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The Population Mean and its Variance in the Presence of Genocide for a Simple Birth-Death-Immigration-Emigration using the Probability Generating Function

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Abstract

This paper discusses on simple birth-death-immigration-emigration (BDIE) processes with genocide to stochastic differential equations (SDE) model. The general Solution for the distribution of the size of the population at any instant in time is obtained in the form of a probability generating function (PGF).The exact solution; mean and variance are derived for constant birth, death, immigration, emigration and catastrophe rates.

Key words: Genocide, Catastrophe, BDIE, SDE, PGF

1. INTRODUCTION

The works on birth-death type processes have been tackled mostly by some scholars such as Yule, Feller, Kendal and Getz among others. These fellows have been formulating the processes to model the behavior of stochastic populations.Recent examples on birth-death processes and stochastic differential equations (SDE) have also been developed. Granita [1], modeled linear growth birth and death processes with immigration and emigration using the stochastic differential equation. Getz [2], generally modeled birth-death processes with positive and negative controls using the probability generating function.The construction of the transient probabilities for a simple birth-death-immigration process under the influence of total catastrophe was approached by Randall[3] and in his paper a total catastrophe that wipes out the total population to size zero was considered. Di Crescenzo [4] also worked on birth-death process subject to catastrophes using the Laplace transform of its probability density function to obtain the mean and variance. As the application of the birth-death process, Sindayigaya [5]used the real data of the population of Rwanda to estimate the population dynamics using the mean and variance derived with the probability generating function. Moreover, Michael [6] modeled the immigration-emigration with catastrophe and found the steady-state solution using the classic recursive methods. However, the generating function model of birth-death-immigration-emigration process with genocide as a partial catastrophe has not been discussed in the previous works. In this paper, we shall consider a genocide which occurs at a constant rate and when it occurs, reduce the population to a certain level. Additionally, it appears that no attempt has been made to derive the mean and variance to obtain the explicit time-varying trajectory of the population. The purpose of this paper is to analyze the BDIE processes by introducing a genocide parameter and upon the differential difference equation; the general solution is obtained by using the probability generating function which finally leads to the determination of the mean and variance of the population.

2. MODEL DEVELOPMENT

The process is formulated by letting $N(t)$ represent the size of the population at time t and

$$P_n(t) = P\left[N(t) = n \middle| N(0) = 0 \right] \tag{2.1}$$

As in the simple birth-death process, births and deaths occur proportional to the population size with a birth rate $\lambda > 0$ and a death rate $\mu > 0$ .Immigration and emigration will occur independent of the population size with rates $\nu > 0$ and respectively $\alpha >$. Further, the occurrence of a genocide is also independent of population size and will occur at a rate $\gamma > 0$. Thus, the process can be described by the following transition rates:

Transition	Rate	
$n \to n+1$	$\lambda n + \nu$,	$n \geq 0$
$n \to n-1$	$\mu n + \alpha$,	$n \geq 1$
$n \to \gamma$	γ,	$n \geq 1$

By that the transient probabilities which is the differential difference equation is given by

$$P_n'(t) = \left[\mu(n+1)+\alpha\right]P_{n+1}(t) + \left[\lambda(n-1)+\nu\right]P_{n-1}(t) - \left[n(\mu+\lambda)+\nu+\alpha+\gamma\right]P_n(t) \tag{2.2}$$

Where $P_n'(t)$ denote the differentiation of $P_n(t)$ with respect to t. Since negative control whe

$n = 0$, we have

$$P_0'(t) = (\mu+\alpha)P_1(t) - \nu P_0(t) - \gamma P_0(t)$$
$$= (\mu+\alpha)P_1(t) - (\nu+\gamma)P_0(t). \qquad (2.3)$$

Letting

$$\phi(s,t) = \sum_{n=0}^{\infty} P_n(t) S^n, \qquad (2.4)$$

be the probability generating function(PGF)for the system, it follows from the standard generating function method that $\phi(s,t)$ satisfies the partial differential equation (PDE).

$$\frac{\partial \phi}{\partial t} = \left[\lambda s(s-1) + s\mu\left(\frac{1}{s}-1\right)\right]\frac{\partial \phi}{\partial s} + \left[\nu(s-1) + \alpha\left(\frac{1}{s}-1\right) - \gamma\right]\phi - \frac{\alpha}{s}P_0(t) \qquad (2.5)$$

The method of solution to (2.5) is sketched and leads to the form of generating function for the distribution of the size of the population at any time t, which here, stated in its most general form for constant parameters, appears to be a new result of the BDIE with genocide/catastrophe.

