A Discrete Parametric Markov-Chain System Model of a Two Unit Standby System with Inspection and Two Types of Repair

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Abstract

The paper deals with cost benefit analysis of a two unit cold standby system model with two possible modes of a unit- normal (N) and total failure (F). A failed unit is inspected by an inspector to decide the kind of failure (major and minor) and then the repair is performed accordingly by a repairman. The priority in repair is given to the minor repair over the major repair. The failure time, inspection time and both types of repair times are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Regenerative point, reliability, MTSF, availability of system, busy period of inspector and repairman, net expected profit.

1. INTRODUCTION

Two unit redundant system models have been widely analyzed including the authors [2-5] by taking continuous distributions of failure and repair times as such type of systems play the vital role in day to day life. But very few authors have analyzed system models by taking discrete distributions of failure and repair times. Recently J. Bhatti et al [1] analyzed a two identical unit cold standby system model with inspection and two types of repair. They have considered geometric distributions of failure time, inspection time and each type of repair time with different parameters. In the above paper[1] we observed a lot of mistakes which reflects the mathematical equations constructed for various characteristics as well as their mathematical solutions. The few of major mistakes are highlighted below-

a) The authors have mentioned two types of repair namely minor and major but in the analysis they have taken same repair rate for both types of repair. In view of
this the consideration of minor and major repair is inconsequential. Moreover for
same repair rate the question of priority in the repair to minor categories has no
meaning.

b) In nomenclature section the authors have mentioned \( p_1 \) as the probability that the
failed unit is inspected satisfactory whereas, the transition diagram given in the
paper indicates that \( p_1 \) is the failure rate of an operating unit. Similarly,
according to the transition diagram \( p_2 \) is inspection rate of a failed unit whereas
in nomenclature section the authors stated it as the probability of failure.

c) In state \( S_1 \) one unit is under inspection and other is operative. But the authors
have not considered the possibility of failure of the operative unit in state \( S_1 \). In
view of this there will be five possible exists from state \( S_1 \) whereas the authors
wrongly considered only two exists from \( S_1 \).

d) In state \( S_1 \) if operative unit fails before the completion of inspection, then the
state \( (F_i,F_{wi}) \) will be generated. This state is missing in the transition diagram
given by the authors.

e) The authors have considered states \( S_7 = (F_{mi}, F_{mr}) \) and \( S_8 = (F_{mr}, F_{miw}) \) whereas for
identical units these two states must be the same.

In view of the above the transition diagram of the system model considered by J.
Bhatti et al is incorrect which results the wrong mathematics for evaluating the
various characteristics.

The purpose of the present paper is to analyze the system model consider by J. Bhatti
et al rectifying the above cumulative mistakes. The following economic related
measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.

ii) Reliability and mean time to system failure.

iii) Point-wise and steady-state availability of the system as well as expected up time
of the system during time \((0, t-1)\).

iv) Expected busy period of inspector and repairman during time \((0, t-1)\).

v) Net expected profit incurred by the system during a finite interval of time epochs
\(0, 1, 2, \ldots \ldots \ldots (t-1) \) and steady-state.

2. MODEL DESCRIPTION AND ASSUMPTIONS

i. The system comprises of two identical units each having two modes- Normal (N)
and total failure (F). Initially one unit is operative and other is kept into cold
standby.

ii. A failed unit is inspected by an inspector to decide the category of failure (major
and minor) and then the repair is performed accordingly by a single repairman.
The minor repair gets the priority over the major repair.

iii. After inspection the unit goes for major repair or minor repair with fixed known
probabilities \( a \) and \( b (=1-a) \) respectively.
iv. An inspector and a single repairman are always available with the system to do their jobs.

v. After minor and major repairs a unit becomes as good as new.

vi. The time to failure, time to inspection and each category of repair time follow geometric distributions with different parameters.

