Numerical Modeling of Transport of Tracers in Two-Phase Flow through Porous Media for Interwell Chemical Tracer Tests

Jayaprakash S. Radhakrishnan¹, Balasubramanian Senthilmurugan¹, and Victor Arana²

¹Research & Development Team
Kuwait Oil Company, P.O Box 9758, Ahmadi, Postal Code 61008, Kuwait
²Rahxion, Monrovia 907, Col Portales, Mexico City 03300, Mexico
*Corresponding Author Email: sr_jp@yahoo.com Phone: +965 23872968

Abstract

Interwell tracer tests are used to obtain key information about a reservoir such as fluid flow characteristics, residual oil saturation and to determine the optimum recovery enhancement strategy. In this study, a 2D numerical model is developed for the flow of tracers in a two-phase flow in hydrocarbon reservoirs. The mathematical model includes viscous, capillary and gravity forces as well as hydrodynamic dispersion (molecular diffusion and mechanical dispersion) of the tracers in the porous medium. A fully implicit formulation is used to solve both the flow equation and the transport equation. Saturation, pressure and phase velocities are calculated implicitly at each time-step. The model is used to estimate the breakthrough times of partitioning and non partitioning tracer and the residual oil saturation in multi well tracer tests. Sensitivity analysis was performed to study the effect of parameters such as partitioning and adsorption of the tracers.

Keywords: Tracers, Inter well tracer test, Enhanced Oil Recovery, Reservoir Simulation, Flow Modeling, Porous Media, Residual Oil Saturation

1. INTRODUCTION

Tracers have been used in the oil field to qualitatively and quantitatively gauge how fluid flows through the reservoir, as well as being a useful tool for estimating residual oil saturations. They are used for tracking the fluid movements during single well and inter-well tracer tests [1], which offers information on the efficacy of the injection method used for enhanced oil recovery. Tracers are used to obtain the reservoir
description that includes volumetric sweep of the injected fluid, directional flow trends and identification of flow barriers [2]. Tracers are compounds injected in a well, with the main aim of producing them from the same well (single well tracer test, SWTT) or from a different well (inter well tracer test, IWTT), and to be detected and quantified in the produced stream. In IWTT, the tracer is injected at one well along with carrier fluid (water in a waterflood or gas in a gasflood) and detected at a producing well after a period of time, which can be anything from days, weeks, or months. The most common tracers are radioactive material such as tritiated water or chemical tracers such as fluorinated benzoic acids. Emerging technologies include use of nanoparticles as tracers which can be engineered to suit the reservoir conditions [3, 4]. A successful tracer application involves the modeling of tracer transport incorporating the pertinent phenomena. In the reservoir, the fluids are distributed based on the characteristics of the formation and according to their particular physical properties. In case of two-phase flow of the oil and water phases, if the saturation conditions of the water and oil allow the flow of each of the phases, they will begin to flow when there is a potential difference, which is caused by the production or injection of fluids into the well. Depending on the characteristics of the rock and the fluids, it may be water or oil, which is the wetting fluid. If natural flow conditions do not allow oil to travel to wells, it is possible to release trapped oil by displacement with water. In the case of water injection, the water behaves as a displacement front that pushes the oil towards the producing wells; As the water saturates the pores of the rock, the oil content decreases until it reaches residual oil saturation. As a secondary recovery process, water injection not only benefits the oil displacement, but also helps to maintain reservoir pressure. Tracer studies help in designing and optimizing a water injection project by evaluating parameters such as sweep efficiencies, break through time, residual oil saturations and water cut. Single well tracer tests could be modeled numerically to obtain the residual oil saturations, involving rigorous simulations [5,6]. The following sections describe the 2-D mathematical modeling of tracer transport in a reservoir porous media in a two-phase flow.

2. THEORY
2.1. Mathematical Model
The flow of oil and water can be modeled from the differential equations that represent their behavior. As with most problems when modeling fluid flow, it is necessary to consider the fundamental principles of conservation of mass and conservation of momentum.