Finally the PDE (2.5) has solution

$$\phi(s,t) = \frac{\left[(\mu+\gamma-\lambda s) + \lambda(s-1)e^{(\lambda-\mu-\gamma)t}\right]^{-\nu/\lambda}}{\mu+\gamma-\lambda}$$
$$\times\left[\frac{(\mu+\gamma-\lambda s) + (\mu+\gamma)(s-1)e^{(\lambda-\mu-\gamma)t}}{s(\mu+\gamma-\lambda)}\right]^{\alpha/\mu+\gamma}$$
$$\times\left\{\sum_{n=0}^{\infty}\Upsilon_n\left[\frac{(\mu+\gamma-\lambda s)+(\mu+\gamma)(s-1)e^{(\lambda-\mu-\gamma)t}}{(\mu+\gamma-\lambda s)+\lambda(s-1)e^{(\lambda-\mu-\gamma)t}}\right]^n\right\} \qquad (2.6)$$

3. DETERMINATION OF THE MEAN AND VARIANCE FOR BDIE

It is known that the two most important moments of a distribution are the mean and variance, and these can be found quite easily if we put $s=1$ in (2.4) and derive twice. Then,

$$\frac{\partial \phi}{\partial s}(1,t) = \sum_{n=0}^{\infty} nP_n(t) = E(n) \qquad (3.1)$$

$$\frac{\partial^2 \phi}{\partial s^2}(1,t) = \sum_{n=0}^{\infty} n(n-1)P_n(t) = E\big(n(n-1)\big) \qquad (3.2)$$

which are the means and second factorial moment of the distribution, respectively. Let $\bar{n}(t)$ denote the mean at time t and $\sigma^2(t)$ the variance at time t. Since the variance is the second moment of the distribution about the mean, we have

$$\sigma^2(t) = E\left[n(n-1)\right] + \bar{n}(t) - \bar{n}^2(t) \qquad (3.3)$$

To facilitate the algebra involved in differentiating (2.6) once and then twice and setting $s=1$ after each differentiation, we can approach the problem step by step in a similar manner of [2] as follows:

Let,

$$f(s)=(\mu+\gamma-\lambda s)+\lambda(s-1)e^{(\lambda-\mu-\gamma)t} \tag{3.4}$$

$$g(s)=(\mu+\gamma-\lambda s)+(\mu+\gamma)(s-1)e^{(\lambda-\mu-\gamma)t} \tag{3.5}$$

Then,

$$f(1)=\mu+\gamma-\lambda$$

$$f'(s)=f'(1)=-\lambda+\lambda e^{(\lambda-\mu-\gamma)t}$$

$$f''(s)=f''(1)=0$$

$$g(1)=\mu+\gamma-\lambda$$

$$g'(s)=g'(1)=-\lambda+(\mu+\gamma)e^{(\lambda-\mu-\gamma)t}$$

$$g''(s)=g''(1)=0$$

Let

$$\psi(s,t)=\frac{f(s)}{\mu+\gamma-\lambda} \tag{3.6}$$

$$\kappa(s,t)=\frac{g(s)}{s(\mu+\gamma-\lambda)} \tag{3.7}$$

$$r(s,t)=\frac{g(s)}{f(s)} \tag{3.8}$$

Then,

$$\psi(1,t)=1$$

$$\psi'(1,t)=\frac{-\lambda}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}\right)$$

$$\psi''(1,t)=0$$

$$\kappa(1,t)=1$$

$$\kappa'(1,t)=\frac{\mu+\gamma}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)$$

$$\kappa''(1,t)=\frac{-2(\mu+\gamma)}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)$$

$$r(1,t)=1$$

$$r'(1,t)=e^{(\lambda-\mu-\gamma)t}$$

$$r''(1,t)=\frac{-2\lambda}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)$$

