

Calculation of Source-detector Solid Angle, Using Monte Carlo Method, for Radioactive Sources with Various Geometries and Cylindrical Detector

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Abstract

In this paper source-detector solid angle calculation has been studied by Monte Carlo method, and a computer program is represented. Since most experimental works in nuclear physics are done by using of cylindrical detectors, the solid angle of this type of detector is calculated for various sources. The source in various shapes of a point, cylinder, sphere, or rectangle, may be located in any position out of the counter. In this calculation, we can obtain both window and lateral face solid angles of detector, as well as addition of them.

Keywords: source-detector solid angle, Monte Carlo calculation, random number

Introduction

In general, the result of decay rate measurement of a radioactive source (by any counting system) is less than the real number of the emission of that source. The main and important factors that cause this difference are: self absorption of the source, absorption and scattering of the particles in air, absorption and scattering of the particles in the window of detector, geometry of counting system, dead time of the detector, and efficiency of the detector.

To find actual number of radiation, we must consider the effect of these factors and apply necessary corrections. Among over parameters, geometric conditions, which is determined a parameter named geometric factor or solid angle, is important. The shape and size of source and detector, distance between them, and their relative position determine the solid angle. Finally, the real solid angle is the angle subtended by the detector at the position of the source divided by 4π . In other words, the solid

angle determines the fraction of particles emitted from the source, which have the chance of being counted by the detector.

There are also some computer codes that they can calculate not only other quantities of transport particles but also the solid angle, such as, the MCNP[7] and the EGS4[8] codes. According to special format of these codes, an user must write an input file that it spends too much time for beginners and experimenters, but this code are written by us, dose not have this difficult, because it asks only five questions from user then it calculates the solid angle.

Analytic calculation of solid angle

Common analytic methods for calculation of solid angle are useful only for sources with simple geometries. Example are: a point source located on the axis of a detector with circular aperture [1,2,3], a point source in front of a detector with rectangular aperture [1], and a disk source parallel to a detector with a circular aperture [1,2]. In all of these limited cases, except the point source located on the axis of circular aperture, not only the presented equations are long, but also the results are almost approximate and with errors. Also, some equations are for special cases, as for a disk source parallel to a detector with circular aperture (Fig. 1), the ratio of both source and detector's aperture radius to distance of each other must be smaller than unity [1].

But for any other arrangement of source-detector, such as point source that is not located on the axis of detector, or a cylindrical, spherical or cubic source which is located in any position, analytic calculation is very complicated, with great error, and sometimes impossible, while we can calculate solid angle for all mentioned geometries with high accuracy by using Monte Carlo method. It is enough to use the definition of solid angle. The only error occurred in this way is statistical errors due to random events. This error can be reduced by increasing the number of random events (number of particles emitted from the source).

Monte Carlo simulation

Since Monte Carlo method is based on random events, the first step of solving a problem using this method is production of random numbers with monotonous distribution and long period (this can be done by using defined computer subroutines such as RAN3 (IDNM), as used in this program). The second step is simulation of both source and detector geometries and their set up, according to experimentalist's offers. For this purpose, in this project a computer subroutine is programmed, which simulates at first a point source, and then with expanding it, a spherical, cylindrical, or cubic source beside a cylindrical detector. Finally all of the subroutines are evoked in a main program and the solid angle is calculated.

In this code user can define the parameters of source and detector arbitrarily and according to the type of experiment. The important point that user must take into account in definition of parameters, is that the detector and source must not overlap in volume.

Simulation of an isotropic point source

Each ray emitted from an isotropic point has a direction distributed monotonously in the whole of the space and in all directions. If we produce random directions with monotonous distribution, we will be able to attribute any direction to a ray emitted from the source.

Since the problem is symmetric in spherical coordinate system, assuming the point source s is in the origin of coordinate system, number of rays incident on unit area of the surface of a sphere with a source at its center radius R , are equal in all of the points on the sphere surface. Then solid angle element $d\Omega$ stated in equation (1) will have monotonous distribution.

$$d\Omega = \sin(\theta)d\theta d\varphi = d(-\cos(\theta))d\varphi \quad (1)$$

Elements $d(-\cos(\theta))$ and $d\varphi$ have isotropic distributions. Because θ and φ are in the range of $[0, \pi]$ and $[0, 2\pi]$ respectively, assuming u and u' as two random numbers with isotropic distribution in the range of $[0, 1]$, θ and φ can be defined as

$$\theta = \cos^{-1}(2u-1) \quad (2)$$

$$\varphi = 2\pi u'$$

Each pair of (θ, φ) having isotropic distribution, is related to a dispatched ray from the source.

