Non-Split Dominating Set of an Interval Graph using an Algorithm.

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Abstract

Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, ecology, biology, computer sciences and particularly useful in cyclic scheduling and computers storage allocation problems etc. In this paper we define the notions new algorithms for non-split domination in graphs. We get many bounds and non-split domination number.

Keywords: Interval family, Interval graph, Connected graph, Dominating Set, Non-split dominating set.

Introduction

Let I = \{I_1, I_2, ..., I_n\} be given interval family. Each interval i in I is represented by \([a_i, b_i]\), for \(i = 1, 2, ..., n\). Here \(a_i\) is called the left endpoint and \(b_i\) the right endpoint of the interval \(I_i\). Without loss of generality we may assume that all end points of the intervals in I which are distinct between 1 and 2n. The intervals are labelled in the increasing order of their right endpoints. Two intervals \(i\) and \(j\) are said to intersect each other, if they have non-empty intersection. A graph \(G(V,E)\) is called an interval graph if there is one-to-one correspondence between \(V\) and \(I\) such that two vertices of \(G\) are joined by an edge in \(E\) if and only if their corresponding intervals in \(I\) intersect. That is, if \(i=[a_i,b_i]\) and \(j=[a_j,b_j]\), then \(i\) and \(j\) intersect means either \(a_i < b_j\) or \(a_j < b_i\). Kulli V.R et all [1] introduced the concept of split and non split domination in graphs and Also in Maheswari, B et all [2]. A non empty set \(D \subseteq V\) of a graph \(G\) is a dominating set[3] of \(G\) if every vertex in \(< V-D >\) is adjacent to some vertex in \(D\). The domination number \(\gamma(G)\) is the minimum cardinality taken over all the minimal dominating sets of \(G\). A graph \(G\) is said to be connected if there is a path between any two vertices of \(G\). A dominating
set D of G is a connected dominating set [4] if the induced sub graph <V-D> is connected. A dominating set D ⊆ V of a graph G is a non-split dominating set (NSDS) if the induced sub graph < V-D > is connected. The non-split domination number is the minimum cardinality of a non-split dominating set. It is denoted by \( \Upsilon_{ns} (G) \). The neighbourhood of a vertex v ∈ V is set consisting all vertices adjacent to v (including v). It is denoted by nbd[v]. Let nbd[i] be defined as the set of vertices adjacent to i including i. Let max(i) denotes the largest interval in nbd[i]. Gurupaksh. C. D., Mallikarjuna Swamy B. P [5], Minimum Matching Dominating Sets and its Applications in Wireless Networks define Next(i)=j if and only if \( b_{i} < a_{j} \) and there does not exist an interval k such that \( b_{i} < a_{k} < a_{j} \). If there is no such j, we define Next(i)=null.

Main Theorems

**Theorem 1:** Let I={I₁, I₂, …., Iₙ} be an interval family I corresponding to an interval graph G. If i and j are any two intervals in I such that i ∈ D, where D is a dominating set of the given interval graph G, j ≠ 1 and j is contained in i and if there is at least one interval to the left of j that intersects j and at least one interval k ≠ i to the right of j that intersects j then non-split occurs in G.

**Proof:** Let I={I₁, I₂, …., Iₙ} be an interval family I corresponding to an interval graph G. Suppose there is at least one interval k ≠ i to the right of j that intersects j. Then it is obvious that j is adjacent to k in the induced sub graph < V-D >, so that there will not be disconnection in < V-D >. Since there is at least one interval to the left of j that intersects j, there will not be any disconnection in < V-D > to its left. Thus we get non-split domination in G. In this regard we will prove the non-split domination set of an interval graph using an algorithm.

First we will define an Algorithm to find minimum dominating set of an interval graph.

**Algorithm MDS**

Input: Interval family Let I= {1, 2, …., n}
Output: Minimum dominating set of an interval graph G.
Step 1: Let S= {max(1)}
Step 2: LI= The largest interval in S.
Step 3: Compute Next(LI)
Step 4: If Next (LI)=null the goto step 8
Step 5: Find max (Next(LI))
Step 6: If max (Next(LI)) does not exist then
\[ \text{Max(Next(LI))}=\text{Next}(LI) \]
Step 7: S=S∪max(Next(LI)) goto step 2
Step 8: End
Algorithm for Finding Non-Split Dominating Set

Input: Interval family Let \( I = \{1, 2, \ldots, n\} \)
Output: Non split dominating set and induced sub graph of an interval graph \( G \) is connected.

