Design and Development of Three Stages Mixed Sampling Plans for Variable – Attribute – Variable Quality Characteristics

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Abstract

The mixed sampling plans are two stages sampling plans in which variable and attribute quality characteristics are used in deciding the acceptance or rejection of the lot. In industries variables play a vital role in making the decision about the process or batches because of its efficiency. Many quality control practitioners insist that in a mixed sampling environment if a batch is not accepted based on first and second stage sample results, then the decision should be based on the results of variable criteria in the third stage. Hence three stages mixed sampling plans are being developed. This paper presents a new algorithm to make a unique decision on the lot for mixed environment with Vari-Attri-Vari combinations for independent plans. The Operating characteristics function and other related measures of the mixed sampling plans are derived by the authors. A new designing methodology for determining the parameters of the sampling plans are given. Comparisons are made between two and three stages of mixed sampling plans. Tables are constructed to facilitate easy application of 3D sampling plans in the Quality Control Section of production industries.

Keywords: Mixed Sampling, Three stages, Variable criteria, Attribute criteria, Designing V-A-V.
1. INTRODUCTION

The mixed sampling plans are product control techniques in which variable and attribute quality characteristics are combined in deciding the acceptance or rejection of the lot. Due to modern quality control systems, mixed sampling plans are widely applied in various stages of production. In industries, variable criteria play a vital role in inspection area. If attribute criteria are used for making a decision, then it is not efficient to control quality of the products when the parameters are measurable. Hence to offset the disadvantage a new three stage mixed sampling plans are developed and termed as Vari-Attri-Vari (VAV) mixed sampling plans. This paper presents a new algorithm to make a unique decision on the lot for mixed environment with Vari-Attri-Vari combinations. The Operating characteristics function and other related measures of the mixed sampling plans are derived. A new designing methodology for determining the parameters of the sampling plans are given. Comparisons are made between two and three stages of mixed sampling plans. Tables are constructed to facilitate easy application of three stages sampling plans in the Quality Control Section of production industries.

Dodge (1932) introduced the concept of mixed sampling plans with variable and attribute quality characteristics. Bowker and Goode (1952) have extended the work of mixed sampling plans. Gregory and Resnikoff (1955) have given a procedure for mixed plans when the standard deviation is known. Savage (1955) has studied the mixed plans, for non-normal distribution. Kao (1966) has used the concept of item variability instead of item central tendency to sentence a lot. In the year 1967, Schilling presented a method for determining the operating characteristics of mixed variables-attributes sampling plans. DevaArul (1996 & 2004) has developed mixed sampling plans by inculcating the blend of process & product control measures to suit specific industrial needs. Suresh and Deva Arul (2003) have developed Multi-Dimensional Mixed Sampling Plans. DevaArul. S (2009) has contributed towards mixed sampling system with tightened inspection in the second stage. Suresh and Devaarul (2002) have designed Mixed Sampling Plans with Chain Sampling as attribute plan. Suresh and DevaArul (2002) have developed mixed sampling plans by combining process and product quality characteristics to reduce the sampling cost. DevaArul and Jemmy Joyce (2010) have developed Mixed Sampling Plans for Second Quality Lots. Asokan and Balamurali (2000) have developed Multi Attribute Single Sampling Plans indexed through AQL and LTPD.
2. FORMULATION OF V-A-V MIXED SAMPLING PLANS