3. NOTATIONS AND STATES OF THE SYSTEM

a) Notations:

- \( p_1q_1 \): P.m.f. of failure time of an operative unit; \( p_1 + q_1 = 1 \).
- \( p_2q_2 \): P.m.f. of inspection time of a failed unit; \( p_2 + q_2 = 1 \).
- \( r_{i} \): P.m.f. of repair time by repairman of minor and major categories respectively for \( i=1, 2 \) and \( t_i + s_i = 1 \).
- \( Q_{i,j} \): P.m.f. and C.d.f. of one step or direct transition time from state \( S_i \) to \( S_j \).
- \( p_{ij} \): Steady state transition probability from state \( S_i \) to \( S_j \).
- \( q_{ij}(t) \): Probability that the system sojourn in state \( S_i \) up to epoch \( (t-1) \).
- \( 
\psi_i \): Mean sojourn time in state \( S_i \).
- \( *, h \): Symbol and dummy variable used in geometric transform e.g.

\[
GT[q_{ij}(t)] = q_{ij}^{*}(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)
\]

b) Symbols for the states of the system:

- \( N_o / N_s \): Unit is in N-mode and operative/into cold standby.
- \( F_i / F_{wi} \): Unit is in total failure (F) mode and under inspection/waits for inspection.
- \( F_{mr} / F_{wmr} \): Unit is in total failure (F) mode and under minor repair/waits for minor repair.
- \( F_{Mr} / F_{wMr} \): Unit is in total failure (F) mode and under major repair/waits for major repair.

With the help of above symbols the possible states of the system are:

- \( S_0 = (N_o, N_s) \), \( S_1 = (F_i, N_o) \), \( S_2 = (F_{Mr}, N_o) \), \( S_3 = (F_{mr}, N_o) \)
- \( S_4 = (F_i, F_{wi}) \), \( S_5 = (F_{Mr}, F_i) \), \( S_6 = (F_{mr}, F_i) \), \( S_7 = (F_{Mr}, F_{wMr}) \)
- \( S_8 = (F_{mr}, F_{wmr}) \), \( S_9 = (F_{mr}, F_{wMr}) \)
The transition diagram of the system model is shown in fig. 1.

4. TRANSITION PROBABILITIES

Let $Q_{ij}(t)$ be the probability that the system transits from state $S_i$ to $S_j$ during time interval $(0, t)$ i.e., if $T_{ij}$ is the transition time from state $S_i$ to $S_j$ then

$$Q_{ij}(t) = P[T_{ij} \leq t]$$

By using simple probabilistic arguments we have

$$Q_{1i}(t) = [1 - q_i t^{1i}], \quad Q_{i2}(t) = \frac{a_{ij} p_j}{1 - q_i q_j} \left[1 - (q_i q_j)^t^{1i}\right]$$

$$Q_{13}(t) = \frac{b_{ij} p_j}{1 - q_i q_j} \left[1 - (q_i q_j)^t^{1i}\right], \quad Q_{i4}(t) = \frac{p_i q_j}{1 - q_i q_j} \left[1 - (q_i q_j)^t^{1i}\right]$$

$$Q_{15}(t) = \frac{a_{ij} p_j}{1 - q_i q_j} \left[1 - (q_i q_j)^t^{1i}\right], \quad Q_{i6}(t) = \frac{b_{ij} p_j}{1 - q_i q_j} \left[1 - (q_i q_j)^t^{1i}\right]$$
The steady state transition probabilities from state $S_i$ to $S_j$ can be obtained from (1-26) by taking $t \rightarrow \infty$, as follows:

$$
P_{01} = 1, \quad P_{12} = \frac{aq_i p_2}{1-q_i q_2}, \quad P_{13} = \frac{bq_i p_2}{1-q_i q_2}, \quad P_{14} = \frac{p_i q_2}{1-q_i q_2}
$$

$$
P_{15} = \frac{ap_i p_2}{1-q_i q_2}, \quad P_{16} = \frac{bp_i p_2}{1-q_i q_2}, \quad P_{20} = \frac{q_i r_2}{1-q_i s_2}, \quad P_{21} = \frac{p_i r_2}{1-q_i s_2}
$$

$$
P_{25} = \frac{p_i s_2}{1-q_i s_2}, \quad P_{30} = \frac{q_i r_2}{1-q_i s_2}, \quad P_{31} = \frac{p_i r_2}{1-q_i s_2}, \quad P_{35} = \frac{p_i s_1}{1-q_i s_1}
$$

$$
P_{45} = a, \quad P_{46} = b, \quad P_{51} = \frac{q_i r_2}{1-q_i s_2}, \quad P_{52} = \frac{ap_i p_2}{1-q_i s_2}
$$