Step 1: Set \( V = \{U[i]\} \) and \( U[i] = i + 1 \) where \( i = 0 \) to \( n-1 \).
Step 2: Find MDS \( D = \{D[i]\} \) for some \( i \) where \( 0 < i < n-1 \)
Step 3: \( Y = |D| \)
Step 4: \( V - D = \{S[i]\} \) where \( i = 0 \) to \( n - Y - 1 \)
Step 5: for \( i = 0 \) to \( n - Y - 1 \)
    
    \[
    D \cup = \quad
    \]
    
    \[
    V - D = \{S[i]\} \text{ where } i=0 \text{ to } n-1
    \]
    
    \[
    \text{If max } S[i]=n \text{ then }
    \]
    
    \[
    p = \text{max } S[i] - Y - 1
    \]
    
    \[
    \text{else }
    \]
    
    \[
    p = \text{max } S[i]-1
    \]
    
    \[
    \text{for } j = i + 1 \text{ to } p
    \]
    
    \[
    \text{If } (S[i], S[j]) \in E \text{ in } G
    \]
    
    \[
    \text{Plot a line from } S[i] \text{ to } S[j]
    \]
    
The induced sub graph \( G_1 \) obtained.

Step 6: If \( w(G_1) = 1 \) then go to step 7
    else go to 14
Step 7: \( S_i = \{\text{max } S[0]\} \) in \( G_1 \).
Step 8: \( L_1 = \text{the largest interval in } S_1 \)
Step 9: Compute Next \((L_1)\) in \( G_1 \)
Step 10: If Next \((L_1) = \text{null} \) the go to step 14
Step 11: Find max \((\text{Next}(L_1))\) in \( G_1 \)
Step 12: If max \((\text{Next}(L_1))\) does not exist then
    
    \[
    \text{Max } (\text{Next}(L_1)) = \text{Next}(L_1) \text{ in } G_1
    \]
    
    \[
    \text{Step 13: } S_1 = S_1 \cup \text{max } (\text{Next}(L_1)) \text{ go to step 8 }
    \]
Step 14: End

**Figure 1:** Interval family

We construct an interval graph from an interval family \( I = \{1, 2, \ldots, 11\} \) as follows
\( \text{nb}d[1] = \{1, 2, 3\} \quad \text{Max}(1) = 3 \quad \text{Next}(1) = 4 \)
nbd[2]=\{1, 2, 3, 4\}  \quad \text{Max}(2)=4  \quad \text{Next}(2)=5
nbd[3]=\{1,2,3,4,5\}  \quad \text{Max}(3)=5  \quad \text{Next}(3)=6
nbd[4]=\{2,3,4,5,7\}  \quad \text{Max}(4)=7  \quad \text{Next}(4)=6
nbd[5]=\{3,4,,5,6,7,9\}  \quad \text{Max}(5)=9  \quad \text{Next}(5)=8
nbd[6]=\{5,6,7,8,9\}  \quad \text{Max}(6)=9  \quad \text{Next}(6)=10
nbd[7]=\{4,5,6,7,8,9\}  \quad \text{Max}(7)=9  \quad \text{Next}(7)=10
nbd[8]=\{6,7,8,9,10,11\} \quad \text{Max}(8)=11 \quad \text{Next}(8)=\text{null}
nbd[9]=\{5,6,7,8,9,10,11\} \quad \text{Max}(9)=11 \quad \text{Next}(9)=\text{null}
nbd[10]=\{8,9,10,11\} \quad \text{Max}(10)=11 \quad \text{Next}(10)=\text{null}
nbd[11]=\{8,9,10,11\} \quad \text{Max}(11)=11 \quad \text{Next}(11)=\text{null}

**Procedure For MDS**

Input: Interval family given in fig.1

Step 1: \(S=3\)

Step 2: \(LI=3\)

Step 3: \(\text{Next}(3)=6\)

Step 4: \(\text{max}(6)=9\)

Step 5: \(S=3 \cup 9=\{3,9\}\) go to step 2

Step 6: \(LI=9\)

Step 7: \(\text{Next}(9)=\text{null}\)

Step 8: End

Output: \(\{3,9\}\) is the minimum dominating set of an interval family in fig.1.