Let

\[ N : \text{Lot size} \]
\[ n_1 : \text{First Stage Sample size} \]
\[ n_2 : \text{Second Stage Sample size} \]
\[ n_3 : \text{Third Stage Sample size} \]
\[ k = \text{Variable factor such that a lot is accepted if } \bar{x} \leq U - k\sigma \]
\[ c = \text{The attributes acceptance number which leads to third stage} \]
\[ \beta = \text{Probability of acceptance} \]
\[ \beta_1 = \text{Probability of acceptance for } P_i \]
\[ \beta'_1 = \text{Probability of acceptance assigned to } (') \text{ stage for percent defective } P_i \]
\[ \sigma = \text{Population standard deviation} \]
\[ U_n = \text{Extreme deviate from the sample mean in studentized form} \]
\[ f_n^{(u)} = \text{Cumulative distribution of the extreme deviate from the sample mean in studentized form from a sample of size } 'n' \]
\[ p = \text{Fraction defective} \]
\[ P(i, x) = \text{Joint probability of } i \text{ and } X \]
\[ P_a(p) = \text{Probability of acceptance} \]
\[ \bar{x} = \text{Sample mean} \]
\[ Z_A = \text{z value for acceptance limit for } \bar{x} \]
\[ Z_{(i)} = \text{z value for the } i^{th} \text{ ordered observation} \]
\[ \mu = \text{Population mean (process)} \]
\[ Z_u = \text{z value of upper specification limit} \]
\[ Z_{(W)} = \text{Standard normal deviate such that } \frac{1}{\sqrt{2\pi}} \int_{e}^{\infty} \frac{-1}{2}\int_{Z_{(W)}}^{w} dt = w \]
\[ Z = \text{Mean of } z \text{ values} \]
\[ z_i = \text{Average of } z \text{ value from } i^{th} \text{ sample} \]
3. Algorithm for three stages (v-a-v) mixed sampling plans

Step 1: Draw a random sample of size $n_1$.

Step 2: Determine $\bar{x}_1$.

Step 3: If $\bar{x}_1 \leq U - k\sigma$, accept the lot.

Step 4: If $\bar{x}_1 \geq U - k\sigma$, go to next step.

Step 5: Draw another sample of size $n_2$.

Step 6: Count the number of defectives $d$.

Step 7: If $d \leq c$, now go to next step else if $d > c$, reject the lot.

Step 8: Draw another sample size $n_3$ and determine $\bar{x}_2$.

Step 9: If $\bar{x}_2 \leq U - k\sigma$, accept the lot otherwise reject the lot.

4. Operating Characteristics and Associated Measures of Mixed Plans

The four principal curves, which describe the performance of an acceptance sampling plan for various fractions defective, are the Operating Characteristic curves, the ASN curves, the AOQ and the ATI curves. The operation of the mixed plans cannot be properly assessed until formulae for the ordinates of each of these measures are defined for the known values of percent defective.

The Operating Characteristics function of VAV mixed sampling plan is defined as

$$P_a(p) = P_{n_1}[\bar{x}_1 \leq A] + \sum_{x=0}^{c} \frac{e^{-n_2p}(n_2p)^x}{x!} P_{n_1}[\bar{x}_1 > A] P_{n_3}[\bar{x}_2 \leq A]$$

Proof:

The lot will be accepted in the following cases

Case (i)

From the sample of size $n_1$ if $\bar{x}_1 \leq A$, accept the lot

Case (ii)

From the sample of size $n_2$ if $d \leq c$ and also from the sample of size $n_3$ if $\bar{x}_2 \leq A$, the lot will be accepted

Case (i) & (ii) are mutually exclusive case. By the law of addition on probability we get
Design and Development of Three Stages Mixed Sampling Plans for Variable...

\[ P_a(p) = p(i) + p(ii) \]
\[ = P[\bar{x}_1 \leq A] + P_{n_1}[\bar{x}_1 > A]P[d \leq c]P[\bar{x}_2 \leq A] \]

In case of type II probability,

\[ P_a(p) = P_{n_1}[\bar{x}_1 \leq U - k_1\sigma] + P_{n_1}[\bar{x}_1 > A] \sum_{x=0}^{c} \frac{e^{-n_2p(n_2p)x}}{x!} P_{n_3}[\bar{x}_2 \leq u - k_2\sigma] \]

In case of Binomial distribution,

\[ P_a(p) = P_{n_1}[\bar{x}_1 \leq U - k_1\sigma] + P_{n_1}[\bar{x}_1 > A] \sum_{x=0}^{c} \binom{n}{x} p^x q^{n-x} P_{n_3}[\bar{x}_2 \leq u - k_2\sigma] \]

In case of lot size is known then

\[ P_a(p) = P_{n_1}[\bar{x}_1 \leq U - k_1\sigma] + P_{n_1}[\bar{x}_1 > A] \sum_{x=0}^{c} \binom{n}{x} \frac{(Np)}{n} \binom{N-Np}{n-np} \{P_{n_3}[\bar{x}_2 \leq u - k_2\sigma]\} \]

5. Designing Through AQL

Step1: Let the three stages be independent. Let the first stage probability acceptance be \( \beta_1' \). Let the second stage probability acceptance be \( \beta_1'' \). Let the third stage probability acceptance be \( \beta_1''' \).

