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Deteriorating Items Production Inventory Model with Different Deterioration Rates Under Stock and Price Dependent Demand

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Abstract

A production inventory model with stock and price dependent demand is developed. Different deterioration rates are considered in a cycle. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis is also carried out for parameters.

Key Words: Production, Inventory model, Varying Deterioration, Price dependent demand, Stock dependent demand, Time varying holding cost

1. INTRODUCTION:

The basic question for making a production inventory model is how much to produce to satisfy customer demand. So, the problem is to find the optimal time to maximize the total relevant profit.

Deterioration is another important factor for controlling the inventory in a production system. Mishra [7] developed the first production lot size model in which both constant and variable rate of deterioration were considered. Choi and Hwang [2] developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Panda et al. [8] developed a single item economic production quantity model with ramp type quadratic demand. Manna and Chiang [6] developed an economic production quantity model for deteriorating item with ramp type demand. Other research work related to deteriorating items can be found in, for instance (Raafat [11], Goyal and Giri [3], Ruxian et al. [13]).

In classical inventory models the demand rate is assumed to be a constant. But in reality demand of physical goods may be price and/ or stock dependent. Gupta and Vrat [4] presented an inventory model for stock dependent consumption rate. Burewell [1] developed economic lot size model for price dependent demand under quantity and freight discounts. A model for a deteriorating item with a stock dependent demand rate was developed by Sana and Chaudhury [15] in which the production rate of the item in stock was partly constant and partly dependent on instantaneous stock and demand. You [17] developed an inventory model when demand for products is price and time dependent. Teng and Chang [16] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Roy and Chaudhury [12] developed two production inventory models (Model I and Model II) for deteriorating items when the demand rate depends on the instantaneous inventory level. Sahoo et al. [14] developed an inventory model for constant deteriorating items with price dependent demand and time varying holding cost. Patra et al. [10] considered a deterministic inventory model with price dependent quadratic demand rate. Kontantaras and Skouri [5] modified the model developed by Roy and Chaudhury [12] using the classical method of Lagrange's multiplier and thereby they converted the constrained problem as unconstrained problem. Patel and Patel [9] developed a deteriorating items production inventory model with demand dependent production rate.

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed a production inventory model with different deterioration rates. Demand of the product is price and stock dependent for the cycle time under time varying holding cost. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

2. ASSUMPTIONS AND NOTATIONS:

NOTATIONS:

The following notations are used for the development of the model:

P(t) : Production rate is a function of price and demand ($P(t) = \eta D(t), \eta>0$)

D(t) : Demand rate is a linear function of price and inventory level ($a + bI(t) - \rho p$, $a>0$, $0<b<1$, $\rho>0$)

Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

B : Setup cost per order

SeC : Setup cost

c : Purchasing cost per unit

p : Selling price per unit

T : Length of inventory cycle

I(t) : Inventory level at any instant of time t, $0 \le t \le T$

Q : Order quantity

θ : Deterioration rate during $\mu_1 \le t \le t_1$, $0< \theta<1$

θt : Deterioration rate during , $t_1 \le t \le T$, $0< \theta<1$

π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of model.

- The demand of the product is declining as a function of price and inventory level.

- Rate of production is a function of demand
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- Deteriorated units neither be repaired nor replaced during the cycle time.