Solid angle calculation for a point source

If the base of the detector is located at $z = 0$ plane and its window at $z = d$, assuming axis of detector is z axis, then equation stating detector volume is:

$$\begin{aligned} x^2 + y^2 &\leq r^2 \\ 0 &\leq z \leq d \end{aligned} \quad (3)$$

For collision of the radiated ray and detector, a hypothetical line that defines the path of a ray from the source, must pass through the detector's volume. Parameter equation of this line is:

$$\begin{aligned} x &= at + x_s \\ y &= bt + y_s \\ z &= ct + z_s \end{aligned} \quad (4)$$

Where x_s , y_s , and z_s are source coordinates and a , b , and c are defined as follows:

$$\begin{aligned} a &= \sin(\theta)\cos(\varphi) \\ b &= \sin(\theta)\sin(\varphi) \\ c &= \cos(\theta) \end{aligned} \quad (5)$$

Combining equations 3, 4, and 5, we can obtain the lines leaving the source toward the detector.

For solid angle calculation, if point source radiates gamma rays, gamma photons can enter not only from detector's window, but also from lateral face of the detector (Fig. 3). In this case the solid angle of the subtended detector from the position of the source is equal to summation of window and lateral face solid angles.

If we assume that N particles emitted from the source, in other words N random directions are produced, N_w particles will enter from the window and N_a will enter from lateral face of the detector. So solid angles Ω_w , Ω_a , and Ω_{total} are defined as:

$$\begin{aligned}
\Omega_w &= N_w/N \\
\Omega_a &= N_a/N \\
\Omega_{total} &= \Omega_w + \Omega_a = (N_w + N_a)/N
\end{aligned}
\tag{6}$$

Cylindrical and plane circular sources simulation

We assume that the axis of cylindrical detector with radius r' and height d' is coincident with z' axis of $x'y'z'$ coordinate system. Now it can be assumed that many point sources are distributed monotonously in the volume of the cylindrical detector and each of them emits isotropic in all directions.

In this case, we must produce random points with monotonous distribution in cylinder volume, then consider each of them as a point source and do as previous section.

First we produce three random numbers (u , u' , and u'') with monotonous distribution in the range of $[0,1]$ then define x' , y' and z' as:

$$\begin{aligned}
x' &= r' (2u - 1) \\
y' &= r' (2u' - 1) \\
z' &= d' u''
\end{aligned}
\tag{7}$$

If $x'^2 + y'^2 \leq r'^2$, then the produced points are in the volume of the source.

Now s' with coordinates of (x',y',z') is defined and we must transmit it to xyz coordinate system. If we assume (x_0,y_0,z_0) is the coordinates of the origin of $x'y'z'$ coordinate system relative to xyz coordinate system, knowing that the two coordinate systems have parallel axis and with same senses, we can write:

$$\begin{aligned}
x &= x_0 + x' \\
y &= y_0 + y' \\
z &= z_0 + z'
\end{aligned}
\tag{8}$$

Then we do calculations the same as a point source.

Note that if the height of the cylindrical source approaches zero, it will be a circular plane source.

Cubic and rectangular sources simulation

We do like to previous section, with defining x' , y' , and z' as:

$$\begin{aligned}
x' &= au \\
y' &= bu' \\
z' &= cu''
\end{aligned}
\tag{9}$$

Where a , b , and c are cubic source dimensions.

If one of the cubic source dimensions approaches zero, cubic source will be a rectangular plane source.

Spherical source simulation

We do like previous sections, with defining x' , y' , and z' as:

$$\begin{aligned}
x' &= r' (2u - 1) \\
y' &= r' (2u' - 1)
\end{aligned}
\tag{10}$$

$$z' = r' (2 u'' - 1)$$
 If $x'^2 + y'^2 \leq r'^2$, then the produced point is in the volume of the source.
 In these equations r' is the radius of the source.
 If r' approaches zero, spherical source will be a point source.

