Designing of Acceptance Double Sampling Plan for Life Test Based on Percentiles of Exponentiated Rayleigh Distribution

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Abstract

In this paper, the attribute characteristic parameter of acceptance double sampling plan based on percentiles is obtained for life testing when the life time of the product follows Exponentiated Rayleigh Distribution (ERD). The minimum sample size which is necessary to ensure a specified life percentile is obtained at various levels of consumer’s risk. The ratio $d_q$ that ensures the life at specified percentile is also obtained which fixes the producer’s risk at 0.05. Further, the operating characteristic values are produced and tabulated for various levels of consumer’s risk.

Keywords: Double sampling plan, percentiles, life test, operating characteristic function, Rayleigh distribution, Exponentiated Rayleigh Distribution.

INTRODUCTION

An acceptance sampling plan is a sampling procedure with a set of rules for making decisions about a lot of products. The decision is based on the number of defectives in a sample. Acceptance sampling plans are classified into two, such as attribute sampling plan and variable sampling plan. Sampling plans are designed based on the counts of defectives, it is called attribute sampling plan, on the other hand, if the sampling plans are based on sample average and standard deviation it is called variable sampling plan. There are different sampling plans such as single sampling plan, double sampling plan, multiple sampling plan, continuous sampling plan, skip-
lot sampling plan etc., which are available in literature and are developed for different real life situations. Acceptance double sampling plan is used in this paper since the advantage in the sampling plan is that, if a lot is good or not good it can be accepted or rejected respectively with small initial sample. Any lot which cannot be decided easily can be examined further by means of additional sample. Therefore in some cases the multiple sampling procedures provide shortcuts to the decision without jeopardizing the incoming or outgoing quality of material.

The sampling inspection plans which are developed for taking decision about a lot based on lifetime of the product through trials are called reliability sampling plans. It’s important for a producer to check whether the lifetime of the product satisfies the customer’s standard or not. Usually, if the lifetime of the products exceeds the specified time then the lot is accepted otherwise it is rejected. However, the limitation of using this method is the time duration spent on testing. That is when the lifetime of the product is assumed too long, it might be a time consuming processes to sentence a lot. Hence, it is normal to terminate the experiment by a pre-assigned time \( t \) and the numbers of failures are noted. If the predetermined mean life reaches a predetermined probability \( p^* \) then the lot is accepted which will protect the consumer. Thus, the life test is terminated when \((c+1)^{\text{th}}\) failure is observed at the pre-assigned time \( t \) whichever is earlier.


Additionally Lio et al (2009) have considered acceptance sampling plans from truncated life tests based on the Brinbaum-Saunders distribution for percentiles and they proposed that the acceptance sampling plan based on mean may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. This explains the material strength of products is deteriorated significantly and may not meet the consumer’s expectations, resulting engineers to pay more attention towards the percentiles of life time than the mean life. Even when the distributions are not symmetric, the percentiles output gives more and clean information regarding the life of the product. And when the distribution is symmetric, 50\(^{\text{th}}\) percentile or the median is equivalent to the mean life. Hence, in such cases, the percentile study is the generalization of acceptance sampling plans based on mean life of items. In this context, many authors have proposed reliability sampling plans based on percentiles.

In this paper, a double sampling plan for life testing is developed when the life of the product follows exponentiated Rayleigh distribution. The purpose of proposing this plan through percentiles may satisfy the customer’s expectation by rejecting the lot of low percentile, even when the lot is accepted when mean life of lifetime is considered. Aslam (2007), Aslam & Jun (2010) designed a double acceptance sampling plan for generalized log-logistic distributions with known shape parameters. Rao (2011) proposed double acceptance sampling plans based on truncated life tests for the Marshall-Olkin extended exponential distribution. Aslam et al. (2012) developed double acceptance sampling plans for Burr type XII distribution percentiles under the truncated life test.