3. THE MATHEMATICAL MODEL AND ANALYSIS:

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

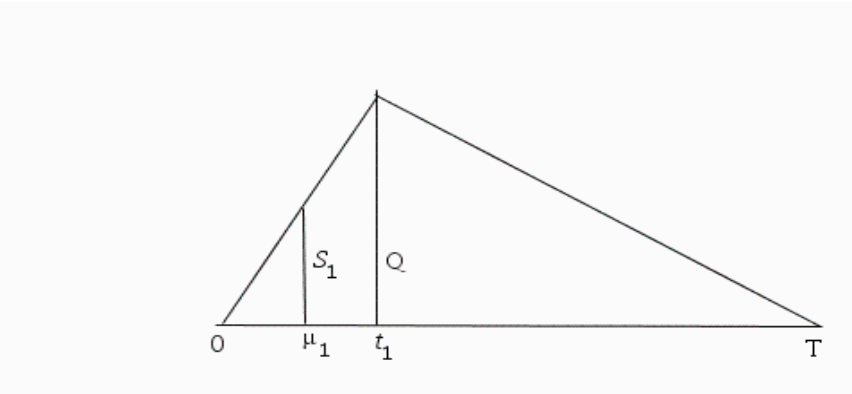


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = (\eta - 1)(a + bI(t) - \rho p), \qquad 0 \le t \le \mu_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = (\eta - 1)(a + bI(t) - \rho p), \qquad \mu_1 \le t \le t_1 \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bI(t) - \rho p), \qquad t_1 \le t \le T \qquad (3)$$

with initial conditions $I(0) = 0$, $I(\mu_1) = S_1$, $I(t_1) = Q$ and $I(T) = 0$.

Solutions of these equations are given by

$$I(t) = (\eta - 1)(a - \rho p)\left(t + \frac{1}{2}(\eta-1)bt^2 \right). \qquad (4)$$

$$I(t) = S_1\left[1 + (\theta+b)(\mu_1 - t) - \eta b(\mu_1 - t)\right]$$
$$- (\eta-1)(a - \rho p)\begin{bmatrix} (\mu_1 - t) + \frac{1}{2}(\theta+b)(\mu_1^2 - t^2) - \frac{1}{2}\eta b(\mu_1^2 - t^2) \\ - (\theta+b)t(\mu_1 - t) + \eta bt(\mu_1 - t) \end{bmatrix} \tag{5}$$

$$I(t) = (a - \rho p)\begin{bmatrix} (T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}\theta(T^3 - t^3) - bt(T - t) \\ - \frac{1}{6}b\theta t(T^3 - t^3) - \frac{1}{2}\theta t^2(T - t) - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{bmatrix}. \tag{6}$$

(by neglecting higher powers of θ)

Putting $t = \mu_1$ in equation (4), we get

$$S_1 = (\eta - 1)(a - \rho p)\left(\mu_1 + \frac{1}{2}(\eta-1)b\mu_1^2\right). \tag{7}$$

Putting $t = t_1$ in equations (5) and (6), we have

$$I(t_1) = S_1\left[1 + (\theta+b)(\mu_1 - t_1) - \eta b(\mu_1 - t_1)\right]$$
$$- (\eta-1)(a - \rho p)\begin{bmatrix} (\mu_1 - t_1) + \frac{1}{2}(\theta+b)(\mu_1^2 - t_1^2) - \frac{1}{2}\eta b(\mu_1^2 - t_1^2) \\ - (\theta+b)t_1(\mu_1 - t_1) + \eta bt_1(\mu_1 - t_1) \end{bmatrix} \tag{8}$$

$$I(t_1) = (a - \rho p)\begin{bmatrix} (T - t_1) + \frac{1}{2}b(T^2 - t_1^2) + \frac{1}{6}\theta(T^3 - t_1^3) - bt_1(T - t_1) \\ - \frac{1}{6}b\theta t_1(T^3 - t_1^3) - \frac{1}{2}\theta t_1^2(T - t_1) - \frac{1}{4}b\theta t_1^2(T^2 - t_1^2) \end{bmatrix}. \tag{9}$$

So, from equations (8) and (9), we have

$$t_1 = \frac{S_1\left(1+(\theta+b)\mu_1-\eta b\mu_1\right)-(a-\rho p)\left((\eta-1)\mu_1+T\right)}{S_1(\theta+b-\eta b)-(a-\rho p)(1+bT)-(\eta-1)(a-\rho p)\left(\mu_1+(\theta+b)\mu_1-\eta b\mu_1\right)} \tag{10}$$

From equation (10), we see that t_1 is a function of μ_1, T and S_1, so t_1 is not a decision variable.