**Procedure For NSDS**

Step 1: \(V=\{1,2,\ldots,11\}\)

Step 2: \(D=\{3,9\}\)

Step 3: \(\Upsilon=2\)

Step 4: \(S[i]=\{1,2,4,5,6,7,8,10,11\}\)

Step 5: for \(i=0\) then \(\text{max } S[0]=3\Rightarrow p=3-1=2 \Rightarrow j=1\) to 2

\((S[0],S[1])=(1,2)\in E\text{ in } G, \text{ Plot a line from 1 to 2}
\quad (S[0],S[1])=(1,2) \notin E\)

for \(i=1\), \(\text{max } S[1]=4 \Rightarrow p=4-1=3 \Rightarrow j=2\) to 3

\((S[1],S[2])=(2,4)\in E\text{ in } G, \text{ Plot a line from 2 to 4}
\quad (S[1],S[2])=(2,5) \notin E\text{ in } G\)

for \(i=2\), \(\text{max } S[2]=7 \Rightarrow p=7-1=6 \Rightarrow j=3\) to 6

\((S[2],S[3])=(4,5)\in E\text{ in } G, \text{ Plot a line from 4 to 5}
\quad (S[2],S[3])=(4,6) \notin E\text{ in } G,
\quad (S[2],S[4])=(4,7) \in E\text{ in } G, \text{ Plot a line from 4 to 7}
\quad (S[2],S[5])=(4,8) \notin E\text{ in } G\)

for \(i=3\), \(\text{max } S[3]=9 \Rightarrow p=9-1=8 \Rightarrow j=4\) to 8

\((S[3],S[4])=(5,6)\in E\text{ in } G, \text{ Plot a line from 5 to 6}
\quad (S[3],S[4])=(5,7) \notin E\text{ in } G, \text{ Plot a line from 5 to 7}
\quad (S[3],S[6])=(5,8) \notin E\text{ in } G, (S[3],S[7])=(5,10) \notin E\text{ in } G\)
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\[(S[3], S[8]) = (5, 11) \notin E \text{ in } G\]

\[\text{for } i = 4, \text{ max } S[4] = 9 \Rightarrow p = 9 - 1 = 8 \Rightarrow j = 5 \text{ to } 8\]

\[(S[4], S[5]) = (6, 7) \in E \text{ in } G, \text{ plot a line from 6 to 7}\]

\[(S[4], S[6]) = (6, 8) \in E \text{ in } G, \text{ plot a line from 6 to 8}\]

\[(S[4], S[7]) = (6, 10) \notin E \text{ in } G, \ (S[4], S[8]) = (5, 11) \notin E \text{ in } G\]

\[\text{for } i = 5, \text{ max } S[5] = 9 \Rightarrow p = 9 - 1 = 8 \Rightarrow j = 6 \text{ to } 8\]

\[(S[5], S[6]) = (7, 8) \in E \text{ in } G, \text{ plot a line from 7 to 8}\]

\[(S[5], S[7]) = (7, 10) \notin E \text{ in } G, \ (S[5], S[8]) = (7, 11) \notin E \text{ in } G\]

\[\text{for } i = 6, \text{ max } S[6] = 11 \Rightarrow p = 11 - 2 - 1 = 8 \Rightarrow j = 7 \text{ to } 8\]

\[(S[6], S[7]) = (8, 10) \in E \text{ in } G, \text{ plot a line from 8 to 10}\]

\[(S[6], S[8]) = (8, 11) \notin E \text{ in } G, \text{ plot a line from 8 to 11}\]

\[\text{for } i = 7, \text{ max } S[7] = 11 \Rightarrow p = 11 - 2 - 1 = 8 \Rightarrow j = 8 \text{ to } 8 \text{ i.e., } j = 8\]

\[(S[7], S[8]) = (10, 11) \in E \text{ in } G, \text{ plot a line from 10 to 11}\]

Step 6: \[w(G_1) = 1\]

Therefore the induced sub graph \(G_1\) is connected.