**ADSP FOR PERCENTILES OF ERD UNDER TRUNCATED LIFE TEST**

Gupta et al. (1998) proposed a model to failure time data by $F^*(t) = [F(t)]^\theta$ where $F(t)$ is a baseline distribution function and $\theta$ is a positive real number which is derived from Lehman alternatives called exponentiated distribution. Abdallah et al (2015) states adding a parameter $\alpha$ (a positive real number) to a cumulative distribution function(cdf) $F$ by exponentiation produces a cdf of the so called Exponentiated Distribution(ED). The cdf of ED can be written as follows

$$G(x) = G(X; \theta) = [F(X; \beta)]^\alpha \equiv [F(X)]^\alpha$$ \hfill (1)

Kundu and Raqab (2005) estimated different estimators for generalized RD. The distribution function of RD is given by,

$$F(t, \tau) = 1 - e^{-\tau (t \tau / \tau)}^\theta, t > 0; 1 / \tau > 0$$ \hfill (2)

Hence, the cumulative distribution function of ERD is given by,

$$F(t; \tau, \theta) = \left[ 1 - e^{-[\tau (t \tau / \tau)]^\theta} \right]^\tau, t > 0; 1 / \tau > 0, \theta > 0$$ \hfill (3)

where $\tau$ and $\theta$ are the scale and shape parameters respectively. The first derivative of any cumulative distribution function is its probability density function. Hence the probability density function of ERD can be written as,
\[ f(t; \tau, \theta) = \frac{d}{dt} \left[ F(t; \tau, \theta) \right] = \frac{d}{dt} \left[ 1 - e^{-t^2/(2\tau^2)} \right] \quad t > 0; \tau > 0; \theta > 0 \]  \quad \ldots (4)

\[ f(t; \tau, \theta) = \theta \left[ 1 - e^{-t^2/(2\tau^2)} \right] \left[ \frac{t}{\tau^2} e^{-t^2/(2\tau^2)} \right] \]  \quad \ldots (5)

The 100\textsuperscript{th} percentile or the \( q \)\textsuperscript{th} quantile of any distribution is given by,

\[ \Pr (T \leq t_q) = q \]

\[ \Rightarrow t_q = \tau \sqrt{-2 \ln(1 - q^\theta)} \]

\( t_q \) and \( q \) are directly proportional. Let,

\[ \eta = \sqrt{-2 \ln(1 - q^\theta)} \]

\[ \Rightarrow \tau = t_q / \eta \]  \quad \ldots (*)

Replacing the scale parameter (\( \tau \)) by (\( * \)), we get the cumulative distribution function of ERD as,

\[ F(t) = \left[ 1 - e^{-1/2\eta^2 (t/t_q)\eta^2} \right]^\theta \quad t > 0, \theta > 0 \]  \quad \ldots (6)

Letting \( \delta = t/t_q \)

\[ F(t; \tau, \theta) = \left[ 1 - e^{-1/2(\eta\delta)^2} \right]^\theta \]  \quad \ldots (7)

Taking partial derivative with respect to \( \delta \), we have

\[ \frac{\partial F(t; \delta)}{\partial \delta} = \theta \eta \left[ 1 - e^{-1/2(\eta\delta)^2} \right]^\theta \left[ e^{-1/2(\eta\delta)^2} \right] \]  \quad \ldots (8)

Assume that a life test is conducted and will be terminated at time \( t_0 \). A probability \( P^* \) to reject a bad lot is used to protect consumers. A bad lot means that the true 100\textsuperscript{th} percentile \( t_q \) is below the supposed 100\textsuperscript{th} percentile \( t_0^0 \) that is, \( t_q < t_0^0 \). The lot is confirmed as a good one if the lifetime data hold the null hypothesis \( H_0 : t_q \geq t_0^0 \) against the alternative \( H_1 : t_q < t_0^0 \). The consumer's risk \( 1 - P^* \) is used as the significance level for this hypothesis testing and \( P^* \) is the consumer's confidence level. The development of DASP (Aslam and Jun, 2010) with a truncated censoring scheme is proposed as follows:
(1) Draw the first random sample of size $n_1$ from the lot and put them on test. If $c_1$ or fewer failures are observed at the pre-determined time $t_0$, the lot is accepted. Otherwise, the life test is truncated to reject the lot before or at $t_0$ if $(c_2 + 1)$ failures are cumulated before or at $t_0$, where $c_1 < c_2$.

(2) If the observed number of failures ($d_1$) by $t_0$ is between $c_1 + 1$ and $c_2$ ($c_2$ included), then draw a second sample of size $n_2$ for life testing till a prescribed termination time $t_0$. The lot is accepted if the cumulated number of failures from two samples ($d_2$) is smaller or equal to $c_2$. Otherwise, the lot is rejected.