$$
P_{53} = \frac{bp_i r_2}{1-q_i s_2}, \quad P_{57} = \frac{ap_i s_2}{1-q_i s_2}, \quad P_{59} = \frac{bp_i s_2}{1-q_i s_2}, \quad P_{61} = \frac{q_i q_2}{1-s_i q_2}
$$

$$
P_{65} = \frac{p_i s_1}{1-q_i s_1}, \quad P_{69} = \frac{ps_2}{1-s_i q_2}, \quad P_{72} = 1-s_2^{t+1}, \quad P_{83} = P_{92} = 1-s_1^{t+1}
$$
\[
p_{62} = \frac{ap_2}{1-s_1q_2}, \quad p_{63} = \frac{bp_2}{1-s_1q_2}, \quad p_{68} = \frac{bs_1p_2}{s_1q_2}, \quad p_{69} = \frac{as_1p_2}{s_1q_2}
\]
\[
p_{72} = p_{83} = p_{92} = 1
\]

We observe that the following relations hold-
\[
p_{01} = p_{72} = p_{83} = p_{92} = 1,
\]
\[
p_{12} + p_{13} + p_{14} + p_{15} + p_{16} = 1, \quad p_{20} + p_{21} + p_{23} = 1
\]
\[
p_{30} + p_{31} + p_{36} = 1,
\]
\[
p_{45} + p_{46} = 1, \quad p_{51} + p_{52} + p_{53} + p_{57} + p_{59} = 1
\]
\[
p_{63} + p_{66} + p_{68} + p_{69} = 1
\]

(27-33)

5. MEAN SOJOURN TIMES

Let \( T_i \) be the sojourn time in state \( S_i \) (i=0-9) then \( \psi_i \) mean sojourn time in state \( S_i \) is given by
\[
\psi_i = \sum_{t=1}^{\infty} P[T \geq t]
\]

In particular,
\[
\psi_0 = \frac{q_1}{p_1}, \quad \psi_1 = \frac{q_1q_2}{1-q_1q_2}, \quad \psi_2 = \frac{q_1s_2}{1-q_1s_2}, \quad \psi_3 = \frac{q_1s_3}{1-q_1s_3}
\]
\[
\psi_4 = \frac{q_2}{p_2}, \quad \psi_5 = \frac{s_1q_2}{1-s_1q_2}, \quad \psi_6 = \frac{s_1q_2}{1-s_1q_2}, \quad \psi_7 = \frac{s_2}{r_2}
\]
\[
\psi_8 = \psi_9 = \frac{s_1}{r_1} = \psi
\]

(34-42)

6. METHODOLOGY FOR DEVELOPING EQUATIONS

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define \( R_i(t) \) as the probability that the system does not fail up to epochs 0, 1, 2, \ldots, (t-1) when it is initially started from up state \( S_i \). To determine it, we regard the failed state \( S_4, S_5, S_6, S_7, S_8 \) and \( S_9 \) as absorbing state. Now, the expression for \( R_i(t) ; i=0, 1, 2, 3; \) we have the following set of convolution equations.
\[
R_0(t) = q_1 + \sum_{u=0}^{t-1} q_{0u}(u)R_i(t-1-u)
\]
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\[
Z_t(t) = q_{12}(t) + q_{13}(t-1)\oplus R_2(t-1)
\]

Similarly,

\[
R_1(t) = Z_1(t) + q_{12}(t-1)\oplus R_2(t-1) + q_{13}(t-1)\oplus R_3(t-1)
\]

\[
R_2(t) = Z_2(t) + q_{20}(t-1)\oplus R_0(t-1) + q_{21}(t-1)\oplus R_1(t-1)
\]

\[
R_3(t) = Z_3(t) + q_{30}(t-1)\oplus R_0(t-1) + q_{31}(t-1)\oplus R_1(t-1)
\]

(43-46)

Where,

\[
Z_1(t) = q_1q_2^t, \quad Z_2(t) = q_1^tq_2^t, \quad Z_3(t) = q_1^tq_3^t
\]

b) Availability of the system-

Let \( A_i(t) \) be the probability that the system is up at epoch (t-1), when it initially started from state \( S_i \). Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for \( A_i(t) \); i=0 to 9.

\[
A_0(t) = Z_0(t) + q_{01}(t-1)\oplus A_1(t-1)
\]

\[
A_1(t) = Z_1(t) + q_{12}(t-1)\oplus A_2(t-1) + q_{13}(t-1)\oplus A_3(t-1) + q_{14}(t-1)\oplus A_4(t-1)
\]

\[+q_{15}(t-1)\oplus A_5(t-1) + q_{16}(t-1)\oplus A_6(t-1) \]

\[
A_2(t) = Z_2(t) + q_{20}(t-1)\oplus A_0(t-1) + q_{21}(t-1)\oplus A_1(t-1) + q_{25}(t-1)\oplus A_5(t-1) \]

\[
A_3(t) = Z_3(t) + q_{30}(t-1)\oplus A_0(t-1) + q_{31}(t-1)\oplus A_1(t-1) + q_{36}(t-1)\oplus A_6(t-1) \]

\[
A_4(t) = q_{43}(t-1)A_3(t-1) + q_{45}(t-1)A_5(t-1)
\]

\[
A_5(t) = q_{51}(t-1)\oplus A_1(t-1) + q_{52}(t-1)\oplus A_2(t-1) + q_{53}(t-1)\oplus A_3(t-1)
\]

\[+q_{57}(t-1)\oplus A_7(t-1) + q_{59}(t-1)\oplus A_9(t-1) \]

\[
A_6(t) = q_{63}(t-1)\oplus A_3(t-1) + q_{62}(t-1)\oplus A_2(t-1) + q_{63}(t-1)\oplus A_3(t-1)
\]

\[+q_{68}(t-1)\oplus A_8(t-1) + q_{69}(t-1)\oplus A_9(t-1) \]

\[
A_7(t) = q_{22}(t-1)\oplus A_2(t-1)
\]

\[
A_8(t) = q_{33}(t-1)\oplus A_3(t-1)
\]

\[
A_9(t) = q_{42}(t-1)\oplus A_2(t-1)
\]

(47-56)

Where,

The values of \( Z_i(t) \); i=0 to 3 are same as given in section 6(a).
c) Busy period of inspector

Let \( B_{i}^{t}(t) \) be the probability that the inspector is busy in the inspection of a failed unit at epoch \( t-1 \), when it initially started from state \( S_{i} \). Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for \( B_{i}^{t}(t) \); \( i=0 \) to 9.

\[
\begin{align*}
B_{0}^{t}(t) &= q_{00}(t-1) \odot B_{0}^{t-1}(t-1) \\
B_{1}^{t}(t) &= Z_{1}(t) + q_{12}(t-1) \odot B_{2}^{t}(t-1) + q_{13}(t-1) \odot B_{3}^{t}(t-1) + q_{14}(t-1) \odot B_{4}^{t}(t-1) \\
& \quad + q_{15}(t-1) \odot B_{5}^{t}(t-1) + q_{16}(t-1) \odot B_{6}^{t}(t-1) \\
B_{2}^{t}(t) &= q_{20}(t-1) \odot B_{0}^{t}(t-1) + q_{21}(t-1) \odot B_{1}^{t}(t-1) + q_{23}(t-1) \odot B_{3}^{t}(t-1) \\
B_{3}^{t}(t) &= q_{30}(t-1) \odot B_{0}^{t}(t-1) + q_{31}(t-1) \odot B_{1}^{t}(t-1) + q_{36}(t-1) \odot B_{6}^{t}(t-1) \\
B_{4}^{t}(t) &= Z_{4}(t) + q_{40}(t-1) \odot B_{0}^{t}(t-1) + q_{45}(t-1) \odot B_{5}^{t}(t-1) \\
B_{5}^{t}(t) &= Z_{5}(t) + q_{50}(t-1) \odot B_{0}^{t}(t-1) + q_{52}(t-1) \odot B_{2}^{t}(t-1) + q_{53}(t-1) \odot B_{3}^{t}(t-1) \\
& \quad + q_{57}(t-1) \odot B_{7}^{t}(t-1) + q_{59}(t-1) \odot B_{9}^{t}(t-1) \\
B_{6}^{t}(t) &= Z_{6}(t) + q_{60}(t-1) \odot B_{0}^{t}(t-1) + q_{62}(t-1) \odot B_{2}^{t}(t-1) + q_{63}(t-1) \odot B_{3}^{t}(t-1) \\
& \quad + q_{68}(t-1) \odot B_{8}^{t}(t-1) + q_{60}(t-1) \odot B_{9}^{t}(t-1) \\
B_{7}^{t}(t) &= q_{72}(t-1) \odot B_{2}^{t}(t-1) \\
B_{8}^{t}(t) &= q_{81}(t-1) \odot B_{1}^{t}(t-1) \\
B_{9}^{t}(t) &= q_{92}(t-1) \odot B_{2}^{t}(t-1)
\end{align*}
\]

