) = \{F(b) \mid b \in B\}\) is a base for \(m(U, A, F)\).

Theorem 3.11 Let \((U, A, F)\) be a soft context. Then \(M = \{F(a) \mid a \in M_I\}\) is a base for \(m(U, A, F)\).

Proof. From Theorem 3.4, a set-valued mapping \(\varphi : M_I \to mI\) defined by \(\varphi(a) = F(a)\) for \(a \in M_I\) is surjective, and by (3) of Theorem 3.10, \(\varphi(M_I) = \{F(a) \mid a \in M_I\} = M\) is a base for \(m(U, A, F)\).

Corollary 3.12 Let \((U, A, F)\) be a soft context. Then \(\cup_{a \in M_I} F(a) = U\).
Proof. It follows from Theorem 3.11. ■

Finally, using Theorem 3.11, we have the following theorem:

**Theorem 3.13** Let \((U, A, F)\) be a soft context and \(F_{M_I} = \{ F(a) \mid a \in M_I \} \). Then
\[
m(U, A, F) = \{ \cup S \mid S \subseteq F_{M_I} \}.
\]

**Example 3.14** For \(U = \{1, 2, 3, 4, 5\} \) and \(A = \{a, b, c, d, e, f, g\} \), let us consider a soft context \((U, A, F)\) as in Example 3.2. In the example, we showed that:
\[
M_D = \{a, b, f\}; \quad M_I = \{c, d, e, g\}.
\]

For \(F(c) = \{2, 4\}, \quad F(d) = \{1, 3\}, \quad F(e) = \{1, 5\}, \) and \(F(g) = \{1\}\),
\[
F_{M_I} = \{\{1\}, \{1, 3\}, \{1, 5\}, \{2, 4\}\}.
\]

So,
\[
m(U, A, F) = \{ \cup S \mid S \subseteq F_{M_I} \}
= \{\emptyset, \{1\}, \{1, 3\}, \{1, 5\}, \{2, 4\}, \{1, 2, 4\}, \{1, 3, 5\}, \{1, 2, 3, 4\}, \{1, 2, 4, 5\}, U\}.
\]

Now, we recall the notion of order on \(m(U, A, F)\) defined in [10] as the following: For \(X, Y \in m(U, A, F)\),
\[
X \preceq Y \text{ if and only if } X \subseteq Y.
\]

\(X\) is called a **sub-m-concept** of \(Y\), and \(Y\) is called a **super-m-concept** of \(X\).

For the ordered set \((m(U, A, F), \preceq)\), the infimum \(\wedge\) and supremum \(\vee\) are defined by:
\[
X \wedge Y = \exists (X \cap Y); \quad X \vee Y = X \cup Y.
\]

Then \((m(U, A, F), \preceq, \wedge, \vee)\) is complete lattice.

The complete lattice \((m(U, A, F), \preceq, \wedge, \vee)\) is called **m-concept lattice** (or **object oriented soft concept lattice**) and simply will be denoted by \(mL(U, A, F)\).

Let \(mL(U, B, F)\) and \(mL(U, C, G)\) be two \(m\)-concept lattices. \(mL(U, B, F)\) is said to be finer than \(mL(U, C, G)\), which is denoted by
\[
mlL(U, B, F) \preceq mL(U, C, G) \iff mL(U, C, G) \subseteq mL(U, B, F)
\]
If \( mL(U, B, F) \leq mL(U, C, G) \) and \( mL(U, C, G) \leq mL(U, B, F) \), then two \( m \)-concept lattices are said to be isomorphic to each other, and denoted by \( mL(U, B, F) \cong mL(U, C, G) \).

**Theorem 3.15 ([10])** Let \((U, A, F)\) be a soft context and \(C \subseteq A\). Then \( mL(U, A, F) \cong mL(U, C, F_C) \) if and only if \( \text{Im}(F) = \text{Im}(F_C) \).

**Theorem 3.16** Let \((U, A, F)\) be a soft context. Then \( mL(U, A, F) \cong mL(U, M_I, F_{M_I}) \).

**Proof.** From Theorem 3.11, \( \text{Im}(F) = \text{Im}(F_{M_I}) \). So, \( mL(U, A, F) \cong mL(U, M_I, F_{M_I}) \).

Finally, by using the family of all \( m \)-independent attributes, we show a reduction process of a soft context concept lattice \( mL(U, A, F) \):

**Remark.** Let us consider a soft context \((U, A, F)\) as shown in Table 2, where \( U = \{1, 2, 3, 4, 5\}, A = \{a, b, c, d, e, f, g\} \).

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Then \((F, A)\) is a soft set as follows:
\( F(a) = \{1, 2\}; \ F(b) = \{1, 3\}; \ F(c) = \{2, 5\}; \ F(d) = F(f) = \{1, 2, 3\}; \ F(e) = \{1, 2, 5\}; \ F(g) = \{1, 2, 3, 4\}. \)

And,
\( M_D = \{d, e, f\}; \ M_I = \{a, b, c, g\}. \)
\( mL(U, A, F) = \{\emptyset, \{1, 2\}, \{1, 3\}, \{2, 5\}, \{1, 2, 3\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{1, 2, 3, 5\}, U\}. \)
Hence, $mL(U, A, F)$ is obtained as shown in the below diagram:

$$A = \{a, b, c, d, e, f, g\}$$

$$U$$

$\{1, 2, 3, 5\} \quad \{1, 2, 3, 4\}$

$\uparrow \quad \leftarrow \uparrow$

$\{1, 2, 5\} \quad \{1, 2, 3\}$

$\uparrow \quad \leftarrow \uparrow \uparrow$

$\{2, 5\} \quad \{1, 2\} \quad \{1, 3\}$

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$\emptyset$

$mL(U, A, F)$

Finally, for $M_I = \{a, b, c, g\}$, by Theorem 3.16, we have $mL(U, A, F) \cong mL(U, M_I, F_{M_I})$ as the following diagram.

$$A = \{a, b, c, d, e, f, g\} \supseteq M_I = \{a, b, c, g\}$$

$$U$$

$\{1, 2, 3, 5\} \quad \{1, 2, 3, 4\}$

$\uparrow \quad \leftarrow \uparrow$

$\{1, 2, 5\} \quad \{1, 2, 3\}$

$\uparrow \quad \leftarrow \uparrow \uparrow$

$\{2, 5\} \quad \{1, 2\} \quad \{1, 3\}$

$\leftarrow \uparrow \uparrow \uparrow$

$\emptyset$

$mL(U, A, F) \cong mL(U, M_I, F_{M_I})$
4. Conclusion

In particular, we showed that every $m$-dependent attribute is generated by some $m$-independent attributes and the family of all the $m$-independent attributes determines all $m$-concepts of a given $m$-context. Also, we showed that a reduction of a soft concept lattice $mL(U, A, F)$ is obtained by the family of all $m$-independent attributes. In the next research, we will study a variety of ways to reduce the soft concept lattices using any family of $m$-independent attributes and investigate how to combine soft concepts and $m$-concepts to efficiently reduce the soft concepts lattices.

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References