Results and discussion

For testing computer program, the results of some of the calculations are compared with the results of simple geometries for which there are analytical relations. These results are tabulated in tables 1 and 2. For a circular plane source, which is coaxial with the detector, variation of solid angle with respect to distance from window of detector is calculated with this computer and compared with the results of two approximate analytical relations [1,2]. These results are shown in (fig. 3). Calculations are done for $r' = 4$ cm (radius of the source) and $r = 2$ cm (radius of the detector). It is seen that the results from each of the three methods are same for large distance from the window of the detector, but in small distance from the window of the detector, the differences are considerable. If the source comes very close to the window of detector and the radius of the detector is smaller than the radius of the source, since all the emitted rays from the back face of the source could not reach the detector, the solid angle will be equal to half of the ratio of square radius of the source to square radius of the detector. Therefore solid angle will be 0.125 for the radii of the detector and the source in this calculation, so the result of computer program is acceptable. In these tables, it is seen that statistical error decreases with increasing the number of particles. Two other sample calculations, the spherical and cubic sources, for which there is no analytical relation, are done and their results are shown in Fig. 4 and Fig. 5. Those are other sample calculations of variation of solid angle with respect to distance from the window of the detector, and they are compared with the point source. As it is shown with increasing distance of sources from the window the detector, the results are comparable with those of the point source.

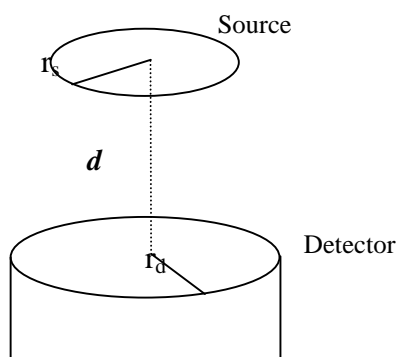


Figure 1: The circle source in front of the cylindrical detector and coaxial with it.

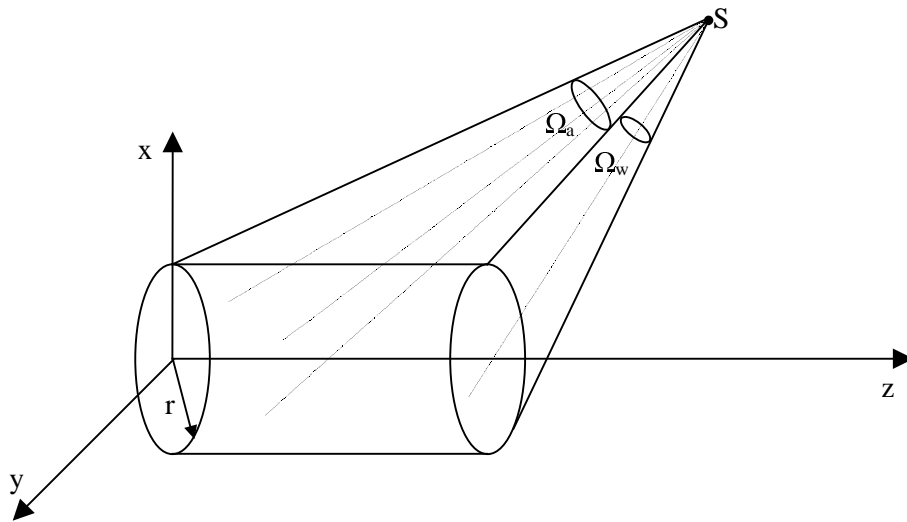


Figure 2: The Solid angle of the window and the around detector.