Step2: Determine the sample size \( n_1 \) for the known \( \beta_1' \)

Step3: Calculate the acceptance limit \( k_1 \) for the existing process average AQL = \( p_1 \)

\[ k_1 = z(p_1) + \frac{z(\beta_1')}{\sqrt{n_1}} \]

where \( z(w) \) is the standard normal variate corresponding to \( w \) such that

\[ \frac{1}{\sqrt{2\pi}} \int_{z(w)}^{\infty} e^{-\frac{1}{2}z^2} dz \]

Step4: Now determine \( \beta_1'' \) the second stage probability of acceptance such that

\[ \beta_1'' = \frac{\beta_1 - \beta_1'}{1 - \beta_1} \]
Step 5: Determine the second sample size $n_2$ and acceptance number “c” such that

$$
\sum_{x=0}^{c} \frac{e^{-n_2 p} (n_2 p)^x}{x!} \approx \beta''
$$

Step 6: Now determine the third stage probability of acceptance $\beta'''$ such that

$$
\beta''' = \frac{\beta - \beta' - (\beta' - \beta'')}{1 - \beta' - (\beta' - \beta'')}
$$

Step 7: Determine the third stage parameters $n_3$ and $k_2$ such that

$$
k_2 = z(p_1) + \frac{z(\beta''')}{\sqrt{n_3}}
$$

For practical reasons we set $n_1 = n_2 = n_3$ the values are known. Hence $k_2$ can be easily determined.

Table 1: Values of the parameters $k_1$, $k_2$ and $c$ for the known $n_1$, $n_2$ and $n_3$ for the known AQL and by assuming first stage probability of acceptance is 65% having total probability of acceptance with 95%

<table>
<thead>
<tr>
<th>AQL</th>
<th>n1=n2=n3=50</th>
<th>n1=n2=n3=100</th>
<th>n1=n2=n3=150</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p1</td>
<td>k1</td>
<td>k2</td>
</tr>
<tr>
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<td>3.14472</td>
<td>3.28047</td>
<td>0</td>
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<td>2.93265</td>
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<td>2.80227</td>
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<td>2.70656</td>
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</tr>
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<td>0.005</td>
<td>2.63032</td>
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</tr>
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<td>0.006</td>
<td>2.56664</td>
<td>2.70238</td>
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<td>2.51176</td>
<td>2.64750</td>
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</tr>
<tr>
<td>0.008</td>
<td>2.46341</td>
<td>2.59915</td>
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<tr>
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<td>2.42011</td>
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<td>0.01</td>
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<td>2.51658</td>
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</table>
Illustration 1:
A production process turns out 0.1% process fraction defective. Determine three stages mixed sampling plan with 95% probability of acceptance.

Solution:
It is given that AQL = 0.1%. From table 1, one can find the required parameters for 95% probability acceptance. For practical reasons if n_1=n_2=n_3=100 then from table 1 we get, \( k_1=3.12876, \ k_2=3.22475, \ C = 0 \).

Operating procedure:
Step 1: Draw a random sample of size \( n_1=100 \).
Step 2: Determine \( \bar{x}_1 \).
Step 3: If \( \bar{x}_1 \leq U - 3.12876\sigma \) , accept the lot.
Step 4: If \( \bar{x}_1 > U - 3.12876\sigma \) , go to next step.
Step 5: Draw another sample of size \( n_2=100 \).
Step 6: Count the number of defectives d. Suppose if \( d > 0 \) , reject the lot.
Step 7: If \( d = 0 \), now go to third stage (i.e) next step.
Step 8: Draw another sample size \( n_3=100 \) and determine \( \bar{x}_2 \).
Step 9: If \( \bar{x}_2 \leq U - 3.22475\sigma \) , accept the lot otherwise reject the lot.