Rewriting (2.6) in terms of (3.6), (3.7) and (3.8) using (3.4) and (3.5) we have

$$\phi(s,t)=\psi^{-\nu/\lambda}\kappa^{\alpha/\mu+\gamma}\left\{\sum_{n=0}^{\infty}\Upsilon_n r^n\right\} \tag{3.9}$$

Having in mind that, if at time $t=0$, we know that the population has a distribution

$$P_i(0)=\Upsilon_i,\ i=0,1,2,...... \tag{3.10a}$$

$$\sum_{i=0}^{\infty}\Upsilon_i=1 \tag{3.10b}$$

(because $P_i(0)$ is a distribution),

$$mean=n_0 \tag{3.10c}$$

$$Variance=\sigma^2 \tag{3.10d}$$

Upon using (3.3) we have that

$$\sum_{n=0}^{\infty}n\Upsilon_n=n_0 \tag{3.11}$$

$$\sum_{n=0}^{\infty}n(n-1)\Upsilon_n=\sigma_0{}^2-n_0+n^2{}_0 \tag{3.12}$$

We can now proceed to get $\bar{n}(t)$ and $\sigma^2(t)$ by differentiating (3.9) once and then twice and setting $s=1$ in order to get the mean and variance respectively.

(i) Derivation of the Mean

$$\frac{\partial\phi}{\partial s}=\frac{-\nu}{\lambda}\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa^{\frac{\alpha}{\mu+\gamma}}\left\{\sum_{n=0}^{\infty}\Upsilon_n r^n\right\}+\frac{\alpha}{\mu+\gamma}\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}-1}\left\{\sum_{n=0}^{\infty}\Upsilon_n r^n\right\}+\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}}\left\{\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right\} \tag{3.13}$$

Using Equation (3.13) we obtain,

$$\bar{n}(t)=\frac{-\nu}{\lambda}\times\frac{\lambda}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)+\frac{\alpha}{\mu+\gamma}\times\frac{\mu+\gamma}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)+n_0e^{(\lambda-\mu-\gamma)t}$$

$$=n_0e^{(\lambda-\mu-\gamma)t}+\frac{\alpha-\nu}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right) \tag{3.14}$$

Hence,

$$\bar{n}(t)=n_0e^{(\lambda-\mu-\gamma)t}+\frac{\alpha-\nu}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right);$$

which is the explicit solution for time-varying trajectory of the mean for the BDIE process with catastrophe.

(ii) Derivation of the Variance

To compute the variance, we take the second derivative of (3.13) and replace $s=1$ .Then by separating the composing terms and take the second derivative separately, we have

$$\frac{\partial\phi_A}{\partial s}=\frac{-\nu}{\lambda}\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa^{\frac{\alpha}{\mu+\gamma}}\left\{\sum_{n=0}^{\infty}\Upsilon_n r^n\right\}$$

$$\frac{\partial \phi_B}{\partial s}=\frac{\alpha}{\mu+\gamma}\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}-1}\left\{\sum_{n=0}^{\infty}\Upsilon_n r^n\right\}$$

$$\frac{\partial \phi_C}{\partial s}=\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}}\left\{\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right\}$$

and

$$\frac{\partial^2 \phi}{\partial s^2}=\frac{\partial^2 \phi_A}{\partial s^2}+\frac{\partial^2 \phi_B}{\partial s^2}+\frac{\partial^2 \phi_C}{\partial s^2}.$$

In detail, we get

$$\frac{\partial \phi_A''}{\partial s^2}=\frac{-\nu}{\lambda}\left\{\begin{array}{l}\left[\left(\frac{-\nu}{\lambda}-1\right)\psi^{\frac{-\nu}{\lambda}-2}(\psi')^2\kappa^{\frac{\alpha}{\mu+\gamma}}\left(\sum_{n=0}^{\infty}\Upsilon_n r^n\right)\right]+\left[\frac{\alpha}{\mu+\gamma}\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa'\kappa^{\frac{\alpha}{\mu+\gamma}-1}\left(\sum_{n=0}^{\infty}\Upsilon_n r^n\right)\right]+\\ \left[\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa^{\frac{\alpha}{\mu+\gamma}}\left(\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right)\right]\end{array}\right\}$$