The continuity equation (conservation of mass) for two phase fluid flow through a porous medium is given by

\[ \nabla \cdot (\rho_\alpha \vec{u}_\alpha) + \vec{q}_{m,\alpha} = -\frac{\partial (\phi S_\alpha \rho_\alpha)}{\partial t} \]  

(1)

where \( \rho_\alpha \) represents the density, \( \phi \) the porosity of the rock, \( S_\alpha \) the saturation of the phase, \( \vec{u}_\alpha \) the velocity, \( \vec{q}_{m,\alpha} \) the mass flow per unit volume, \( t \) the time and \( \alpha \) is the subscript that represents any of the phases (water or oil). The mass flow \( \vec{q}_{m,\alpha} \) can represent the
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source or sink term which takes into account the input or output of mass in the system. The density of the phases \( \rho_\alpha \) can be expressed explicitly or implicitly through an equation of state.

From Eq. 1, the mass flow rate \( \dot{m}_\alpha \) of phase \( \alpha \) in terms of the rock volume \( V_b \) is given as

\[
\dot{m}_\alpha = \frac{q_{m,\alpha}}{V_b}
\]

The velocity through the porous medium can be written using Darcy’s law based on the relative permeabilities of the various phases as

\[
\bar{u}_\alpha = -\frac{k k_{r,\alpha}}{\mu_\alpha} \left( \bar{\nabla} p_\alpha - \gamma_\alpha \bar{\nabla} D \right)
\]

where \( k \) is the absolute permeability, \( k_{r,\alpha} \) is the relative permeability of the ‘\( \alpha \)’ phase, \( \mu_\alpha \) the viscosity, \( p_\alpha \) the pressure, \( \gamma_\alpha \) the specific gravity, \( \phi \) the porosity, \( S_\alpha \) the saturation, \( D \) the depth or vertical distance and \( \alpha \) the subscript that represents any of the phases (water or oil).

The formation volume factor \( (B) \) and the shrinkage factor \( (b) \) of the fluid are given by

\[
B_\alpha = \frac{\rho_{a,c.s.}}{\rho_\alpha}
\]

(4)

\[
b_\alpha = \frac{1}{B_\alpha}
\]

(5)

Substituting equations 3, 4 and 5 in equation 1 and dividing the equation by the density at standard conditions \( (\rho_{a,c.s.}) \), the general model that represents the flow of phase ‘\( \alpha \)’ is obtained as

\[
\bar{\nabla} \cdot \left[ -\frac{k k_{r,\alpha}}{B_\alpha \mu_\alpha} \left( \bar{\nabla} p_\alpha - \gamma_\alpha \bar{\nabla} D \right) \right] + \frac{q_{a,c.s.}}{V_b} = -\frac{\partial}{\partial t} (\phi S_\alpha b_\alpha)
\]

(6)

Equation 6 can be used to model the flow of oil \( (\alpha = 'o') \) and water \( (\alpha = 'w') \) phases through the porous medium. The first term in the left side of the equation represents the flow term, the second term represents the source or sink term. The right side of the equation represents the accumulation or the mass change per pore volume in the reservoir.

2.2. Model Constraints

In case of two phase flow (oil and water), the sum of the water saturation \( (S_w) \) and oil saturation \( (S_o) \) will cover 100% the porous medium (Equation 7). It is also necessary to represent the interaction that fluids have with each other and with the rock; This interaction is due to the effects of interfacial tension and wettability of the rock that give rise to capillary effects. The capillary water-oil pressure \( (P_{cwo}) \) is is the difference between the oil and water phase pressures (Equation 8).

\[
S_w + S_o = 1
\]

(7)

\[
P_{cwo} = p_o - p_w
\]

(8)

2.3. Numerical Modeling

Equation 6 can be solved numerically to define the flow of oil and water phases through the porous medium. A numerical solution for equation 6 was developed using finite
difference method. The equation is discretized in space and time using central differencing. The nodes are placed in the center of each cell, as shown in Figure 1. At the boundaries (x=0 and x=L), flow and pressure gradients of both phases are set to zero as boundary conditions. A fully implicit formulation is used to solve the flow equations. The non-linear set of equations are solved using Newton's method and Taylor series approximation with pressure and saturation as the unknown variables. The system of equations is then represented in the form of a matrix of derivatives (Jacobian). The well model of Peaceman [7] is used for the relationship between the cell pressure and the bottom pressure, defining an equivalent radius according to the geometry of the cell where the well is placed.

