FDD Based Decentralized Recovery of Sparse Signals in Wireless Sensor Network Applications

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Abstract

Wireless sensor networks have revolutionized the ability to sense and understand phenomena in a region of interest. Sensor networks offer a great potential with wide scope and find applications in military surveillance, environmental observation, building or infrastructure monitoring and health care, to name a few. This paper concentrates only on structural health monitoring application in which the damage detection of civil engineering structures can be done by using a frequency decomposition based decentralized recovery of sensor measurements.

Keywords:-compressed sensing, decentralized approach, FFT, SVD, WSN

INTRODUCTION

Wireless Sensor Networks (WSN)[1, 2] is an upcoming technology which has a wide range of applications [3] including infrastructure protection, industrial sensing and diagnostics, environment monitoring, context-aware computing (for example intelligent home and responsive environment) and so on. This kind of network usually consists of a large number of nodes that communicate together to form a wireless network. It is however essential to improve the energy efficiency for WSNs as the energy designated for sensor nodes is usually extremely limited.

Recently most of the applications in signal processing need to find out a sparse solution to a linear system of equations[4, 5]. For finding out the sparse solution the process involved is to minimize the objective function such as

\[ \arg\min_x \frac{1}{2} ||y - Ax||_2^2 + \lambda ||x||_1 \] (1)

Where \( x \in \mathbb{R}^D \), \( y \in \mathbb{R}^M \), \( A \) is an \( M \times D \) measurement matrix with \( M \ll D \). The formulation (1) corresponds to the maximum a posteriori [6, 7] estimate of \( x \) given the
observations of the form $y = Ax + n$ when the prior on $x$ is Laplacian and $n$ represents white Gaussian noise of variance $\sigma^2$.

The proposed method is concentrates on structural health monitoring [8, 9] which is the process of implementing a damage detection and characterization strategy for engineering structures. Here damage is defined as changes to the material and/or geometric properties of a structural system, including changes to the boundary conditions and system connectivity, which adversely affect the system’s performance. The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of system health.

This paper proposes a decentralized approach [10, 11] for collecting the noisy observations from a large set of sensor nodes distributed in a large area. Because of the difficulty to see directly from the time domain data representation with what frequencies the bridge vibrates it is better to convert the characteristics of a signal into a frequency domain representation. Therefore, Fast Fourier transform, a mathematical operation that decomposes a signal into its constituent frequencies is used here. Here FFTacts as data compression technique and the singular value decomposition [12, 13] is applied to those values for reducing the data redundancy.

Here the wireless sensor network is considered as a non-bipartite connectivity graph which encodes the information flow between the sensors, and assume that each sensor measures only a few projections of the unknown vector $x$. The rest of the paper is organized as Section 2 describes the system setup and formulation, and Section 3 presents the decentralized method, FFT and SVD. Section 4 analyzes the performance of the proposed method using simulations, and Section 5 provides conclusions.

2. SYSTEM SETUP AND FORMULATION

The sensor network whose topology is described using an undirected graph $G(S, L)$ where $S$ represents the set of $S$ sensor nodes and $L$ represents the set of all edges. An edge $(i, j) \in L$ exists if and only if information flow is possible between the nodes (direct neighbors). The measurement model across the sensors is given by,

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_S \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_S \end{bmatrix} \begin{bmatrix} x \\ w_1 \\ \vdots \\ w_S \end{bmatrix} = Ax + w$$

(2)

where $y_s \in \mathbb{R}^{M_s}$ denotes the measurement made by sensor $s$ using the $M_s \times D$ measurement matrix $A_s$, $x \in \mathbb{R}^D$ is the signal of interest to be recovered, $w_s \in \mathbb{R}^{M_s}$ is the additive Gaussian noise with zero mean and covariance $\sigma^2 I$. $y \in \mathbb{R}^{M}$ is the overall measurement vector.
Let $n_s$ represents the set of neighbors of sensor $S$ then the network is deployed to estimate the $D \times 1$ vector $x$. Assume that observations $y_s$ collected at different sensors are conditionally independent and that the conditional probability density function (pdf) $P(y|Ax)$ is known at each sensor. The goal is to estimate the vector $x$ by stacking the observation samples $\{y_1, y_2, ..., y_s\}$ at all sensor nodes. The estimated vector $Ax$ can be defined as

$$A\hat{x} = \arg \max_x p(Ax|y) = \arg \max_x p(y|Ax)p(Ax)$$

(3)

Recalling the conditional independence of the observations at different sensors, the conditional probability in (3) can be rewritten as

$$p(y|Ax) = \prod_{s=1}^{S} p(y_s|Ax)$$

(4)

The estimation of $Ax$ is obtained through the maximization of objective function given in (3)

3. PROPOSED METHOD

In this section a detailed description of estimating a sparse signal from the sensory measurements is considered. A decentralized method is used for collecting the sensor measurements and after receiving the measurement data matrix they are converted into frequency domain using fast fourier transform and the singular value decomposition is performed for avoiding the data redundancy. Here FFT acts as a data compression technique and SVD eliminates data redundancy and hence reduce the transmitted and received data volume and hence increase the speed of operations.