Step 7: \(S_1 = \{\text{max } S[0]\} = 2\)

Step 8: \(L1_1 = 2\)

Step 9: \(\text{Next}(2) = 5\)

Step 10: \(\text{Max}(5) = 7\)

Step 11: \(S_1 = 2 \cup 7 = \{2, 7\} \text{ go to step 8}\)

Step 12: \(L1_1 = 7\)

Step 13: \(\text{Next}(7) = 10\)

Step 14: \(\text{Max}(10) = 11\)

Step 15: \(S_1 = \{2, 7\} \cup 11 = \{2, 7, 11\} \text{ go to step 8}\)

Step 16: \(L1_1 = 11\)

Step 17: \(\text{Next}(11) = \text{null}\)

Step 18: \(\text{End}\)

Out put: \(\{2, 7, 11\}\) is the non-split dominating set of the interval family as in fig. 1.

In this regard there is no any isolated vertex in \(G\).

**Theorem 2:** If \(i\) and \(j\) are two intervals in \(I\) such that \(i \in D\), where \(D\) is a minimum dominating set of \(G\), \(j = 1\) and \(j\) is contained in \(i\) and if there is one more interval other than \(i\) that intersects \(j\) then non-split domination occurs in \(G\).

**Proof:** Let \(I = \{I_1, I_2, \ldots, I_n\}\) be an interval family. Let \(j = 1\) be the interval contained in \(i\) where \(i \in D\), where \(D\) is a minimum dominating set of \(G\). Suppose \(k\) is an interval, \(k \neq i\) and \(k\) intersect \(j\). Since \(i \in D\), the induced sub graph \(< V - D >\) does not contain \(i\). Further in \(< V - D >\), the vertex \(j\) is adjacent to the vertex \(k\) and hence there will not be any disconnection in \(< V - D >\). Therefore we get non-split domination in \(G\). In this connection as follows an algorithm.
Figure 2: Interval Family

Procedure For NSDS
Input: Interval family Let I = \{1,2,…..,10\} similarly the above method as follows nbd[i], max(i)and Next(i).

Step1: V=\{1,2,…..,10\}
Step2: D=\{3,7,10\}
Step3: Y = 3
Step4: S[ ]=\{1,2,4,5,6,8,9\}
Step5: for i=0 then max S[0]= 3  \Rightarrow  p= 3-1= 2  \Rightarrow  j = 1 to 2
   (S[0],S[1])=(1,2)\in E in G, plot a line from 1 to 2
   (S[0],S[2])=(1,4)\in E in G
for i=1, max S[1]= 4  \Rightarrow  p= 4-1= 3  \Rightarrow  j = 2 to 3
   (S[1],S[2])=(2,4)\in E in G, plot a line from 2 to 4
   (S[1],S[3])=(2,5)\in E in G
for i=2, max S[2] = 7  \Rightarrow  p= 7-1= 6  \Rightarrow  j = 3 to 6
   (S[2],S[3])=(4,5) \notin E in G
   (S[2],S[4])=(4,6) \in E in G, plot a line from 4 to 6
   (S[2],S[5])=(4,8) \notin E in G, (S[2],S[6])=(4,9) \notin E in G
for i=3, max S[3] = 7  \Rightarrow  p= 7-1= 6  \Rightarrow  j = 4 to 6
   (S[3],S[4])=(5,6)\in E in G, plot a line from 5 to 6
   (S[3],S[5])=(5,8) \notin E in G,(S[3],S[6])=(5,9)\notin E in G
for i=4, max S[4] = 7  \Rightarrow  p= 7-1= 6  \Rightarrow  j = 5 to 6
   (S[4],S[5])=(6,8)\in E in G, plot a line from 6 to 8
   (S[4],S[6])=(6,9) \notin E in G
For i=5, max S[5] =10  \Rightarrow  p= 10-3= 7  \Rightarrow  j = 6 to 6 i.e.,6
   (S[5],S[6])=(8,9)\in E in G, plot a line from 8 to 9
Step6: w(G_i)=1
Therefore the induced sub graph G_i is connected.
Step7: S_i = \{max S[0]\}= 2 in G_i
Step8: LI_i=2
Step9: Next (2)=5
Step10: Max (5)=6
Step11: S_i = 2\cup 6 = \{2,6\} go to step 8
Step12: LI_i=6
Step13: Next(6)=9
Step14: Max(9)=9
Step 15: $S_1 = \{2, 6\} \cup\nolimits_9 = \{2, 6, 9\}$ go to step 8

Step 16: $L_{I_1} = 9$

Step 17: $\text{Next}(9) = \text{null}$

Step 18: End

Output: $\{2, 6, 9\}$ is the non-split dominating set of the interval family as in fig. 3.