Putting value of S_1 from equation (7) in equation (5), we have

$$I(t) = (\eta - 1)(a - \rho p)\begin{pmatrix} \mu_1 - \frac{1}{2}(\eta-1)\mu_1^2 + (\theta+b)\mu_1(\mu_1 - t) - \eta b(\mu_1 - t) \\ - \frac{1}{2}(\eta-1)\mu_1^2(\theta+b)(\mu_1 - t) \end{pmatrix}$$
$$- (\eta-1)(a - \rho p)\begin{bmatrix} (\mu_1 - t) + \frac{1}{2}(\theta+b)(\mu_1^2 - t^2) - \frac{1}{2}\eta b(\mu_1^2 - t^2) \\ - (\theta+b)t(\mu_1 - t) + \eta bt(\mu_1 - t) \end{bmatrix} \tag{11}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Setup cost (SeC) = B (12)

(ii) $HC = \int_0^T (x+yt)I(t)dt$

$$= \int_0^{\mu_1} (x+yt)I(t)dt + \int_{\mu_1}^{t_1} (x+yt)I(t)dt + \int_{t_1}^{T} (x+yt)I(t)dt$$

$$= \begin{bmatrix} \frac{1}{4}y(\eta-1)(a-\rho p)\left(\frac{1}{2}\eta-\frac{1}{2}\right)\mu_1^4 + \frac{1}{2}x(\eta-1)(a-\rho p)\mu_1^2 \\ + \frac{1}{3}\left(x(\eta-1)(a-\rho p)\left(\frac{1}{2}\eta-\frac{1}{2}\right) + y(\eta-1)(a-\rho p)\right)\mu_1^3 \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{1}{6}x(\eta-1)(a-\rho p)(\theta+b-\eta b)(t_1^3-\mu_1^3) \\ + \frac{1}{2}x\left(S_1(-\theta-b+\eta b) - (\eta-1)(a-\rho p)(-1-(\theta+b)\mu_1+\eta b\mu_1)\right)(t_1^2-\mu_1^2) \\ + x\left(S_1(1+(\theta+b)\mu_1-\eta b\mu_1) - (\eta-1)(a-\rho p)\left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^2-\frac{1}{2}\eta b\mu_1^2\right)\right)(t_1-\mu_1) \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{1}{8}y(\eta-1)(a-\rho p)(\theta+b-\eta b)(t_1^4-\mu_1^4) \\ + \frac{1}{3}y\left(S_1(-\theta-b+\eta b) - (\eta-1)(a-\rho p)(-1-(\theta+b)\mu_1+\eta b\mu_1)\right)(t_1^3-\mu_1^3) \\ + \frac{1}{2}y\left(S_1(1+(\theta+b)\mu_1-\eta b\mu_1) - (\eta-1)(a-\rho p)\left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^2-\frac{1}{2}\eta b\mu_1^2\right)\right)(t_1^2-\mu_1^2) \end{bmatrix}$$

$$+\left[\begin{array}{l}\frac{1}{12}x(a-\rho p)b\theta\left(T^5-t_1^{\,5}\right)+\frac{1}{12}x(a-\rho p)\theta\left(T^4-t_1^{\,4}\right)\\+\frac{1}{6}x(a-\rho p)\left(b-\theta T-\frac{1}{2}b\theta T^2\right)\left(T^3-t_1^{\,3}\right)\\+\frac{1}{2}x(a-\rho p)\left(-1-bT-\frac{1}{6}b\theta T^3\right)\left(T^2-t_1^{\,2}\right)+x(a-\rho p)\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)(T-t_1)\end{array}\right]$$

$$+\left[\begin{array}{l}\frac{5}{72}y(a-\rho p)b\theta\left(T^6-t_1^{\,6}\right)+\frac{1}{15}y(a-\rho p)\theta\left(T^5-t_1^{\,5}\right)\\+\frac{1}{8}y(a-\rho p)\left(b-\theta T-\frac{1}{2}b\theta T^2\right)\left(T^4-t_1^{\,4}\right)\\+\frac{1}{3}y(a-\rho p)\left(-1-bT-\frac{1}{6}b\theta T^3\right)\left(T^3-t_1^{\,3}\right)+\frac{1}{2}y(a-\rho p)\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)\left(T^2-t_1^{\,2}\right)\end{array}\right] \quad (13)$$

(by neglecting higher powers of θ)

