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Pebbling Number of the Sequential Join of Complete Graphs

Anil Kumar M. S.

Department of Mathematics, VTMNSS College,
Dhanuvachapuram, University of Kerala, Trivandrum, Kerala, India
E mail : animankulam@yahoo.co.in

Abstract

Given a configuration of pebbles on the vertices of a graph G, a *pebbling* move consists of taking two pebbles off some vertex v and putting one of them back on a vertex adjacent to v. A graph is called p*ebbleable* if for each vertex v there is a sequence of pebbling moves that would place at least one pebble on v. *The pebbling number* of graph G, is the smallest integer m such that G is pebbleable for every *configuration* of m pebbles on G. Let G_1 and G_2 be graphs such that G_1 and G_2 have disjoint vertex sets V_1 and V_2 and edges sets E_1 and E_2 respectively. Their join G_1+G_2 consists of $G_1 U G_2$ and all edges joining V_1 with V_2. For three or more disjoint graphs G_1, G2….., G_n , the *sequential join* $G_1 + G_2 +…..+ G_n$ is the graph $(G_1 + G_2) \cup (G_2 + G_3) \cup….. \cup (G_{n-1} + G_n)$. In this paper we find the pebbling number of the sequential join of complete graphs.

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Introduction

A pebbling configuration of a graph G is a distribution of pebbles on G. A pebbling move consists of removing two pebbles lying on the some vertex v and placing one of these pebbles on some vertex that is adjacent to v. A distribution (configuration) p is *"v-solvable"* (v is reachable under p), if v has a pebble after some (possibly empty) pebbling sequence (sequence of pebbling moves) starting from p. The pebbling number of a graph G is the minimum number *m* of pebbles that ensure that every vertex of G is pebbleable, no matter what initial distribution of *m* pebbles we start with. Let f (G, v) denotes the pebbling number of a vertex v of G, and f (G) the pebbling number of G.

T. Clarke et al [3] defined class 0 and class 1 graphs. They defined a graph G to be class 0 if $f(G) = n(G)$, the number of vertices in G and of class 1 if $f(G) = n(G) + 1$.

Let G_1 and G_2 be graphs such that G_1 and G_2 have disjoint vertex sets V_1 and V_2 and edges sets E_1 and E_2 respectively. Their join $G_1 + G_2$ consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2. For three or more disjoint graphs $G_1, G_2, \ldots, G_n$, the *sequential join* , $G_1 + G_2 + \ldots + G_n$ is the graph $(G_1 + G_2) \cup (G_2 + G_3) \cup \ldots \cup (G_{n-1} + G_n)$.

Theorem 1[1] : If diam (G) $= 2$ and K (G) ≥ 3, then G is of class 0.
The graph H in figure 1 is $K_1+K_2+K_2+K_1$. It is clear that H is not class 0. More generally, the graph $K_1+K_r+K_r+K_1$ is class 0 if $r \geq 3$ and not otherwise.

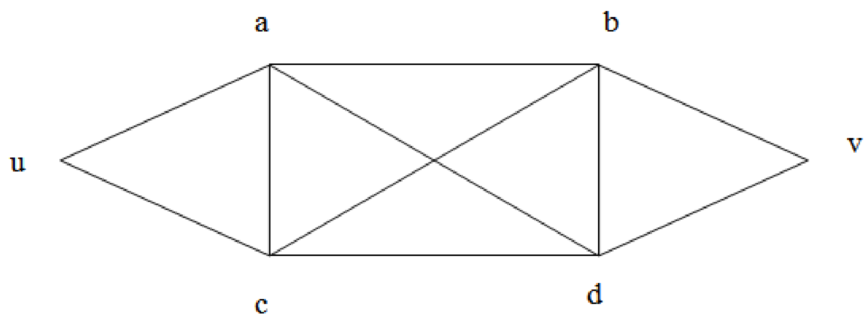

Figure 1: A graph H which is not class 0.

Theorem 2: The graph $K_1 + K_{n-2} + K_1$ is class 0, if $n \geq 4$.