![Figure 1. Convention of nodes in the x direction (nodes centered).](image)

### 2.4. Interwell tracer test model (2D Cartesian model)

Modeling the transport of tracers through the porous medium of oil reservoirs helps in the validation and interpretation of tests inter-well tracer tests, and to determine heterogeneities, connectivity between wells and carry out studies prior to the implementation of secondary and improved recovery methods such as the estimation and validation of residual oil saturation ($S_{wr}$).

The transport of the tracer through the oil and water phase is defined by equation 9.

$$\nabla \left[ \phi S_w D \ast \nabla C_{w}^{nw} + \phi S_o D \ast \nabla \left( K_{q}^{\phi} C_{q}^{nw} \right) \right] - \nabla \left( v_w C_{w}^{nw} + v_o K_{q}^{\phi} C_{q}^{nw} \right)$$

$$= \frac{\partial}{\partial t} \left( \phi S_w C_{w}^{nw} + \phi K_{q}^{\phi} S_o C_{q}^{nw} + \phi K_{ad} C_{q}^{nw} \right), \tag{9}$$

Where $C$ is the concentration of the tracer, $D^*$ is the diffusion or dispersion coefficient, $K_{ad}$ is the coefficient of adsorption, $K_{q}^{\phi}$ is the partition coefficient of the tracer particles (q) between the oil and water phases, $S$ is the saturation and $\phi$ is the porosity. Subscript ‘o’ and ‘w’ denote the parameter in the oil and water phase respectively. Equation 9 is solved using central finite differences in space and regressive differences in time.

The geometry of the reservoir grid is shown in figure 2. It simulates a 5-point pattern with 10 cells each in the x and y direction (10 nodes), with the injector in the (1,1) location and the producers at (10,10) location. Figure 2 shows the top-right quadrant of the 5-point pattern. The geometry has one cell in the z direction. The numerical model is solved using Matlab.
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3. RESULTS AND DISCUSSION

3.1. Problem Setup

Injection and production of a tracer particles were simulated to estimate residual oil saturation ($S_{or}$). The simulation process was performed on a 2D model in directions of flow x and y. Tables 1 and 2 provide the dimensions of the simulation mesh and the characteristics of the reservoir and the fluids.

Table 1. Characteristics of the Reservoir and problem setup

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length in x direction</td>
<td>450 ft</td>
<td>ft</td>
<td></td>
</tr>
<tr>
<td>Length in y direction</td>
<td>450 ft</td>
<td>ft</td>
<td></td>
</tr>
<tr>
<td>Length in z direction (vertical)</td>
<td>30 ft</td>
<td>ft</td>
<td></td>
</tr>
<tr>
<td>Initial water saturation</td>
<td>0.25</td>
<td>fraction</td>
<td>All cells</td>
</tr>
<tr>
<td>Critical water saturation</td>
<td>0.25</td>
<td>fraction</td>
<td>All cells</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.20</td>
<td>fraction</td>
<td>All cells</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>$3 \times 10^{-6}$</td>
<td>psia$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Initial reservoir pressure (cells)</td>
<td>3000</td>
<td>psia</td>
<td>All cells</td>
</tr>
<tr>
<td>No. of cells in x direction</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of cells in y direction</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of cells in z direction</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation time</td>
<td>2</td>
<td>years</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. 2-D Reservoir grid used in the model (I: Injector, P: Producer)
Table 2. Characteristics of the wells

<table>
<thead>
<tr>
<th>Well</th>
<th>Condition</th>
<th>Value</th>
<th>Units</th>
<th>Cell Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector</td>
<td>Constant pressure injection</td>
<td>3200</td>
<td>psia</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Producer</td>
<td>Constant pressure production</td>
<td>2600</td>
<td>psia</td>
<td>(10, 10, 1)</td>
</tr>
<tr>
<td></td>
<td>Injection period</td>
<td>100-101</td>
<td>days</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Injection concentration</td>
<td>0.01</td>
<td>%vol</td>
<td></td>
</tr>
</tbody>
</table>

The procedure consists of injecting water continuously for an adequate time to reach the residual oil saturation of the reservoir, and subsequently injecting a water patch with two tracers, one of them soluble in both water and oil (partitioning, $K > 0$) and the other only soluble in water (non-partitioning, $K = 0$). This is followed by continuous water injection until the tracer is recovered in the producing wells. An initial time step of $Dt = 1$s is used, which is incremented at 5% every time step, to reduce the number of time steps. As the cumulative time reaches the start and end of the tracer injection, the time step is reset to 1s and allowed to increase again.