(iii) $DC = c\left(\int_{\mu_1}^{t_1}\theta I(t)dt + \int_{t_1}^{T}\theta t I(t)dt\right)$

$$=\left[c\theta\left(\begin{array}{l}-\frac{1}{6}(\eta-1)(a-\rho p)(\theta+b-\eta b)\left(t_1^{\,3}-\mu_1^{\,3}\right)+S_1\left(1+(\theta+b)\mu_1-\eta b\mu_1\right)(t_1-\mu_1)\\+\frac{1}{2}\left(S_1(-\theta-b+\eta b)-(\eta-1)(a-\rho p)\left(-1-(\theta+b)\mu_1+\eta b\mu_1\right)\right)\left(t_1^{\,2}-\mu_1^{\,2}\right)\\-(\eta-1)(a-\rho p)\left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^{\,2}-\frac{1}{2}\eta b\mu_1^{\,2}\right)(t_1-\mu_1)\end{array}\right)\right]$$

$$+\left[c\theta\left(\begin{array}{l}\frac{5}{72}(a-\rho p)b\theta\left(T^6-t_1^{\,6}\right)+\frac{1}{15}(a-\rho p)\theta\left(T^5-t_1^{\,5}\right)\\+\frac{1}{8}(a-\rho p)\left(b-\theta T-\frac{1}{2}b\theta T^2\right)\left(T^4-t_1^{\,4}\right)\\+\frac{1}{3}(a-\rho p)\left(-1-bT-\frac{1}{6}b\theta T^3\right)\left(T^3-t_1^{\,3}\right)\\+\frac{1}{2}(a-\rho p)\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)\left(T^2-t_1^{\,2}\right)\end{array}\right)\right] \quad (14)$$

(iv) $SR = p\left(\int_0^T (a+bI(t) - \rho p)dt\right)$

$$
=\left[\begin{array}{l}
p\left(\frac{1}{3}b(\eta-1)(a-\rho p)\left(\frac{1}{2}\eta-\frac{1}{2}\right)\mu_1^{\,3}+\frac{1}{2}b(\eta-1)(a-\rho p)\mu_1^{\,2}+a\mu_1-\rho p\mu_1\right) \\
p\left(\begin{array}{l}
-\frac{1}{6}b(\eta-1)(a-\rho p)(\theta+b-\eta b)\left(t_1^{\,3}-\mu_1^{\,3}\right) \\
+\frac{1}{2}b\left(S_1(-\theta-b+\eta b)-(\eta-1)(a-\rho p)\left(-1-(\theta+b)\mu_1+\eta b\mu_1\right)\right)\left(t_1^{\,2}-\mu_1^{\,2}\right) \\
+a(t_1-\mu_1)+b\left(\begin{array}{l}
S_1\left(1+(\theta+b)\mu_1-\eta b\mu_1\right)- \\
(\eta-1)(a-\rho p)\left(\mu_1+\frac{1}{2}(\theta+b)\mu_1^{\,2}-\frac{1}{2}\eta b\mu_1^{\,2}\right)
\end{array}\right)(t_1-\mu_1) \\
-\rho p(t_1-\mu_1)
\end{array}\right)
\end{array}\right]
$$

$$
+\left[p\left(\begin{array}{l}
\frac{1}{12}(a-\rho p)b^2\theta\left(T^5-t_1^{\,5}\right)+\frac{1}{12}(a-\rho p)b\theta\left(T^4-t_1^{\,4}\right) \\
+\frac{1}{6}(a-\rho p)b\left(b-\theta T-\frac{1}{2}b\theta T^2\right)\left(T^3-t_1^{\,3}\right)
\end{array}\right)\right]
$$

$$
+\left[p\left(\begin{array}{l}
+\frac{1}{2}(a-\rho p)b\left(-1-bT-\frac{1}{6}b\theta T^3\right)\left(T^2-t_1^{\,2}\right)+a(T-t_1) \\
+(a-\rho p)b\left(T+\frac{1}{2}bT^2+\frac{1}{6}\theta T^3\right)(T-t_1)-\rho p(T-t_1)
\end{array}\right)\right] \tag{15}
$$