Proof: The proof is obvious.
We say that a vertex v in graph G is *substituted* by H if v is replaced by H and if uv is an edge in G, $\{uw / w \in V(H)\}$ will be edges in the new graph. If G^1 is the graph obtained, then

$$G^1 = \{G - v\} \cup H \cup \{ux : x \in V(H), u \in N(v)\}$$

Lemma 3: Let G be a connected graph. Let $v \in G$ be substituted by K_n, a complete graph on a vertices. Let G^1 be the new graph obtained. Let $V^1 = V(K_n)$ and $w \in v^1$, then $f(G^1, w) = f(G, v) + n - 1$.

Proof: Let f(G, v)=r. Let p be a non v-solvable distribution of pebbles for G with |p| =r-1. Clearly p(v)=0.

Let p^1 be the pebbling distribution of G^1 with $p/_{G} -\{v\} = p1/_{G} -\{v\}$ and one pebble on each vertex of K_n except w. Then $|p^1|=r+(n-2)$ and is, clearly, not w solvable.

Let Q be any distribution of r+n-1 pebbles on the vertices of G^1.

Then, either $Q/|V^1| \geq n$ or $Q/\{V(G) - V^1| \geq f(G, v)$

So, Q is w solvable.

Hence the lemma.

Lemma 4: Let G, V, G^1 be as in lemma 3. Let $uv \in E(G)$. Then

$f(G^1,u) \leq f(G, u) +|V(H)|$.

If there is at least one non u- solvable distribution of f(G, u)-1 pebbles on G, with p(v) = 1, then $f(G^1,u) \geq f(G, u) +|V(H)|-1$.

(In particular, if deg v=1, the equality holds)

Theorem 5: The $K_{n1}+k_{n2}+K_{n3}$ is class of 0, if $\sum_{i=1}^{3} n_i \geq 4$ and $n_2 \geq 2$.

Proof: the graph is 3 –connected and is of diameter 2 and hence a class 0 graph.

Theorem 6: Let G denotes the sequential join of the four complete graphs

K_{n1}, k_{n2}, K_{n3}. K_{n4} Then $f(G) = \sum_{i=1}^{4} n_I$, where $n_2 \geq 2$,

$n_3 \geq 2$, $n_2 + n_3 \geq 5$. That is, G is a diameter 3, class 0, graph.

Proof: Consider the path P_4 with vertex set {u,v,w,x}. We have

$G \cong K_{n1}+ k_{n2}+K_{n3} +K_{n4}$.

G can be obtained by substituting u,v,w and x by K_{n1}, k_{n2}, K_{n3} and K_{n4} respectively. We calculate the pebbling number of G in four steps.

***Step 1*:** Replace one pendant vertex of P_4 say u by K_{n1},. Let G_1 be the new graph obtained. It is easy to see that $f(G_1) = n_1+7$.

***Step 2*:** We now replace x in G_1 by K_{n4}. Let G_2 be the new graph obtained.

Then $f(G_2) = n_1+n_4+6$.

***Step 3*:** In G_2, we substitute K_{n2} in the place of v. Let G_3 be the new graph obtained.

$$\text{We have } f(G_3) = \begin{cases} n+3, & \text{if} \quad n_2 = 2 \\ n+2, & \text{if} \quad n_2 = 3 \\ n+1, & \text{if} \quad n_2 \geq 4, \end{cases}$$

Step 4**:** The vertex w in G_3 is replaced by K_{n3}. Let G_4 be the new graph obtained. We see that G_4 is required graph G. Again it is easy to prove that

$$f(G) = \begin{cases} n & \text{if} \quad n_2+n_3 \geq 6 \\ n+1 & \text{if} \quad n_2+n_3 = 5 \\ n+2 & \text{if} \quad n_2=n_3 = 2 \end{cases}$$

Hence the theorem

We now have the following conjecture.
Conjecture 7: If $G = K_{n1}+ K_{n2}+ \ldots. + K_{nr}$, $r \geq 4$, then
G is class 0 if $n_2, n_3, \ldots.., n_{r-1} \geq \frac{2r}{r-1}$

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