**Theorem 3:** If $i, j, k$ are 3 consecutive intervals such that $i < j < k$ and $j \in D$, where $D$ is a dominating set of $G$, $i$ intersects $j$, $j$ intersects $k$ and $i$ intersects $k$ then non-split domination occurs in $G$.

**Proof:** Suppose $I = \{I_1, I_2, \ldots, I_n\}$ be an interval family. Let $i, j, k$ be three consecutive intervals satisfying the hypothesis. Now $i$ and $k$ intersect implies that $i$ and $k$ are adjacent in induced sub graph $\langle V \setminus D \rangle$. So that there will not be any disconnection in $\langle V \setminus D \rangle$ an algorithm as follows.

We construct an interval graph from an interval family $I = \{1, 2, \ldots, n\}$ which are

**Procedure For NSDS**

Input: Interval family Let $I = \{1, 2, \ldots, 10\}$ as the above method follows $\text{nbd}[i]$, $\text{max}(i)$ and $\text{Next}(i)$.

Step 1: $V = \{1, 2, \ldots, 10\}$

Step 2: $D = \{3, 8\}$

Step 3: $Y = 2$

Step 4: $S[\_] = \{1, 2, 4, 5, 6, 7, 9, 10\}$

Step 5: for $i = 0$ then max $S[0] = 3 \Rightarrow p = 3 - 1 = 2 \Rightarrow j = 1$ to 2

$\quad (S[0], S[1]) = (1, 2) \in E$ in $G$, plot a line from 1 to 2

$\quad (S[0], S[2]) = (1, 4) \notin E$ in $G$

for $i = 1$, max $S[1] = 4 \Rightarrow p = 4 - 1 = 3 \Rightarrow j = 2$ to 3

$\quad (S[1], S[2]) = (2, 4) \in E$ in $G$, plot a line from 2 to 4

$\quad (S[1], S[3]) = (2, 5) \in E$ in $G$

for $i = 2$, max $S[2] = 5 \Rightarrow p = 5 - 1 = 4 \Rightarrow j = 3$ to 4

$\quad (S[2], S[3]) = (4, 5) \in E$ in $G$, plot a line from 4 to 6
(S[2],S[4])=(4,6) $\notin E$ in G, (S[2],S[6])=(4,9) $\notin E$ in G
for $i=3$, max $S[3]=6 \Rightarrow p=6-1=5 \Rightarrow j=4$ to $5$
(S[3],S[4])=(5,6)$\in E$ in G, plot a line from 5 to 6
(S[3],S[5])=(5,7)$\in E$ in G, plot a line from 5 to 7
for $i=4$, max $S[4]=8 \Rightarrow p=8-1=7 \Rightarrow j=5$ to $7$
(S[4],S[5])=(6,7)$\in E$ in G, plot a line from 6 to 7
(S[4],S[6])=(6,9) $\notin E$ in G, (S[4],S[7])=(6,10) $\notin E$ in G
for $i=5$, max $S[5]=9 \Rightarrow p=9-1=8 \Rightarrow j=6$ to $8$
(S[5],S[6])=(7,9)$\in E$ in G, plot a line from 7 to 9
for $i=6$, max $S[6]=10 \Rightarrow p=10-2=8 \Rightarrow j=7$ to $7$ i.e., $j=7$
(S[5],S[6])=(9,10)$\in E$ in G, plot a line from 9 to 10
Step6: w(G1)=1

Therefore the induced sub graph $G_1$ is connected
Step7: $S_1 = \{\text{max } S[0]\}=2$
Step8: LI1=2
Step9: Next(2)=5
Step10: Max(5)=7
Step11: $S_1 = 2 \cup 7 = \{2,7\}$ go to step 8
Step12: LI1=7
Step13: Next(7)=10 in S
Step14: Max(10)=10 in S
Step15: $S_1 = \{2,7\} \cup 10 = \{2,7,10\}$ go to step 8
Step16: LI1=10 in S
Step17: Next(10)=null in S
Step18: End
Output: $\{2,7,10\}$ is the non-split dominating set of the interval family as in fig.1.

References