Let us represent the acceptance double sampling plan as $(n_1, n_2, c_1, c_2, \delta_0)$. Here, $n_i$ and $c_i$ are the sample size and acceptance number associated with the $i^{th}$ sample respectively, $i=1, 2$. For the proposed acceptance double sampling plan, the probability of acceptance of lot is given by,

$$L(p) = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1-d_1} \sum_{d_2=0}^{c_2} \binom{n_2}{d_2} p^{d_2} (1-p)^{n_2-d_2}$$

Where, $p$ is the failure probability before the time $t$, given a specified 100$\theta$th percentile lifetime $t_{\theta}^0$, is obtained from $p = F(t; \delta_0) = \theta \eta \left(1 - e^{-\theta t_{\theta}^0} \right)^{\theta-1} \left(e^{-\theta t_{\theta}^0} \right)^{\theta-1}$.

Where, $\delta_0 = t/t_{\theta}^0$ and $F(t; \delta_0)$ is a non-decreasing function of $\delta$ since $\frac{\partial F(t; \delta)}{\partial \delta} > 0$ from (8). Accordingly, we have $F(t; \delta) \leq F(t; \delta_0) \Leftrightarrow t \geq t_{\theta}^0$. The first sample size $n_1$ is supposed to be the minimum sample size obtained from single sampling plan hence the second sample size $n_2$ is simulated for the developed sampling plan by satisfying the condition $L(p) \leq (1-P^*)$.

**EXAMPLE**

Assume that the life distribution is an Exponentiated Rayleigh Distribution and the experimenter is interested in showing that the true unknown 10$\theta$th percentile life $t_{0.1}^0$ is at least 1000hrs. Let the shape parameter, $\theta=2$ and the consumer risk is set to $3-(1-P^*)=0.25$. It is desire to stop the experiment at time $t=1000hrs$. Then for the acceptance numbers $c_2$ and $c_2$ as 0 and 2 respectively then from table 1, the double sampling plan $(n_1, n_2, c_1, c_2, t/t_{0.1}^0) = (14, 43, 0, 2, 1)$.

This explains, the experiment is done up to 1000hrs and the following decision is made

1) $d=0,1$ the lot is accepted.

2) $d \geq 3$ , the lot is rejected and the inspector should advice the management to concentrate on the production process for better quality products.
3) \( d=2 \), the inspector is suggested to go for second sample.

The operating characteristic curve for the plan obtained from table 1 is given as

<table>
<thead>
<tr>
<th>( t_{0.1}/t_{0.1}^{0} )</th>
<th>0.75</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2</th>
<th>2.25</th>
<th>2.5</th>
<th>2.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>OC</td>
<td>0.0209</td>
<td>0.2509</td>
<td>0.6521</td>
<td>0.8876</td>
<td>0.9663</td>
<td>0.9892</td>
<td>0.9961</td>
<td>0.9985</td>
<td>0.9993</td>
</tr>
</tbody>
</table>

It is observed from the above table that if the true 10\(^{th}\) percentile is 0.75 times the required 10\(^{th}\) percentile \( (t_{0.1}/t_{0.1}^{0} = 0.75) \) the producer’s risk is approximately 0.9791 which nears to zero as the true 10\(^{th}\) percentile approaches 2.25 times the required 10\(^{th}\) percentile. And the OC curve representing the above table is as follows:

![OC curve for c_1=0 and c_2=1 at p^*=0.75, d_0=1 based on the 10^{th} percentile, d=d_{0.1} of exponentiated Rayleigh distribution with \theta=2.](image)

**Fig 1:** OC curve for \( c_1=0 \) and \( c_2=1 \) at \( p^*=0.75 \), \( d_0=1 \) based on the 10\(^{th}\) percentile, \( d=d_{0.1} \) of exponentiated Rayleigh distribution with \( \theta=2 \).

The respective \( d_{0.1} \) from the table 2 is 1.6686 which ensures the producer’s risk at 0.05. This explains that the product can have a 10\(^{th}\) percentile life of 1.6686 times the specified 10\(^{th}\) percentile. That is we can say that the probability of the product to be accepted is at least 0.95.
Table 1: Minimum Sample Sizes and OC values for Acceptance double sampling plan \((n_1, n_2, c_1, c_2, \delta_0)\) when \(c_1=0\), and \(c_2=2\) for \(10^{th}\) percentile of Exponentiated Rayleigh distribution when \(\theta=2\)

