A Methodology of Fatigue Analysis of Pressure Vessels by FEA

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Abstract
Pressure vessels contains fluid under pressure and temperature varying with time, which makes it important to analyse pressure vessel for fatigue loading. The design should ensure the structural integrity of the pressure vessel during several transients. This study gives a methodology to perform fatigue analysis of a pressure vessel by ASME code. ANSYS software is used to perform all the analyses. Transient thermal and pressure analysis is performed and results are used to determine cumulative usage factor. Fatigue curves are used to determine cycles and hence usage factor is calculated. Cumulative usage factor of fatigue are investigated to determine the adequacy of the design by using Miner’s law.

Keywords: Pressure Vessel, Fatigue, ASME BPVC codes, ANSYS, Transient Thermal analysis, Fatigue curves, Cumulative usage factor, Miner’s Law

I. INTRODUCTION
A pressure vessel is a closed container designed to hold gases or liquids at a pressure substantially different from the atmosphere pressure. Pressure vessels have a wide application in thermal and nuclear power plants. The fluid may be at elevated temperatures and in a pressurized state varying with respect to time. The paper of MYUNG JO JHUNG[7], gives details about the procedure of analysis and variable to be considered during different analysis. The paper of MULLA NIYAMAT[6] describes about analytical approach towards design of pressure vessel and validation.
of design. The complete analysis is done in stages. First step is pressure analysis and it is done under design and hydrostatic pressure. Second step is performing a transient thermal analysis. For the transient thermal analysis, the heat transfer coefficients are determined based on the operating environment and the thermal transient data are simplified to prepare a straightforward input deck. The most severe instances are found considering the total stress intensity range and the stress levels at those times are obtained along with the applied pressure. These values are then used in a fatigue analysis to determine the final cumulative usage factor. The usage factors are used to determine the adequacy of the pressure vessel by Miner’s rule.

II. DESIGN OF PRESSURE VESSEL

A. Determination of shape and type of pressure vessel

![Dimensions of a typical pressure vessel](Fig 1. Dimensions of a typical pressure vessel)

B. Determination of shell thickness

Design calculation [3]-(From ASME sec.8 div. 1)

- For thickness of cylindrical shell

  \[ T_c = \frac{PR}{SE + 0.4P} \]

  \( T_c = 309.15 \text{ mm}, \) provided thickness = 320 mm
Elliptical Magnetic Binary Problem when the Primaries are Oblate Spheroids

- For thickness of hemispherical vessel heads
  \[ T_s = \frac{PL}{2SE - 0.2P} \]
  \( T_s = 165 \text{ mm}, \) provided thickness =180mm

- For tapered joints instructions have been followed from [3] ASME BPVC 2015 Section VIII part 1 Fig UW 13.1.

C. Axisymmetric Modeling in ANSYS

Axis symmetric plane 183 element is chosen in analysis.

D. Design Parameters

Properties are taken for SA-508 Grade 3 Class-1. Data is referred from [2]ASME Section- II D Properties (Metric).

<table>
<thead>
<tr>
<th>Table 1. Design parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, ( P ) \quad 17\text{MPa}</td>
</tr>
<tr>
<td>Hydrostatic Pressure \quad 23 \text{MPa}</td>
</tr>
<tr>
<td>Working Pressure \quad 4.136-15.51 \text{MPa}</td>
</tr>
<tr>
<td>Design Temperature ,( T_D ) \quad 360 ^\circ \text{C}</td>
</tr>
<tr>
<td>Working Temperature \quad 70- 450^\circ F</td>
</tr>
<tr>
<td>Allowable Stress, ( S ) \quad 158\text{MPa}</td>
</tr>
<tr>
<td>Young’s Modulus \quad 171\times10^3 \text{MPa}</td>
</tr>
<tr>
<td>Poisson’s Ratio \quad 0.3</td>
</tr>
<tr>
<td>Density \quad 7750 \text{Kg/m}^3</td>
</tr>
<tr>
<td>Internal Diameter \quad 5200 \text{mm}</td>
</tr>
<tr>
<td>External Diameter \quad 5840 \text{mm}</td>
</tr>
</tbody>
</table>

E. Meshing

The meshing of the pressure vessel by [4] ANSYS. Total number of nodes after meshing is 8983.
III. PRESSURE ANALYSIS

Pressure run is performed on the pressure vessel. Two pressure- design and hydrostatic are selected for this run. Path operation is performed on three paths at different position in the vessel. 1-1 is in hemisphere 2-2 is the junction of the hemispherical and cylindrical shell and 3-3 is center of the cylindrical shell.

A. Boundary Conditions

Two boundary conditions are provided for this structural analysis also shown in figure-
1. Symmetric Boundary Condition at axis of symmetry
2. Displacement Boundary Condition at bottom line of symmetry for hemisphere

![Defined boundary conditions](image)

**Fig 4.** Defined boundary conditions

**B. Design pressure structural analysis**

Design pressure is applied on the inner walls of the vessel with the boundary conditions.