$$=\frac{-\nu}{\lambda}\left[\begin{array}{l}\left(\frac{-\nu}{\lambda}-1\right)\frac{\lambda^2}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2-\frac{\alpha}{\mu+\gamma}\times\frac{\lambda}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(\frac{\mu+\nu}{\lambda-\mu-\gamma}\right)\left(e^{(\lambda-\mu-\gamma)t}-1\right)\\ +\frac{\lambda}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)(1+n_0)\end{array}\right]$$

$$=\left[\begin{array}{l}\frac{\nu^2+\nu\lambda}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{\nu\alpha}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2-\frac{\nu n_0}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)\\ +\frac{\nu}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right] \quad (3.14a)$$

$$\frac{\partial \phi_B''}{\partial s^2}=\frac{\alpha}{\mu+\gamma}\left\{\begin{array}{l}\left[\frac{-\nu}{\lambda}\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa^{\frac{\alpha}{\mu+\gamma}-1}\left(\sum_{n=0}^{\infty}\Upsilon_n r^n\right)\right]+\left[\psi^{\frac{-\nu}{\lambda}}\left(\frac{\alpha}{\mu+\gamma}-1\right)\kappa^{\frac{\alpha}{\mu+\gamma}-2}(\kappa')^2\left(\sum_{n=0}^{\infty}\Upsilon_n r^n\right)\right]\\ +\left[\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}-1}\kappa''\left(\sum_{n=0}^{\infty}\Upsilon_n r^n\right)+\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}-1}\kappa'\left(\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right)\right]\end{array}\right\}$$

$$=\frac{\alpha}{\mu+\gamma}\left[\begin{array}{l}\frac{-\nu}{\lambda}\frac{\lambda}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2(\mu+\gamma)+\frac{\alpha-\mu-\gamma}{\mu+\gamma}\times\frac{(\mu+\gamma)^2}{(\mu+\gamma-\lambda)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2\\ +\frac{2(\mu+\gamma)}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)+\frac{\mu+\gamma}{\mu+\gamma-\lambda}(1+n_0)\left(e^{(\lambda-\mu-\gamma)t}\right)\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right]$$

$$=\left[\begin{array}{l}\frac{-\nu\alpha}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{\alpha^2-\alpha\mu-\alpha\gamma}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{2\alpha}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\\ -\frac{\alpha n_0}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)-\frac{\alpha}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right] \quad (3.14b)$$

$$\frac{\partial \phi_C''}{\partial s^2}=\left[\begin{array}{l}\frac{-\nu}{\lambda}\psi^{\frac{-\nu}{\lambda}-1}\psi'\kappa^{\frac{\alpha}{\mu+\gamma}}\left(\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right)+\frac{\alpha}{\mu+\gamma}\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}-1}\kappa'\left(\sum_{n=0}^{\infty}n\Upsilon_n r^{n-1}r'\right)\\ +\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}}\left\{\sum_{n=0}^{\infty}n(n-1)\Upsilon_n r^{n-2}(r')^2+\psi^{\frac{-\nu}{\lambda}}\kappa^{\frac{\alpha}{\mu+\gamma}}\left(n\Upsilon_n r^{n-1}r''\right)\right\}\end{array}\right]$$

$$=\left[\begin{array}{l}\frac{-\nu}{\lambda}\times\frac{\lambda}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)(1+n_0)\left(e^{(\lambda-\mu-\gamma)t}\right)-\frac{\alpha}{\mu+\gamma}\times\frac{\mu+\gamma}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)(1+n_0)\left(e^{(\lambda-\mu-\gamma)t}\right)\\ +\left(\sigma_0^{\ 2}+n^2{}_0-n_0\right)\left(e^{2(\lambda-\mu-\gamma)t}\right)+\frac{(1+n_0)}{\lambda-\mu-\gamma}2\lambda\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right]$$

$$=\left[\begin{array}{l}-\frac{(\nu-\alpha)}{\lambda-\mu-\gamma}n_0\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)+\left(\sigma_0^{\ 2}+n^2{}_0\right)\left(e^{2(\lambda-\mu-\gamma)t}\right)-n_0\left(e^{2(\lambda-\mu-\gamma)t}\right)\\ +\left\{(1+n_0)+(\mu+\gamma)(1-n_0)\left(e^{(\lambda-\mu-\gamma)t}-1\right)\right\}\end{array}\right] \quad (3.14c)$$