7. Designing through LQL
Step 1: Let the first stage probability acceptance be \( \beta_2' \)
Let the second stage probability acceptance be \( \beta_2'' \)
Let the third stage probability acceptance be \( \beta_2''' \)
Step 2: Determine the sample size \( n_1 \) for the known \( \beta_2' \)
Step 3: Calculate the acceptance limit \( k_1 \) for the existing process average LQL= \( p_2 \)

\[
k_1 = z(p_2) + \frac{z(\beta_2')}{\sqrt{n_1}}
\]
where \( z(w) \) is the standard normal variate corresponding to \( w \) such that

\[
\frac{1}{\sqrt{2\pi}} \int_{z(w)}^{\infty} e^{-\frac{1}{2}z^2} \, dz
\]
Step 4: Now determine \( \beta_1'' \) the second stage probability of acceptance such that

\[
\beta_1'' = \frac{\beta_1 - \beta_1'}{1 - \beta_1'}
\]
Step 5: Determine the second sample size $n_2$ and acceptance number “c” such that

$$\sum_{x=0}^{c} \frac{e^{-n_2p} (n_2p)^x}{x!} \approx \beta_1''$$

Step 6: Now determine the third stage probability of acceptance $\beta_1'''$ such that

$$\beta_1''' = \frac{\beta_1 - \beta_1' - (\beta_1' - \beta_1'')}{1 - \beta_1' - (\beta_1' - \beta_1'')}$$

Step 7: Determine the third stage parameters $n_3$ and $k_2$ such that

$$k_2 = z(p_3) + \frac{z(\beta_1''')}{\sqrt{n_3}}$$

Since $n_1 = n_2 = n_3$ the values are known. $k_2$ can be easily determined.

**Table 2:** Values of the parameters $k_1$, $k_2$ and $c$ for the known $n_1$, $n_2$ and $n_3$ for the known AQL and by assuming first stage probability of acceptance is 65% having total probability of acceptance with 95%

<table>
<thead>
<tr>
<th>LQL</th>
<th>n1=n2=n3= 50</th>
<th>n1=n2=n3= 100</th>
<th>n1=n2=n3= 150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_2$</td>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$c$</td>
</tr>
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<td>0</td>
</tr>
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<td>0</td>
</tr>
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<tr>
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<td>1.63032</td>
<td>0</td>
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<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>1.51417</td>
<td>1.50680</td>
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</tr>
</tbody>
</table>

**Illustration 2**

A production process turns out 3% process average fraction defective. Determine three stages mixed sampling plans for the known LQL at 10% Probability of acceptance.

Solution:

It is given that LQL = 3%.
Let the probability of acceptance at LQL is 3%. From table 2, one can find the required parameters for 10% probability acceptance. For practical reasons if \( n_1 = n_2 = n_3 = 200 \), then from table 2, \( k_1 = 1.99710, k_2 = 1.99342, c = 2 \).

8. Average Sample Number (ASN)

The expected number of sample size required to make a decision on the lot is defined as

\[
E(n) = n_1 + n_2 \left[ P_{n_1}( \bar{x} > A) \right] + (n_1 + n_2) P_{n_1} (\bar{x} > A) P(d \leq c)
\]

9. Average Outgoing Quality (AOQ)

The expected outgoing quality after the inspection is defined as

\[
AOQ = p P_a(p)
\]

CONCLUSION:

In this research article, a new three stages mixed sampling plans are developed with final decision based on variable criteria which will lead to exact and efficient decision. The operating procedure is given and the related measures such as OC, ASN, AOQ are derived for the first time. The new algorithm is easy to operate in quality control section especially when the decision should be taken based on the basis of variable quality characteristics. Utilizing the new designing procedure tables are constructed to facilitate quality control engineers for easy selection of sampling plans. It is found that the 3-stage mixed sampling plan is more sensitive towards deterioration of the quality. Whenever lot quality is not maintained then there is a decrease in probability of the acceptance of the lot. Hence this sampling plan coerces the producer to maintain the standard quality so that the customer is satisfied. The comparison between 2stages and 3stages mixed sampling plan shows that the average sample number is more economical with respect to sample size in case of newly developed mixed sampling plans.

REFERENCES:


