Determination of Quick Switching System by Attributes under the Conditions of Zero-Inflated Poisson Distribution

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Abstract

Sampling plans, systems and schemes are used to accept/reject the lots when inspecting a series of lots. A sampling system is a grouping of two or more sampling plans with specified rules for switching between the plans for selecting the lots of finished products. Quick Switching System (QSS) is used to make accept or reject decisions when inspecting a series of lots. Quick Switching System requires fewer samples for processes running at low levels of defects. QSS is a sampling system with reference to single sampling plan that involves normal and tightened plans by incorporating a switching rule. Quick Switching System can be applied to protect from poor quality and to reduce the cost of inspection. When the production process is monitored well, occurrence of defects in the manufactured products would be rare. The information related to number of defects will have more number of zeros. Under such circumstances, the appropriate probability distribution of the number of defects is a zero-inflated probability distribution rather than the usual probability distribution. A zero-inflated probability distribution (ZIPD) can be defined by a mixture of two distributions. The component distribution associated with the larger mixing proportion would be a distribution which degenerates at zero. In this paper, a study of QSS with reference to Single Sampling Plan using ZIPD is designed and tables are constructed for some set of values of \((p_1, \alpha, p_2, \beta)\). Comparison is made with the existing plan and its advantages are highlighted with example.

Keywords: QSS, ZIPD, AQL, LQL, producer’s risk, consumer’s risk, OC.
INTRODUCTION AND REVIEW OF LITERATURE:
Acceptance sampling is a major field/tool in Statistical Quality Control to inspect the quality of the product or raw material at various stages against the specified quality standards. Various sampling plans, systems and schemes were developed and applied in the industries based on the need of the shop floor situations.

The Quick Switching System (QSS) by attributes is the most common and easiest of all acceptance sampling procedures. A Quick Switching System by attributes is defined as \((N, n, c_N, c_T)\), where \((N)\) is the lot size, \((n)\) is the sample size and \((c_N)\) is the acceptance number for normal level and \((c_T)\) is the acceptance number for tightened level. The sample size \((n)\), \((c_N)\) and \((c_T)\) are determined for specified requirements of the producer and consumer.

Guenther (1969) proposed an iterative procedure to determine the parameters of single sampling plan by attributes for two specified points on the operating characteristic (OC) curve, namely, \((p_1, 1-\alpha)\) and \((p_2, \beta)\), where \(p_1\) is the producer’s quality level, \(p_2\) is the consumer’s quality level, \(\alpha\) is the producer’s risk and \(\beta\) is the consumer’s risk.

Quick Switching System explored in this article consists of two sampling plans along with a set of rules for switching between them. The first sampling plan, called the Normal plan, is intended for use during periods of good quality. It has a smaller sample size in order to reduce inspection costs. The second sampling plan, called the Tightened plan, is intended for use when problems are encountered. It is designed to give a high level of protection. The switching rules ensure that the correct and appropriate plan is used. Sampling plans are designed to be easy to use and to react quickly to changes in quality. QSSs concentrate one’s inspection effort where it will do the most good. Further, for processes running at low level of inspection but that react severely to the first hint of a problem.

Romboski (1969) has introduced sampling inspection system QSS-1 \((n, kn; c_0)\) which is QSS-1 with single sampling plan as a reference plan \((n, c_0)\) and \((kn, c_0), k>1\) are respectively the normal and tightened single sampling plans. Devaraj Arumainayagam (1991) has studied Quick Switching System with reference to Double Sampling Plan and it is termed as QSDSS for both acceptance number tightening and sample size tightening and constructed tables for various parameters. Devaraj Arumainayagam and Uma (2010) have constructed Quick Switching System using Weighted Poisson distribution – sample size tightening.

The ZIP distribution has been used as an appropriate probability distribution in diversified fields. Lambert (1992) fitted a ZIP regression model to the data concerning the number of defects in a manufacturing process. Gurma and Trivedi (1996) applied ZIP model for studying recreational trips. Saei and McGilchrist (1997) used this distribution for studying chemotherapy use. Ridout, Demetrio and Hinde (1998) have studied the problem of modeling count data with many zeros in horticultural research and investigated the appropriateness of the ZIP \((\varphi, \lambda)\) model over the zero-inflated
negative binomial model. Dankmar Bohning, Ekkehart Detz and Peter Schlattmann (1999) have used ZIP ($\phi, \lambda$) regression model to study a set of dental epidemiological data. Hall (2000) has carried out a case study on irrigation of greenhouse crops. Xiang, Lee, Yau and McLachlan (2007) have studied the problem of pancreas disorder length of stay that comprised mainly same-day separations and simultaneous prolonged hospitalizations.

Xie et al. (2009) developed a score test in the case of fitting zero-inflated generalized Poisson mixed regression model to a count data and obtained estimates of the model parameters based on best linear unbiased prediction log-likelihood. Yang et al. (2010a) recommended the application of score test for testing zero-inflation in correlated count data and subsequently, Yang et al. (2010b) argued with reference to score test that zero-inflated generalized Poisson model is a reasonable alternative to zero-inflated generalized negative binomial model. Yang et al. (2010c) extended their studies to ZIP and zero-inflated binomial regression models. Xiang and Teo (2011) showed using ZIP model that null-Wald test and likelihood ratio test are more powerful than score test for testing zero-inflation in the correlated count data. Li (2011) proposed a semi parametric ZIP regression model for count data containing many zeros and developed a procedure for testing the adequacy of the fitted model.


In recent years, production process is designed in such a way that occurrence of defects is a rare phenomenon. The number of zero defects will be found more. The probability distribution which is appropriate to describe such situations is a zero-inflated distribution. In this paper, the number of defects is assumed to be distributed according a zero-inflated Poisson distribution and designed as QSS_{ZIPD}(n; C_{N}, C_{T}).

**Operating Procedure for QSS (n, c_{N,CT}) Under Zero Inflated Poisson Distribution**

**Step 1:** From a lot, take a random sample of size $n$ at the normal level and count the number of defectives $d$.

(a) If $d \leq c_{N}$, accept the lot and repeat step 1.
(b) If $d > c_{N}$, reject the lot and go to step 2.

**Step 2:** From the next lot, take a random sample of size $n$ at the tightened level and count the number of defectives $d$.

(a) If $d \leq c_{T}$, accept the lot and go to step 1.
(b) If $d > c_{T}$, reject the lot and repeat step 2.
The performance of a sampling system can be assessed using its OC function. The OC function of a QSS is defined as

\[ P_a = \frac{p_T}{1-P_N+p_T} \]  

where \( P_a \) is the Probability of acceptance
\( p_T \) is the Probability of acceptance under tightened plan
\( P_N \) is the Probability of acceptance under normal plan.

When a random sample is drawn from a production process in a continuous stream the observed number of defects in the sample is distributed according to Poisson distribution with parameter \( \lambda = np \), which is the average number of defects per unit. Schilling (1982) has pointed out when \( n/N \leq 0.10 \), \( n \) is large, \( p \) is small such that \( np < 5 \), the Poisson distribution is appropriate.

In many practical situations, a Poisson random variable can take the value “0” quite frequently. For instance, when the manufacturing equipment is properly aligned, number of defects may be nearly zero. Under such circumstances, the suitable probability distribution of the number of defects is a zero-inflated Poisson distribution rather than the usual Poisson distribution.

A zero-inflated Poisson distribution (ZIP) is defined by the following probability mass function.

\[ P(X = x|\phi, \lambda) = \phi f_1(x) + (1 - \phi)P(X = x|\lambda) \]  

where
\[ f_1(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0 \end{cases} \]

and
\[ P(X = x|\lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad \text{when } x = 0, 1, 2, ... \]

In this distribution, \( \phi \) (0<\( \phi \) < 1) is known as the mixing proportion. Also, \( \phi \) and \( \lambda \) are the parameters of the ZIP distribution. Thus, a ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero, and a Poisson distribution. The above probability mass function can also be expressed as

\[ P(X = x|\phi, \lambda) = \begin{cases} \phi + (1 - \phi)e^{-\lambda}, & \text{when } x = 0 \\ (1 - \phi)\frac{e^{-\lambda} \lambda^x}{x!}, & \text{when } x = 1, 2, ... \end{cases} \]

The operating characteristics (OC) of the QSS under the condition of ZIP distribution is defined in eqn 1,

where \( P_N \) represents
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\[ P_N = \varphi + (1 - \varphi)e^{-\lambda} + (1 - \varphi)\sum_{x=1}^{c_N} \frac{e^{-\lambda} \lambda^x}{x!} \]

and \( P_T \) represents

\[ P_T = \varphi + (1 - \varphi)e^{-\lambda} + (1 - \varphi)\sum_{x=1}^{c_T} \frac{e^{-\lambda} \lambda^x}{x!} \]

**Determination of \((n, c_N, c_T)\) for Zero-Inflated Poisson Quick Switching System**

The objective of the determination of an optimum QSS is to find a pair \((n; c_N, c_T)\) for the specified strength \((p_1, \alpha, p_2, \beta)\). The conditions are:

(i) \( P_a(p_1) = 1 - \alpha \)

(ii) \( P_a(p_2) = \beta \)

Since \( n, c_N \) and \( c_T \), need to be integers, the conditions (i) and (ii) may not be exactly satisfied. Hence, optimum plan \((n, c_N, c_T)\) could be defined by satisfying the conditions:

(iii) \( P_a(p_1) \geq 1 - \alpha \)

(iv) \( P_a(p_2) \leq \beta \)

So that the maximum producer’s risk and maximum consumer’s risk under the plan will be at \( \alpha \) and \( \beta \) respectively. The optimum QSS\(_{ZIPD}\) by attributes for given \((p_1, 1- \alpha)\) and \((p_2, \beta)\) is determined by using the iterative procedure proposed by Guenther (1969). On making use of this procedure, the optimal Quick Switching System are determined for a fixed \( \varphi \) and a wide range of \( p_1 \) and \( p_2 \) with specified maximum producer’s risk of 5% and a maximum consumer’s risk of 10%. The plans are presented in Table 1 and Table 2 for two different values of \( \varphi=0.0001 \) and 0.01.