Where,

The values of \( Z_{i}(t) \) is same as given in section 6(a), \( Z_{1}(t) = q_{1}^{i} \), \( Z_{2}(t) = q_{2}^{i} s_{2}^{i} \) and \( Z_{6}(t) = s_{1}^{i} q_{2}^{i} \).

\[\text{(57-66)}\]

\[\text{d) Busy period of repairman-}\]

Let \( B_{i}^{b_{m}}(t) \) and \( B_{i}^{b_{m}}(t) \) be the respective probabilities that the repairman busy in the minor and major repair of a failed unit at epoch \( t-1 \), when it initially started from state \( S_{i} \). Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for \( B_{i}^{t}(t) \); \( i=0 \) to 9.

\[
\begin{align*}
B_{0}^{b_{m}}(t) &= q_{00}(t-1) \odot B_{0}^{b_{m}}(t-1) \\
B_{1}^{b}(t) &= q_{12}(t-1) \odot B_{2}^{b}(t-1) + q_{13}(t-1) \odot B_{3}^{b}(t-1) + q_{14}(t-1) \odot B_{4}^{b}(t-1) \\
& \quad + q_{15}(t-1) \odot B_{5}^{b}(t-1) + q_{16}(t-1) \odot B_{6}^{b}(t-1)
\end{align*}
\]
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\[ B_2^j (t) = (1 - \delta)Z_2^j (t) + q_{20} (t - 1)\oplus B_0^j (t - 1) + q_{21} (t - 1)\oplus B_1^j (t - 1) + q_{25} (t - 1)\oplus B_5^j (t - 1) \]

\[ B_3^j (t) = 8Z_3^j (t) + q_{30} (t - 1)\oplus B_0^j (t - 1) + q_{31} (t - 1)\oplus B_1^j (t - 1) + q_{35} (t - 1)\oplus B_5^j (t - 1) \]

\[ B_4^j (t) = q_{45} (t - 1)\oplus B_5^j (t - 1) + q_{46} (t - 1)\oplus B_6^j (t - 1) \]

\[ B_5^j (t) = (1 - \delta)Z_5^j (t) + q_{51} (t - 1)\oplus B_1^j (t - 1) + q_{52} (t - 1)\oplus B_2^j (t - 1) + q_{53} (t - 1)\oplus B_3^j (t - 1) \]

\[ B_6^j (t) = 8Z_6^j (t) + q_{61} (t - 1)\oplus B_1^j (t - 1) + q_{62} (t - 1)\oplus B_2^j (t - 1) + q_{63} (t - 1)\oplus B_3^j (t - 1) + q_{65} (t - 1)\oplus B_5^j (t - 1) + q_{66} (t - 1)\oplus B_6^j (t - 1) \]

\[ B_7^j (t) = (1 - \delta)Z_7^j (t) + q_{72} (t - 1)\oplus B_2^j (t - 1) \]

\[ B_8^j (t) = Z(t) + q_{83} (t - 1)\oplus B_3^j (t - 1) \]

\[ B_{83}^j (t) = 8Z(t) + q_{82} (t - 1)\oplus B_2^j (t - 1) \]

(67-76)

Where,

\[ \delta = 1 \text{ and } 0 \text{ respectively for } j = m_i \text{ and } M_a. \] The values of \( Z_i (t); i = 2, 3, 5, 6 \) are same as given in section 6(a) and (b), and \( Z_r (t) = s_2^i, Z(t) = s_1^i. \)

7. ANALYSIS OF RELIABILITY AND MTSF

Taking geometric transforms of (43-46) and simplifying the resulting set of algebraic equations for \( R_0^*(h) \) we get

\[ R_0^*(h) = \frac{N_1(h)}{D_1(h)} \] (77)