Recall that the variance is given as

$\sigma^2(t)=E\left[n^2(t)\right]-E\left[\left(\bar{n}(t)\right)\right]^2$ or $\sigma^2=E\left[n(n-1)\right]+\bar{n}(t)-\left[\bar{n}(t)\right]^2$ but

$E\left[n(n-1)\right]=\left.\frac{\partial^2\phi}{\partial s^2}\right|_{s=1}$. Then, $\sigma^2=\left.\frac{\partial^2\phi}{\partial s^2}\right|_{s=1}+\bar{n}-(\bar{n})^2$ (3.14d)

hence, by assembling (3, 14a), (3.14b)and (3.14c) and apply into (3.14d) we obtain

$$\sigma^2=\left\{\begin{array}{l}\left[\begin{array}{l}\frac{\nu^2+\nu\lambda}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{\nu\alpha}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2-\frac{\nu n_0}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)\\ +\frac{\nu}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right]\\ +\left[\begin{array}{l}\frac{-\nu\alpha}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{\alpha^2-\alpha\mu-\alpha\gamma}{(\lambda-\mu-\gamma)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{2\alpha}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\\ -\frac{\alpha n_0}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)-\frac{\alpha}{\lambda-\mu-\gamma}\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right]\\ +\left[\begin{array}{l}-\frac{(\nu-\alpha)}{\lambda-\mu-\gamma}n_0\left(e^{(\lambda-\mu-\gamma)t}-1\right)\left(e^{(\lambda-\mu-\gamma)t}\right)+\left(\sigma_0^{\ 2}+n^2{}_0\right)\left(e^{2(\lambda-\mu-\gamma)t}\right)-n_0\left(e^{2(\lambda-\mu-\gamma)t}\right)\\ +\left\{(1+n_0)+(\mu+\gamma)(1-n_0)\left(e^{(\lambda-\mu-\gamma)t}-1\right)\right\}\end{array}\right]\\ +\left[n_0\left(e^{(\lambda-\mu-\gamma)t}\right)\right]+\left[\begin{array}{l}\frac{\alpha-\nu}{\mu+\gamma-\lambda}\left(e^{(\lambda-\mu-\gamma)t}-1\right)-n^2{}_0\left(e^{2(\lambda-\mu-\gamma)t}\right)-\frac{(\alpha-\nu)^2}{(\mu+\gamma-\lambda)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2\\ -\frac{2(\alpha-\nu)}{\mu+\gamma-\lambda}n_0\left(e^{(\lambda-\mu-\gamma)t}\right)\left(e^{(\lambda-\mu-\gamma)t}-1\right)\end{array}\right]\end{array}\right\}$$

And finally, upon simplification we get

$$\sigma^2(t)=\sigma^2{}_0e^{2(\lambda-\mu-\gamma)t}+\frac{\nu\lambda-\alpha\mu-\alpha\gamma}{(\mu+\gamma-\lambda)^2}\left(e^{(\lambda-\mu-\gamma)t}-1\right)^2+\frac{\lambda+\mu+\gamma}{\lambda-\mu-\gamma}n_0e^{(\lambda-\mu-\gamma)t}\left(e^{(\lambda-\mu-\gamma)t}-1\right) \quad (3.15)$$

Equation (3.15) is the explicit solution for time-varying trajectory of the variance for the BDIE processes with genocide or catastrophe. In the case that $\gamma=0$, Equations

(3.14) and (3.15) will turn up to BDIE without catastrophe and will have a close relationship with the results obtained by [1] and [2].

SUMMARY

In this work, we have established the partial differential equation (PDE)for the BDIE processes with genocide/catastrophe from which the general form was obtained using the probability generating function (PGF).Upon derivation of the PGF, the mean and Variance functions for BDIE processes with genocide/catastrophe were effectively determined.

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