![Stress contour in the Z direction](image)

**Fig 5.** Stress contour in the Z direction

Following are linearized stress plots for hoop stress in all three paths 1-1, 2-2, 3-3.
Fig 6. Linearized stress curve for path 1-1 in z dir.

Fig 7. Linearized stress curve for path 2-2 in z dir.

Fig 8. Linearized stress curve for path 3-3 in z dir.
Table 2. Final results for Design pressure structural analysis and Comparison with calculated values

<table>
<thead>
<tr>
<th>Path 3-3(cyli.)</th>
<th>Calculated Values (lame’s theory)</th>
<th>ANSYS linearized values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_i$</td>
<td>$R_m$</td>
</tr>
<tr>
<td>Hoop Stress $\sigma_h$</td>
<td>146.1</td>
<td>136.95</td>
</tr>
<tr>
<td>Radial Stress $\sigma_r$</td>
<td>-17</td>
<td>-7.816</td>
</tr>
<tr>
<td>Axial Stress $\sigma_a$</td>
<td>65</td>
<td>65</td>
</tr>
</tbody>
</table>

All stress units are in MPa

C. Hydrostatic pressure structural analysis

Hydrostatic pressure is applied on the inner walls of the vessel with the boundary conditions.

![Stress contour in the Z direction](image1)

**Fig 9.** Stress contour in the Z direction

Following are linearized stress plots for hoop stress in all three paths 1-1, 2-2, 3-3.

![Linearized stress curve for path1-1 in z dir.](image2)

**Fig 10.** Linearized stress curve for path1-1 in z dir.
Fig 11. Linearized stress curve for path2-2 in z dir.

Fig 12. Linearized stress curve for path3-3 in z dir.

Table 3 Final results for Hydrostatic pressure structural analysis and comparison with calculated values.

<table>
<thead>
<tr>
<th>Path 3-3 (cylinder)</th>
<th>Calculated Values (Lamé’s theory)</th>
<th>ANSYS Linearized value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ri</td>
<td>Rm</td>
</tr>
<tr>
<td>Hoop Stress $\sigma_c$</td>
<td>199.04</td>
<td>186.5</td>
</tr>
<tr>
<td>Radial Stress $\sigma_r$</td>
<td>-23</td>
<td>-10.5</td>
</tr>
<tr>
<td>Axial Stress $\sigma_a$</td>
<td>88.02</td>
<td>88.02</td>
</tr>
</tbody>
</table>

All stress units are in MPa
IV. THERMAL TRANSIENT ANALYSIS

A. Loading Conditions

Following plot shows transient loading from 0 to 20000 seconds for heat up process of the vessel. The pressure variation is shown by light spotted line and temperature is shown by dark line.

![Loading condition for transients](image)

**Fig 13.** Loading condition for transients

B. Heat Transfer Coefficient Calculation

The heat transfer coefficient for natural convection on a vertical surface was calculated by using Heat and Mass Transfer data book. The methodology for calculation of heat transfer coefficient is as follows [9]:

\[ Nu_D = C \times (Ra)^n \]

\[ Gr = \frac{D^3 \times \rho \times g \times \beta \times \Delta T}{\mu^3} \]

\[ Pr = \frac{C_p \times \mu}{k} \]

\[ h = \frac{Nu_D \times k}{D} \]

Constant for use with these equations for isothermal surfaces are \( C = 0.590 \)

\( Nu_D = \) Nusselt Number

\( Gr = \) Grashoff’s Number

\( Pr = \) Prandtl Number
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D = Diameter of the vessel (m)
ρ = Density (Kg/m$^3$)
g = Acceleration due to gravity (m/s$^2$)
$C_p$ = Specific Heat (J/Kg.°C)
μ = Dynamic Viscosity (N/m$^2$-s)

**Table 4** Thermal Properties at loading temperatures

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Density (Kg/m$^3$)</th>
<th>Dynamic Viscosity(N/m$^2$-s)</th>
<th>Specific Heat(J/Kg.°C)</th>
<th>Thermal conductivity (W/m-°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.1</td>
<td>999.86</td>
<td>0.000973</td>
<td>4171.9</td>
<td>0.6022</td>
</tr>
<tr>
<td>51.6</td>
<td>989.06</td>
<td>0.00053</td>
<td>4170.7</td>
<td>0.6473</td>
</tr>
<tr>
<td>232.2</td>
<td>835.89</td>
<td>0.00011</td>
<td>4591</td>
<td>0.6526</td>
</tr>
<tr>
<td>343.3</td>
<td>601.6</td>
<td>6.927</td>
<td>8557</td>
<td>0.4628</td>
</tr>
</tbody>
</table>

Following values of heat transfer coefficient (h) came for different points of time.

