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Stress Analysis of Variable Thickness Rotating FG Disc

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***Abstract*— An accurate solution is presented to investigate the stresses and strains in the rotating FGM disc having variable thickness. The material properties of the FGM disc are assumed to vary along the radial direction. The effect of varying disc thickness profile and distribution of reinforcement gradient in the disc are investigated on the elastic stresses and strains in the disc. It is observed that the stresses and strains in the disc are significantly affected by varying disc thickness profile and reinforcement gradient. The FGM disc having decreasing thickness along radial direction exhibits the lowest strains, which reduces further with the rise in disc thickness gradient.**

***Index Terms*— Variable thickness, Disc, Stress, Functionally Graded Material.**

I. INTRODUCTION

FUNCTIONALLY Graded Material (FGM) is a kind of composite material in which the constituent and/or their structure vary gradually along certain directions so as to achieve the desired variation in properties [5-6]. Rotating disc is a widely used component in many engineering applications. In most of these applications, the disc is subjected to severe mechanical loadings, thereby causing significant deformations. Therefore, it attracts the attention of many researchers [1-3].

Horgan and Chan [7] investigated the effects of material inhomogeneity on the stresses in constant thickness solid circular discs rotating. They observed that the stresses in inhomogeneous disc are significantly different from those observed in their homogeneous counterparts. Durodala and Attia [8] performed elastic analysis for rotating hollow and solid FGM discs of constant thickness, subjected to centrifugal load. The study also reveals that the different forms of reinforcement gradation modify the stresses and displacements in the FGM discs as compared to uniform composite disc. Zenkour [9] presented a solution to estimate the distribution of elastic stresses in rotating FGM disc of constant thickness. The study indicates that the stresses and deformations in the FGM disc are lower when the elastic modulus and density are more near the inner radius than those noticed towards the outer radius. Callioglu [10] obtained closed-form solutions for rotating annular FGM disc subjected to internal as well as external pressure. The elastic modulus and density of the disc were assumed to vary radially according to different power law functions. It is observed that with the increase in property gradient, the stresses and deformations in the disc change significantly. Peng and Li [11] performed elastic analysis for a constant thickness rotating sandwich solid disc made of FGM. The numerical results obtained are observed to be in excellent agreement with the exact solution, for some specific power-law gradients.

The literature consulted so far reveals that attempts have been made to analyze elastic stresses in FGM disc having constant thickness. The variable thickness disc performs better than a constant thickness disc [12]. Therefore it has been decided to investigate the effects of varying disc thickness gradient on elastic stresses and strains in a variable thickness rotating FGM disc.If your paper is intended for a conference, please contact your conference editor concerning acceptable word processor formats for your particular conference.

II. DISC PROFILE AND MATERIAL PROPERTIES

When Let us consider a variable thickness functionally graded rotating disc with the inner radius a (=0.04 m) and the outer radius b (=0.1 m). The thickness h(r) of the disc is assumed to vary radially, according to the following relation,

$$h(r) = h_b\left(\frac{r}{b}\right)^k \qquad (1)$$

where h_b is thickness of disc at the outer radius and k is the disc thickness gradation index.

The volume content of silicon carbide particles (SiCp) in the FGM disc is assumed to decrease along the radial direction,

$$V(r) = V_o\left(\frac{r}{b}\right)^n \qquad (2)$$

Where V_0 is the content of SiCp at the outer disc radius and n is SiCp gradation index. The value of n is chosen -0.5 in this study.

The density and young's modulus denoted respectively by and at any radius (r) of the FGM disc are given by,

$$\rho(r) = \rho_0\left(\frac{r}{b}\right)^{n_1},\ E(r) = E_0\left(\frac{r}{b}\right)^{n_2} \qquad (3)$$

where ρ_o and E_o are respectively the values of density and young's modulus at the outer radius of FGM disc. The exponents n_1 and n_2 are the gradation indices.