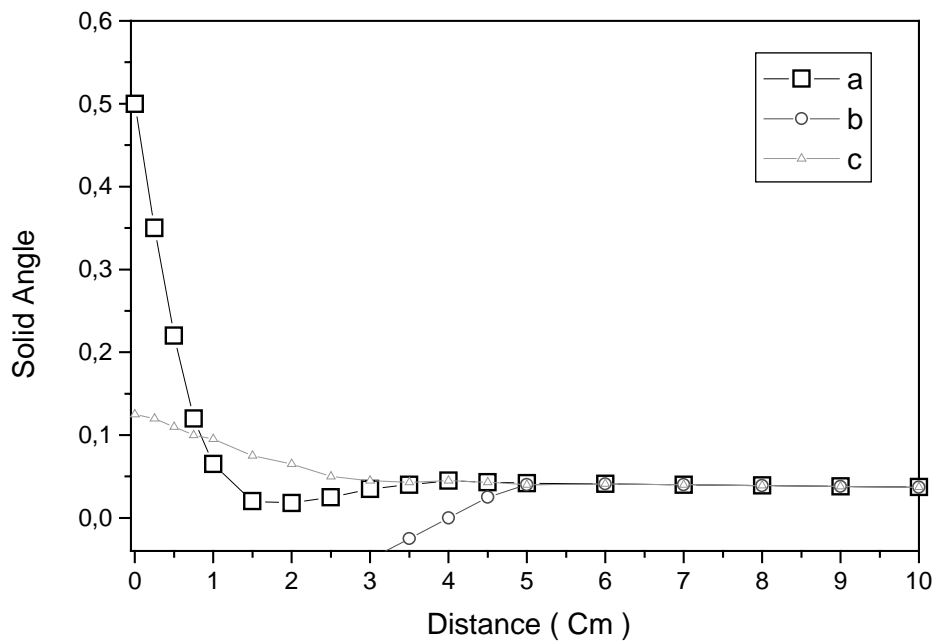


Figure 3: The variation of the solid angle of the circle source versus distance of the window of detector

(a) Using of Analytical relation of reference (2); (b) Using of Analytical relation of reference (1); (c) Using of this program ($r = 2$, $r' = 4$).

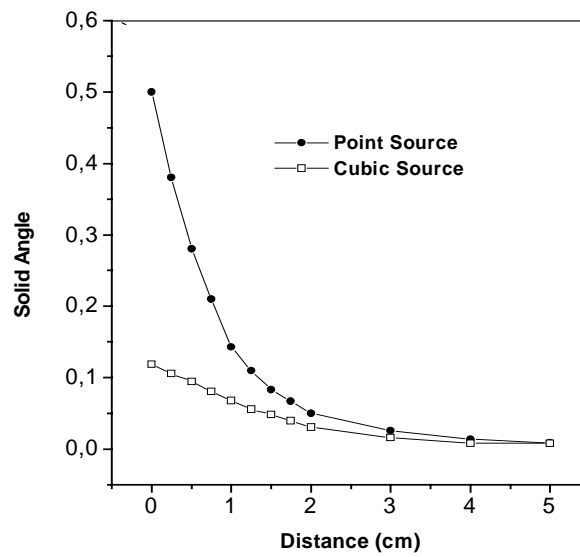


Figure 4: Comparing of the variation the solid angle of the point source and the cube source versus the distance of the window detector.

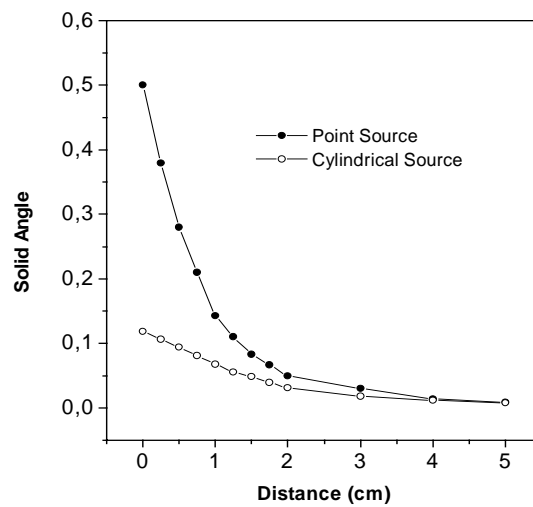


Figure 5: Comparing of the variation the solid angle of the point source and the cylindrical source versus the distance of the window detector.

Table 1: The solid angle subtended by a cylindrical detector from a point source, analytical equation and computer program $r = 1$, $h = 10$.

Coordinates of the source XYZ	Solid angle of the window of the detector	Solid angle of the lateral face of the detector	Solid angle using analytical equation
(0,0,10)	0.499992 ± 0.000071	0	0.5
(0,0,20)	0.002484 ± 0.000068	0	0.002481
(0,0,0)	0	0.500314 ± 0.000071	nothing
(0,0,-10)	0	0.002479 ± 0.000066	nothing

Table 2: The solid angle subtended by a cylindrical detector from a circular source, analytical equation and computer program $r = 1$, $h = 10$.

Origin of x'y'z' coordinate system	Radius of the source	Solid angle of the window of the detector	Solid angle of the lateral face of the detector	Solid angle using analytical equation
(0,0,10)	0.5	0.49993 ± 0.00022	0	0.5
(0,0,20)	0.5	0.00246 ± 0.00001	0	0.002012
(0,0,10)	2	0.12507 ± 0.00011	0.08944 ± 0.00094	nothing
(0,0,10 ⁻⁶)	0.5	0	0.49923 ± 0.00022	nothing
(0,0,10 ⁻⁶)	2	0	0.21375 ± 0.00014	nothing

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