Where,

\[ N_1(h) = [1 - h^2 q_{12} q_{21} - h^2 q_{13} q_{31}] Z_0^* + h q_{01} Z_1^* + h^2 q_{02} q_{12} Z_2^* + h^2 q_{03} q_{13} Z_3^* \]

\[ D_1(h) = 1 - h^2 q_{12}^* q_{21}^* - h^2 q_{13}^* q_{31}^* - h^3 q_{02}^* q_{12}^* q_{20}^* - h^3 q_{03}^* q_{13}^* q_{30}^* \]

Collecting the coefficient of \( h^i \) from expression (77), we can get the reliability of the system \( R_0^*(t) \). The MTSF is given by-

\[ E(T) = \lim_{h \to 1} \sum_{i=1}^{\infty} h^i R(t) = \frac{N_1(1)}{D_1(1)} - 1 \] (78)

\[ N_1(1) = [1 - p_{12} p_{21} - p_{13} p_{31}] \psi_0 + \psi_1 + p_{12} \psi_2 + p_{13} \psi_3 \]

\[ D_1(1) = 1 - p_{12} (p_{21} + p_{20}) - p_{13} (p_{31} + p_{30}) \]
8. AVAILABILITY ANALYSIS

On taking geometric transforms of (47-56) and simplifying the resulting equations for we get

\[ A'_{0}(h) = \frac{N_{2}(h)}{D_{2}(h)} \]  

(79)

Where,

\[ N_{2}(h) = \]

\[
\begin{bmatrix}
1 & -hq^{*}_{01} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -hq^{*}_{12} & -hq^{*}_{13} & -hq^{*}_{14} & -hq^{*}_{15} & -hq^{*}_{16} & 0 & 0 & 0 \\
-hq^{*}_{20} & -hq^{*}_{21} & 1 & 0 & 0 & -hq^{*}_{25} & 0 & 0 & 0 & 0 \\
-hq^{*}_{30} & -hq^{*}_{31} & 0 & 1 & 0 & 0 & -hq^{*}_{36} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -hq^{*}_{45} & -hq^{*}_{46} & 0 & 0 & 0 \\
0 & -hq^{*}_{41} & -hq^{*}_{52} & -hq^{*}_{53} & 0 & 1 & 0 & -hq^{*}_{57} & 0 & -hq^{*}_{59} \\
0 & -hq^{*}_{61} & -hq^{*}_{62} & -hq^{*}_{63} & 0 & 0 & 1 & 0 & -hq^{*}_{68} & -hq^{*}_{69} \\
0 & 0 & -hq^{*}_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -hq^{*}_{83} & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -hq^{*}_{92} & 0 & 0 & 0 & 0 & 0 & 0 & 1
\]

and

\[ D_{2}(h) = \]

\[
\begin{bmatrix}
Z_{0}^{*} & hq^{*}_{01} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_{1}^{*} & 1 & hq^{*}_{12} & hq^{*}_{13} & hq^{*}_{14} & hq^{*}_{15} & hq^{*}_{16} & 0 & 0 & 0 \\
Z_{2}^{*} & hq^{*}_{21} & 1 & 0 & 0 & hq^{*}_{25} & 0 & 0 & 0 & 0 \\
Z_{3}^{*} & hq^{*}_{31} & 0 & 1 & 0 & 0 & hq^{*}_{36} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & hq^{*}_{45} & hq^{*}_{46} & 0 & 0 \\
0 & hq^{*}_{51} & hq^{*}_{52} & hq^{*}_{53} & 0 & 1 & 0 & hq^{*}_{57} & 0 & hq^{*}_{59} \\
0 & hq^{*}_{61} & hq^{*}_{62} & hq^{*}_{63} & 0 & 0 & 1 & 0 & hq^{*}_{68} & hq^{*}_{69} \\
0 & 0 & hq^{*}_{72} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & hq^{*}_{83} & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & hq^{*}_{92} & 0 & 0 & 0 & 0 & 0 & 0 & 1
\]
The steady state availabilities of the system due to operation of unit -

\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{h \to 1} \frac{N_2(h)}{D_2(h)} \]

But \( D_2(h) \) at \( h=1 \) is zero, therefore by applying L. Hospital rule, we get

\[ A_0 = -\frac{N_2(1)}{D_2(1)} \quad (80) \]