3.2. Residual Oil Saturation

The separation of the tracer curves is a function of both the amount of oil present in the swept zone and the partition coefficient. Hence, residual oil saturation ($S_{or}$) can be determined from the difference between the first temporal moments of the respective tracer breakthrough curves [8, 9]. The residence time of partitioned tracer in the reservoir is higher than that of the unpartitioned tracer and the breakthrough of the unpartitioned tracer will be earlier. The residual oil saturation is calculated from the difference in the arrival times between the partitioned ($t_p$) and the unpartitioned ($t_{np}$) tracer, as follows:

$$S_{or} = \frac{1}{1 + \frac{K}{\Delta tr}}$$

(10)

where

$$\Delta tr = \frac{t_p - t_{np}}{t_{np}}$$

(11)

Figures 3, 4 and 5 show the rate of production of oil and water and the injection of water. Breakthrough of water through the injector is observed at 23 days. The injection and production processes were carried out with wells producing under the condition of constant bottom pressure. Figure 6 shows the concentration of the tracer recovered from the producing well for the partitioning ($K=3$) and the non-partitioning ($K=0$) tracer. The arrival times of the tracers are obtained from the peak time of the concentration plot. Table 3 shows the calculated arrival times of the tracers and the resulting residual oil saturation.
Table 3. Tracer arrival times and Residual oil saturations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival time for partitioned tracer, $t_p$</td>
<td>350.2108</td>
<td>days</td>
</tr>
<tr>
<td>Arrival time for non-partitioned tracer, $t_{np}$</td>
<td>206.6896</td>
<td>days</td>
</tr>
<tr>
<td>Arrival time difference, $\Delta t_r$</td>
<td>0.6944</td>
<td>-</td>
</tr>
<tr>
<td>Residual Oil Saturation, $S_{or}$</td>
<td>0.188</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 3. Oil production rate

Figure 4. Water production rate
Conservation of mass balance of the tracer was checked by verifying that the amount of tracer injected is equal to the amount of tracer produced plus tracer remaining in the reservoir. Figures 7, 8, 9 and 10 show the distribution of water saturation and concentration of non-partitionable tracer at different simulation times.
Figure 7. Water saturation and concentration of non-partitionable tracer at 54.2357 days

Figure 8. Water saturation and concentration of non-partitionable tracer at 110.7546 days

Figure 9. Water saturation and concentration of non-partitionable tracer at 150.3049 days
Figure 10. Water saturation and concentration of non-partitionable tracer at 206.6896 days

3.3. Effect of Tracer Partition
The partition coefficient strongly influences the tracer flow behavior, increasing the difference between arrival times. This is due to the fact that the higher the partition coefficient, the more tracer is propagated in the oil phase. This results in an increased arrival times of the partitioned tracer (indicated by the shift of the concentration peaks to the right). Figure 11 shows the effect of partition coefficient on the production behavior of the tracer. Table 4 shows the results in the calculation of residual oil saturation for each test.

Figure 11. Effect of partition coefficient on tracer production concentration
Table 4: Residual oil saturation calculated for different tracer partition coefficients.

<table>
<thead>
<tr>
<th>Test</th>
<th>( S_{or} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original, ( K = 3 )</td>
<td>0.188</td>
</tr>
<tr>
<td>( K = 2 )</td>
<td>0.1952</td>
</tr>
<tr>
<td>( K = 1 )</td>
<td>0.1918</td>
</tr>
<tr>
<td>( K = 0.5 )</td>
<td>0.1768</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS
A 2-D simulation model was constructed to study the transport of tracers through a porous medium during an inter-well tracer test. The model was solved numerically using finite difference method with implicit formulation. The effect of partitioning of the tracer in oil and water phases were studied. The arrival time of the partitioning tracer was longer than the non-partitioning tracer. The residual oil saturation after the water injection process was calculated from the model. The model can be utilized to study the injection of engineered nanoparticles into the reservoir and the interaction of the particles with the rock and fluids.

Declaration of interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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