**Table 5** Heat transfer coefficients at different loading condition

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Pressure (MPa)</th>
<th>Temperature (°C)</th>
<th>Heat transfer coefficient(h) (W/m$^2$k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.12</td>
<td>21.1</td>
<td>49.54</td>
</tr>
<tr>
<td>2800</td>
<td>4.136</td>
<td>51.6</td>
<td>179.3</td>
</tr>
<tr>
<td>7800</td>
<td>15.5</td>
<td>232.2</td>
<td>2030.11</td>
</tr>
<tr>
<td>10200</td>
<td>15.51</td>
<td>343.3</td>
<td>2311.45</td>
</tr>
</tbody>
</table>

C. Thermal Transient Analysis

**Table 6** Parameters for thermal transient analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature ,T</td>
<td>21.1° C - 343.3°C</td>
</tr>
<tr>
<td>Allowable Stress, S</td>
<td>158MPa</td>
</tr>
<tr>
<td>Young’s Modulus, E</td>
<td>171×10$^3$ MPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Density, ρ</td>
<td>7750 Kg/m$^3$</td>
</tr>
<tr>
<td>Thermal Conductivity, α</td>
<td>37 W/m-k</td>
</tr>
<tr>
<td>Specific Heat, $C_p$</td>
<td>5.84×10$^5$ J/kg.°C</td>
</tr>
</tbody>
</table>
Transient thermal analysis is performed from 0 to 20000 seconds and temperature is changed according to time as load steps.

**D. Thermal+Pressure Transient Analysis**

The results of thermal transient analysis is taken as input for combined pressure and thermal analysis.

**Fig 14** Nodal temperature solution

**Fig 15** Nodal stress solution for Thermal + Pressure analysis
From solutions of this combined run it is obsevered that node 1100 is the severe node with peak value of stress.

On this severe node time history plots are obtained for different stresses $\sigma_1$, $\sigma_2$, $\sigma_3$, $(\sigma_2 - \sigma_1)$, $(\sigma_3 - \sigma_2)$, and $(\sigma_3 - \sigma_1)$.

**Fig 16** Time history solution for stress For $(\sigma_2 - \sigma_1)$

**Fig 17** Time history solution for stress For $(\sigma_3 - \sigma_2)$
V. FATIGUE ANALYSIS

Graphical Representation of Principal Stresses W.R.T. Time at Severe Node 1100 for \((\sigma_3 - \sigma_1)\) shows maximum stress range and hence chosen to find max alternating stress at service loading.

A. Calculation for Range and Alternating Stress

- For Hydrostatic Loading
  \[ \text{Range} = \sigma_{\text{max}} - \sigma_{\text{min}} = 179.817 - 0 = 179.817 \]
  \[ \sigma_{\text{alt}} = \text{range}/2 = 179.817/2 = 89.9 \]

- For Combined loading
  \[ \text{Range} = \sigma_{\text{max}} - \sigma_{\text{min}} = 322 - (-50) = 372 \]
  \[ \sigma_{\text{alt}} = \text{range}/2 = 372/2 = 186 \]

All stress values in MPa

B. S-N CURVE FOR MATERIAL

S-N curve for SA 508 Grade 3 Class 1 is taken from [1] ASME BPVC 2015 Section III A 2015 Figure I-9.5M.

C. Calculation for Modified Alternating Stress

- Stress concentration factor = 1.2 (ASME Section VIII Div-2 Table 5.11)
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- Modified $\sigma_{alt} = k_s \cdot \sigma_{alt} = 1.2 \cdot 186 = 223.2$
- Effect of elastic modulus [ASME section 3, nb3222.4,e,4]
  $= (195/171) \cdot 223.2 = 254.5$
- Similarly for hydrostatic conditions $\sigma_{alt} = 123.03$

All stress values are in MPa

D. Determination of number of cycles
- For Hydrostatic conditions, $\sigma_{alt} = 123.03$ MPa
  From SN curve $N_1 = 1584893$ cycles
- For Combined service Loading, $\sigma_{alt} = 254.5$ MPa
  From SN curve $N_2 = 39180$ cycles
- Applied Loading Cycles for hydrostatic condition = $n_1 = 15$ cycles
- Applied Loading Cycles for Combined Service Loading = $n_2 = 300$ cycles

E. Calculation for usage factor
- $U_2$ (For Hydrostatic conditions)
  $= n_2/N_2 = 15/1584893 = 0.000009$
- $U_1$ (For Combined service Loading)
  $= n_1/N_1 = 300/39180 = 0.075$

VI. VERIFICATION FROM MINER’S LAW

Miner’s law is given by

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} \leq 1$$

By substituting values given above

$0.075 + 0.000009 = 0.075009$

$0.075009 \leq 1$

Hence, the design is safe.
VII. CONCLUSION

- This paper provides the methodology to perform fatigue analysis on a typical pressure vessel to ASME.
- Likewise, other transient cycles, if any, can be included in the same way.
- The stress concentration factor is one of the most important factors affecting the fatigue usage factor.

REFERENCES


[3] ASME Section 8 division 1 for rules of construction of boiler and pressure vessels.