Where,

\[ N_2(1) = u_0\psi_0 + u_1\psi_1 + u_2\psi_2 + u_3\psi_3 \]

and

\[ D_2'(1) = -\left[ u_0\psi_0 + u_1(\psi_0 + p_{14}\psi_4) + u_2\psi_2 + u_3\psi_3 + u_4(\psi_4 + p_{59}\psi_9 + p_{57}\psi_7) + u_6(\psi_6 + (p_{68} + p_{60})\psi) \right] \]

where,

\[ u_i = U_i^*(0) \] and \( U_i(h) \); \( i=0, 1, 2, 3, 5, 6 \) are the minors of 1st, 2nd, 3rd, 4th, 6th and 7th elements lying in first column of the matrix corresponding to determinant \( D_2(h) \).

Now the expected up time of the system due to operative unit upto epoch \((t-1)\) is given by

\[ \mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x) \]

so that

\[ \mu_{up}^*(h) = \frac{A_0^*(h)}{1-h} \quad (81) \]

7. BUSY PERIOD ANALYSIS

a) Busy period of inspector-

On taking geometric transforms of (57-66) and simplifying the resulting equations for \( B_0^i(h) \) we get

\[ B_0^i(h) = \frac{N_3(h)}{D_2(h)} \quad (82) \]

Where,

\[ N_3(h) = hq_{01}^* \left[ U_i^* \left( Z_1^* + hq_{14}^* Z_4^* \right) + U_i^* Z_3^* + U_i^* Z_6^* \right] \]

and \( D_2(h) \) is same as in availability analysis.

In the long run the respective probability that the inspector is busy in the inspection of failed unit given by-
\[ B_0^i = \lim_{t \to h} B_0^i(t) = \lim_{h \to 1} \left( 1 - h \right) \frac{N_4(h)}{D_2(h)} \]

But \( D_2(h) \) at \( h=1 \) is zero, therefore by applying L. Hospital rule, we get

\[ B_0^i = \frac{N_4(1)}{D_2(1)} \]  

(83)

Where,
\[ N_4(1) = u_1(\psi_1 + p_4\psi_4) + u_5\psi_5 + u_8\psi_6 \]

and \( D_2'(1) \) is same as in availability analysis.

Now the expected busy period of inspector is in the inspection of a failed unit up to epoch \((t-1)\) is given by -

\[ \mu_b'(t) = \sum_{x=0}^{t-1} B_0^i(x) \]

So that,

\[ \mu_b^*(h) = \frac{B_0^*(h)}{1-h} \]  

(84)

b) Busy period of repairman-

On taking geometric transforms of (67-76) and simplifying the resulting equations for \( B_0^{mi} \) and \( B_0^{Ma} \) we get

\[ B_0^{mi}(h) = \frac{N_4(h)}{D_2(h)} \quad \text{and} \quad B_0^{Ma*}(h) = \frac{N_5(h)}{D_2(h)} \]  

(85-86)

Where,
\[ N_4(h) = hq_0^* \left[ U_3 Z_3 + U_5 \left( hq_{60}^* + hq_{69}^* \right) Z_1^* \right] + U_5^* hq_{57}^* Z_1^* \]

\[ N_5(h) = hq_0^* \left[ U_2^* Z_2 + U_5^* \left( Z_6^* + hq_{57}^* Z_1^* \right) \right] \]

and \( D_2(h) \) is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the minor repair and major repair of a failed unit are given by-

\[ B_0^{mi} = \lim_{t \to \infty} B_0^{mi}(t) = \lim_{h \to 1} (1-h) \frac{N_4(h)}{D_2(h)} \]

\[ B_0^{Ma} = \lim_{t \to \infty} B_0^{Ma}(t) = \lim_{h \to 1} (1-h) \frac{N_5(h)}{D_2(h)} \]

But \( D_2(h) \) at \( h=1 \) is zero, therefore by applying L. Hospital rule, we get
A Discrete Parametric Markov-Chain System Model of a Two Unit Standby System...

\[ B_0^m = \frac{-N_2(1)}{D'_2(1)} \]  \text{ and } \[ B_0^{Ma} = \frac{-N_2(1)}{D'_2(1)} \] (87-88)

Where,

\[ N_4(1) = u_3 \psi_3 + u_5 p_{99} \psi + u_6 \left( \psi_6 + (p_{68} + p_{69}) \psi \right) \]

\[ N_5(1) = u_2 \psi_2 + u_5 (\psi_5 + p_{57} \psi_7) \]

and \( D'_2(1) \) is same as in availability analysis.