3.1. DECENTRALIZED METHOD

In a large scale sensor network the estimation of sparse signal is possible only through the collaboration and processing of the sensor measurements. Most of the energy consumption in WSN is used for exchanging collected data between nodes and among nodes and substations. This energy consumption can be reduced through a decentralized method in which nodes need to communicate only with their local neighbors. So the amount of data to be processed by each sensor is reduced and the sensors no longer need to transmit data to the fusion center as in the centralized approaches. Hence the communication cost is reduced and this requires very low bandwidth.

Fig 1:centralized processing and distributed processing
3. 2FAST FOURIER TRANSFORM
In bridge monitoring, accelerations, strains, deflections, temperature and applied forces, etc., are the important signals interesting to measure for analysis. These measurements are recorded for a given period which shows the time flow of a parameter under the study. It is difficult to see directly from the time domain data representation with what frequencies the bridge vibrates. The characteristics of a signal can be better understood in a frequency domain representation rather than the intuitive time domain representation. Therefore, it’s better to use a Fourier transform, a mathematical operation that decomposes a signal into its constituent frequencies.

Fast Fourier Transform (FFT) is an efficient mechanism for transforming time domain data into frequency domain. This fast implementation of discrete fourier transform reduces the number of computations needed for N point DFT from \( O(N^2) \) to \( O(N\log_2 N) \). The only requirement of the most popular implementation of this algorithm is that the number of points in the series has to be a power of 2. The FFT considerably reduces the computational requirements of the DFT.

3. 3 SINGULAR VALUE DECOMPOSITION
In linear algebra, the singular value decomposition (SVD) is a factorization of a real or complex matrix, with many useful applications in signal processing and statistics. Formally, the singular value decomposition of an \( m \times n \) real or complex matrix \( M \) is a factorization of the form
\[
M = U \Sigma V^* \]
where \( U \) is a \( m \times m \) real or complex unitary matrix, \( \Sigma \) is an \( m \times n \) rectangular diagonal matrix with nonnegative real numbers on the diagonal, and \( V^* \) (the conjugate transpose of \( V \), or simply the transpose of \( V \) if \( V \) is real) is an \( n \times n \) real or complex unitary matrix. The diagonal entries \( \Sigma_{i, i} \) of \( \Sigma \) are known as the singular values of \( M \). The \( m \) columns of \( U \) and the \( n \) columns of \( V \) are called the left-singular vectors and right-singular vectors of \( M \), respectively.

The SVD is quite similar to the eigen value decomposition technique. However, the SVD is very general in the sense that it can be applied to any \( m \times n \) matrix. Additionally, the SVD has got several advantages compared to other decomposition methods as listed below
i. more robust to numerical error;
ii. exposes the geometric structure of a matrix an important aspect of many matrix calculations; and
iii. quantify the resulting change between the underlying geometry of those vector spaces.

Assume that compression converts two sensor measurements of size \( B \) bits each to an output stream of size \( B + b \) where \( b < B \). The SVD operator, as discussed above converts \( s \) sensor measurements of size \( B \) bits each into \( s \) eigenvectors of size \( b \) bits each with \( b < B \). Thus the redundant values in neighboring nodes can be avoided by performing SVD on sensor measurements.
4. PERFORMANCE ANALYSIS
To simulate the proposed algorithm and to demonstrate its performance the MATLAB software is used. In this simulation, 8 nodes are considered as distributed in a civil structure which resonates with a natural frequency. A decentralized approach is used to collecting the sensor measurements and for analyzing those datas convert those into frequency domain representation using FFT. After that SVD of those datas are done for detecting the sensor node which measures the variation from the natural frequency signal.

![Fig 2: Time domain representation of signals from different nodes](image1)

![Fig 3: Frequency domain representation of signals from sensor nodes](image2)

5. CONCLUSION
A FDD based algorithm for recovering the natural frequency in a civil structure is developed and validate the algorithm using numerical experiments. Here a decentralized method is used for collecting the sensor measurements and singular value decomposition on those values in frequency domain is performed for removing the data redundancy. This work can be extended for finding out the damage of the civil structures.
6. REFERENCES


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