In some cases very large values were obtained for \( n \), which are not possible to apply in practice. To those data sets, the optimum Quick Switching Systems are not presented and are denoted as ***.

**Example 1**

Suppose that the strength of the system is specified as \((p_1=0.005, \alpha=0.05, p_2=0.05 \text{ and } \beta=0.10)\) and the mixing proportion is 0.01. Corresponding to these specifications, the parameters of the optimum QSS\(_{ZIPD}\) \((85; 2, 1)\) is found from Table 2.

**Comparison**

In recent years, production process is designed in such a way that occurrence of defects is a rare phenomenon. The number of zero defects will be found more. Considering determination of SSP under conditions of ZIPD as a base paper, one can adopt the system procedure to have the minimum sample size and can use this application.
Example 2

For the parameters $AQL = 0.025$, $p_1$ and $p_2$, $LQL = 0.07$, $\varphi = 0.01$, the $SSP_{ZIPD}$ gives the sample size and acceptance number as $(207, 9)$ whereas for the same parameters $QSS_{ZIPD}$ generates the sample size and acceptance number as $(89; 5, 2)$. In this, the sample size is three times reduced with minimum acceptance number. Hence it is concluded that system is more efficient than that of the sampling plans with minimum sample size.

Table 1: Optimum $QSS_{ZIPD}$ by attributes for given $(p_1, \alpha = 0.05)$ and $(p_2, \beta = 0.10)$ and the process parameter $\varphi = 0.0001$

<table>
<thead>
<tr>
<th>AQL($p_1$)</th>
<th>LQL($p_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>0.005</td>
<td>(82;2,1)</td>
</tr>
<tr>
<td>0.010</td>
<td>(110;3,2)</td>
</tr>
<tr>
<td>0.015</td>
<td>(164;5,4)</td>
</tr>
<tr>
<td>0.020</td>
<td>(147;5,1)</td>
</tr>
<tr>
<td>0.025</td>
<td>(***)</td>
</tr>
<tr>
<td>0.030</td>
<td>(***)</td>
</tr>
<tr>
<td>0.035</td>
<td>(***)</td>
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<tr>
<td>0.040</td>
<td>(***)</td>
</tr>
<tr>
<td>0.045</td>
<td>(***)</td>
</tr>
<tr>
<td>0.050</td>
<td>(***)</td>
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</tbody>
</table>
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Table 2: Optimum QSS\(_{ZIPD}\) by attributes for given \((p_1, \alpha=0.05)\) and \((p_2, \beta=0.10)\) and the process parameter \(\varphi=0.01\)

<table>
<thead>
<tr>
<th>AQL((p_1))</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>(85;2,1)</td>
<td>(71;2,1)</td>
<td>(61;2,1)</td>
<td>(53;2,1)</td>
<td>(47;2,1)</td>
<td>(43;2,1)</td>
</tr>
<tr>
<td>0.010</td>
<td>(113;3,2)</td>
<td>(71;2,1)</td>
<td>(61;2,1)</td>
<td>(53;2,1)</td>
<td>(47;2,1)</td>
<td>(43;2,1)</td>
</tr>
<tr>
<td>0.015</td>
<td>(167;5,4)</td>
<td>(75;3,1)</td>
<td>(81;3,2)</td>
<td>(71;3,2)</td>
<td>(47;2,1)</td>
<td>(43;2,1)</td>
</tr>
<tr>
<td>0.020</td>
<td>(104;5,1)</td>
<td>(121;5,3)</td>
<td>(119;5,4)</td>
<td>(56;3,1)</td>
<td>(63;3,2)</td>
<td>(57;3,2)</td>
</tr>
<tr>
<td>0.025</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(89;5,2)</td>
<td>(78;5,2)</td>
<td>(93;5,4)</td>
<td>(45;3,1)</td>
</tr>
<tr>
<td>0.030</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(91;5,3)</td>
<td>(81;5,3)</td>
<td>(84;5,4)</td>
<td></td>
</tr>
<tr>
<td>0.035</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(58;5,1)</td>
<td>(62;5,2)</td>
<td></td>
</tr>
<tr>
<td>0.040</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(*** *)</td>
<td>(52;5,1)</td>
<td></td>
</tr>
<tr>
<td>0.045</td>
<td>(*** *)</td>
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<tr>
<td>0.050</td>
<td>(*** *)</td>
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</table>

CONCLUSION

When the process parameter increases, the sample size also increases. Tables providing the determination of the parameters using QSS are presented for some specified strength. QSS\(_{ZIPD}\) reduces both producer’s risk as well as consumer’s risk. Since the sample size is minimum in QSS than in SSP, it is suggested that, we can adopt this system to predict the minimum sample size as well as the cost reduction. Based on this system, the better outcome can be achieved in the shop floor situations.

REFERENCES