The total profit (π) during a cycle, T consisted of the following:

$$
\pi=\frac{1}{T}\left[SR - SeC - HC - DC\right] \tag{16}
$$

Substituting values from equations (12) to (15) in equation (16), we get total profit per unit. Putting $\mu_1 = v_1T$ and value of t_1 from equation (10) in equation (16), we get profit in terms of T and p. Differentiating equation (16) with respect to T and p and equate it to zero, we have

$$
\text{i.e. } \frac{\partial\pi(T,p)}{\partial T}=0,\ \frac{\partial\pi(T,p)}{\partial p}=0 \tag{17}
$$

provided it satisfies the condition

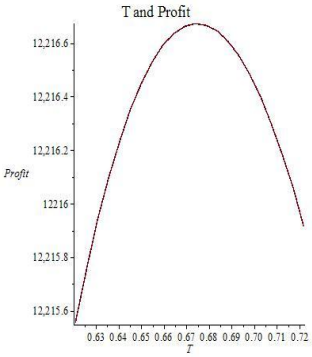
$$
\frac{\partial^2\pi(T,p)}{\partial T^2}<0,\ \frac{\partial^2\pi(T,p)}{\partial p^2}<0 \ \text{ and } \left[\frac{\partial^2\pi(T,p)}{\partial T^2}\right]\left[\frac{\partial^2\pi(T,p)}{\partial p^2}\right]-\left[\frac{\partial^2\pi(T,p)}{\partial p\partial T}\right]^2>0. \tag{18}
$$

4. NUMERICAL EXAMPLE:

Considering A= Rs.100, a = 500, b=0.05, η=2, c=Rs. 25, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, ρ =5, in appropriate units. The optimal value of T* =0.6745, p* = 50.5106 and Profit*= Rs. 12216.6735.

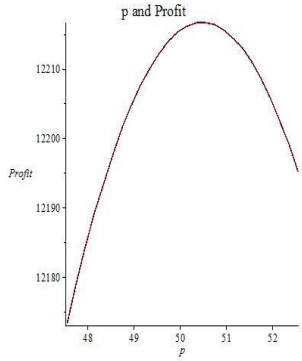
The second order conditions given in equation (18) are also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



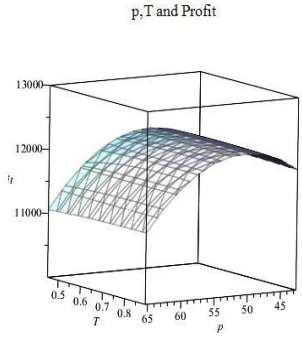
Graph 1

p and Profit



Graph 2

T, p and Profit



Graph 3

5. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Parameter	%	T	p	Profit
A	+20%	0.6660	60.4796	17715.2006
	+10%	0.6667	55.4814	14839.8685
	-10%	0.6858	45.5189	9845.6843
	-20%	0.7029	40.5315	7727.0147
θ	+20%	0.6588	50.5157	12211.1568
	+10%	0.6665	50.5131	12213.8924

Parameter	%	T	p	Profit
	-10%	0.6830	50.5082	12219.5026
	-20%	0.6918	50.5058	12222.3825
x	+20%	0.5978	50.5139	12176.7500
	+10%	0.6337	50.5186	12196.1082
	-10%	0.7237	50.5036	12238.6803
	-20%	0.7842	50.4981	12262.4456
B	+20%	0.7330	50.5555	12188.2487
	+10%	0.7045	50.5336	12202.1661
	-10%	0.6427	50.4863	12231.8617
	-20%	0.6088	50.4606	12247.8469
ρ	+20%	0.6403	42.1506	10116.1622
	+10%	0.6552	45.9501	11070.8187
	-10%	0.7006	56.0865	13617.5869
	-20%	0.7376	63.0603	15369.4532

From the table we observe that as parameter a increases/ decreases average total profit increases/ decreases.

Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/ increase in total profit.

From the table we observe that as parameter B and ρ increases/ decreases average total profit decreases/ increases.

6. CONCLUSION:

In this paper, we have developed a production inventory model for deteriorating items with price and inventory dependent demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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