Now the expected busy period of the repairman in minor repair and major repair of a failed unit up to epoch (t-1) are respectively given by-

\[ \mu_0^m(t) = \sum_{x=0}^{\infty} B_0^m(x) \]

\[ \mu_0^{Ma}(t) = \sum_{x=0}^{\infty} B_0^{Ma}(x) \]

So that,

\[ \mu_0^m(h) = \frac{B_0^m(h)}{1-h} \]

\[ \mu_0^{Ma}(h) = \frac{B_0^{Ma}(h)}{1-h} \] (89-90)

8. \textbf{PROFIT FUNCTION ANALYSIS}

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

\[ K_0 = \text{revenue per-unit time by the system due to operative unit.} \]

\[ K_1 = \text{cost per-unit time when inspector is busy in the inspection of failed unit.} \]

\[ K_2 = \text{cost per-unit time when repairman is busy in the minor repair of a failed unit.} \]

\[ K_3 = \text{cost per-unit time when repairman is busy in the major repair of a failed unit.} \]

Then, the net expected profit incurred up to epoch (t-1) given by

\[ P(t) = K_0 \cdot u_p(t) - K_1 \cdot b^i(t) - K_2 \cdot b^m(t) - K_3 \cdot b^{Ma}(t) \] (91)

The expected profit per unit time in steady state is given by-

\[ P = \lim_{t \to \infty} \frac{P(t)}{t} = \lim_{h \to 1} (1-h)^2 P'(h) \]

\[ = K_0 \lim_{h \to 1} (1-h)^2 \frac{A^*_0(h)}{1-h} - K_1 \lim_{h \to 1} (1-h)^2 \frac{B^*_0(h)}{1-h} - K_2 \lim_{h \to 1} (1-h)^2 \frac{B^*_0(1-h)}{1-h} \]

\[ + K_3 \lim_{h \to 1} (1-h)^2 \frac{B^*_0(1-h)}{1-h} \]
\[ I_{Mi} = K_0 A_0 - K_1 B_{01}^1 - K_2 B_{02}^{ii} - K_3 B_{03}^{iii} \]  

(92)

9. GRAPHICAL REPRESENTATION

The curves for MTSF and profit function have been drawn for different values of failure parameters. Fig. 2 depicts the variation in MTSF with respect to inspection rate \( (p_2) \) for different values of minor repair rate \( (r_1) \) of a unit and failure rate \( (p_1) \) when values of other parameters are kept fixed as \( r_2 = 0.02 \) and \( a = 0.25 \). From the curves we conclude that expected life of the system increases as the values of \( r_1 \) and \( p_2 \) increases and decrease with increase in \( p_1 \).

![Behavior of MTSF with respect to \( p_1, p_2 \) and \( r_1 \)](image)

Similarly, Fig. 3 reveals the variations in profit \( (P) \) with respect to \( p_2 \) for varying values of \( r_1 \) and \( p_1 \), when other parameters are kept fixed as \( r_2 = 0.02 \), \( a = 0.25 \), \( K_0 = 150 \), \( K_1 = 150 \), \( K_2 = 180 \) and \( K_3 = 220 \). From the figure it is clearly observed from the smooth curves, that the system is profitable if the value of parameter \( p_2 \) is greater than 0.0550, 0.0590 and 0.0690 respectively for \( r_1 = 0.10 \), 0.15 and 0.20 for fixed value of \( p_1 = 0.025 \). From dotted curves, we conclude that system is profitable
only if value of parameter $p_2$ is greater than 0.0825, 0.0925 and 0.1200 respectively for $r_1 = 0.10, 0.15$ and 0.20 for fixed value of $p_1 = 0.030$.

**Behavior of Profit (P) with respect to $p_1, p_2$ and $r_1$**

![Behavior of Profit (P) with respect to $p_1, p_2$ and $r_1$](image)

**Fig. 3**

**REFERENCES**