<table>
<thead>
<tr>
<th>(p^*)</th>
<th>(n_1)</th>
<th>(n_2)</th>
<th>(t_{0.1}/t_{0.1}^0)</th>
<th>(t_{0.1}/t_{0.1}^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75</td>
<td>0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75</td>
</tr>
<tr>
<td>0.75</td>
<td>48</td>
<td>208</td>
<td>0.0188 0.2506 0.6302 0.8778 0.9644 0.9892 0.9963 0.9986 0.9994</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>20</td>
<td>66</td>
<td>0.0200 0.2503 0.6503 0.8879 0.9669 0.9896 0.9963 0.9986 0.9994</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>14</td>
<td>43</td>
<td>0.0209 0.2509 0.6521 0.8876 0.9663 0.9892 0.9961 0.9985 0.9993</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>4</td>
<td>10</td>
<td>0.0232 0.2436 0.6338 0.8707 0.9576 0.9854 0.9945 0.9977 0.9990</td>
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</tr>
<tr>
<td>0.75</td>
<td>2</td>
<td>4</td>
<td>0.0185 0.2312 0.6119 0.8513 0.9471 0.9806 0.9924 0.9967 0.9985</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>3</td>
<td>2.5</td>
<td>0.0315 0.2456 0.5951 0.8347 0.9403 0.9790 0.9925 0.9972 0.9989</td>
</tr>
<tr>
<td>0.75</td>
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<td>3</td>
<td>3</td>
<td>0.0046 0.0754 0.3022 0.5951 0.8057 0.9156 0.9646 0.9852 0.9936</td>
</tr>
<tr>
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<td>3</td>
<td>3.5</td>
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</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>0.0000 0.0046 0.0449 0.1695 0.3767 0.5951 0.7637 0.8710 0.9320</td>
</tr>
<tr>
<td>0.75</td>
<td>79</td>
<td>230</td>
<td>0.0410 0.1009 0.4449 0.7809 0.9286 0.9766 0.9917 0.9967 0.9986</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>32</td>
<td>86</td>
<td>0.0019 0.1006 0.4409 0.7746 0.9244 0.9745 0.9907 0.9963 0.9984</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>22</td>
<td>64</td>
<td>1</td>
<td>0.0023 0.1008 0.4229 0.7557 0.9154 0.9712 0.9895 0.9958 0.9982</td>
</tr>
<tr>
<td>0.75</td>
<td>6</td>
<td>13</td>
<td>1.5</td>
<td>0.0135 0.1009 0.4292 0.7516 0.9081 0.9663 0.9869 0.9945 0.9975</td>
</tr>
<tr>
<td>0.75</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>0.0025 0.1055 0.4475 0.7504 0.8994 0.9593 0.9827 0.9921 0.9962</td>
</tr>
<tr>
<td>0.75</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>2.5</td>
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<tr>
<td>0.9</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0.0000 0.0754 0.3022 0.5951 0.8057 0.9156 0.9646 0.9852 0.9936</td>
</tr>
<tr>
<td>0.9</td>
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<td>2</td>
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<tr>
<td>0.9</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>0.0007 0.0136 0.1163 0.3404 0.5876 0.7724 0.8831 0.9420 0.9715</td>
</tr>
<tr>
<td>0.95</td>
<td>103</td>
<td>224</td>
<td>0.0002 0.0505 0.3499 0.7208 0.9033 0.9667 0.9877 0.9950 0.9978</td>
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<tr>
<td>0.95</td>
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<td>0.95</td>
<td>8</td>
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<td>3</td>
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<tr>
<td>0.95</td>
<td>1</td>
<td>2</td>
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<td>0.0015 0.0557 0.2685 0.5550 0.7724 0.8941 0.9527 0.9790 0.9906</td>
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<tr>
<td>0.95</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>0.0001 0.0136 0.1163 0.3404 0.5876 0.7724 0.8831 0.9420 0.9715</td>
</tr>
<tr>
<td>0.99</td>
<td>158</td>
<td>188</td>
<td>0.7</td>
<td>0.0000 0.0110 0.2189 0.6139 0.8499 0.9432 0.9773 0.9902 0.9954</td>
</tr>
</tbody>
</table>
Table 2: Gives the ratio $d_{0.1}$ for accepting the lot with the producer’s risk of 0.05 when $\theta=2$

CONCLUSION

In this paper, the acceptance double sampling plan based on percentiles of Exponentiated Rayleigh Distribution is designed for life testing when the life test is truncated for a pre-defined time. This plan will be useful when the life of the product follows Exponentiated Rayleigh Distribution. Useful tables are provided and applied for establishment of the developed plan.

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