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III. MATHEMATICAL FORMULATION

The radial strain (ε_r) and tangential strain (ε_θ) are related by the following Eq.

$$\varepsilon_r = \varepsilon_\theta + r\frac{d\varepsilon_\theta}{dr} \quad (4)$$

For elastic deformations, the constitutive equations between elastic stresses and strains for an isotropic FGM disc under plane stress condition are given by [5],

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_\theta) + \alpha(r)T(r) \quad (5)$$

$$\varepsilon_\theta = \frac{1}{E(r)}(\sigma_\theta - \nu\sigma_r) + \alpha(r)T(r) \quad (6)$$

where σ_r and σ_θ are respectively the radial and tangential stresses in the disc and ν (=0.3) is the Poisson's ratio.

The force equilibrium equation for a variable thickness rotating FGM disc rotating at angular velocity (ω = 1570 *rad/s*) is given as [4],

$$\frac{d}{dr}\left[rh(r)\sigma_r\right] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0 \quad (7)$$

The equilibrium relation (14) is satisfied by the stress function *F*, defined as,

$$\sigma_r = \frac{F}{r\,h(r)}, \quad \sigma_\theta = \frac{1}{h(r)}\frac{dF}{dr} + \rho(r)\omega^2 r^2 \quad (8)$$

Substituting the values of ε_r and ε_θ from (5-6) and using (7-8) into the (4), we get,

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr}\left[1 - k - n_2\right] + F\left[\nu k + \nu n_2 - 1\right] = -\frac{\rho_o\omega^2 h_b r}{b^{n_1+k}}(3 + \upsilon + n_1 - n_2)r^{3+n_1+k} \quad (9)$$

Solving the above this differential equation, the stress function (*F*) can be obtained as,

$$F = C_1 r^{\frac{n_2+k+m}{2}} + C_2 r^{\frac{n_2+k-m}{2}} + Ar^{3+n_1+k} \quad (10)$$

where C_1 and C_2 are the constants of integration and *A*, *B*, *C* and *m* are given by,

$$A = \frac{-\rho_o\omega^2 h_b(3 + \nu + n_1 - n_2)}{b^{n_1+k}(8 + n_1{}^2 + n_1 k + 3k + 6n_1 - n_1 n_2 - n_2 k - 3n_2 + \nu n_2 + \nu k)}$$

and $m = \sqrt{(n_2 + k)^2 - 4(\nu n_2 + \nu k - 1)}$

The radial and tangential stresses can be obtained by substituting the values of stress function (*F*) in (8) as,

$$\sigma_r = \frac{1}{h(r)}\left[C_1 r^{\frac{n_2+k+m-2}{2}} + C_2 r^{\frac{n_2+k-m-2}{2}} + Ar^{2+n_1+k}\right] \quad (11)$$

$$\sigma_\theta = \frac{1}{h(r)}\left[\left(\frac{n_2+k+m}{2}\right)C_1 r^{\frac{n_2+k+m-2}{2}} + \left(\frac{n_2+k-m}{2}\right)C_2 r^{\frac{n_2+k-m-2}{2}} + (3 + n_1 + k)Ar^{2+n_1+k}\right] + \rho(r)r^2\omega^2 \quad (12)$$

It is assumed that the FGM disc under investigation is fitted on a splined shaft where small axial movement is permitted. Thus, we may use the following free-free boundary conditions (σ_r =0 at *r* = *a* and σ_r =0 at *r* = *b*) in (11),

$$C_1 = \frac{D_2 b^{\frac{2+m-n_2-k}{2}} - D_1 a^{\frac{2+m-n_2-k}{2}}}{(b^m - a^m)}$$

and $$C_2 = \frac{D_1 b^m a^{\frac{2+m-n_2-k}{2}} - D_2 b^{\frac{2+m-n_2-k}{2}} a^m}{(b^m - a^m)} \quad (13)$$

where,

$$D_1 = -Aa^{2+n_1+k} \quad D_2 = -Ab^{2+n_1+k} \quad (14)$$

IV. RESULTS AND DISCUSSION

A computer code has been developed to estimate the stresses and strains in FGM disc with different thickness profile (refer Table 1). The values of various constants and properties used during the computation process are summarized in Table 2. Knowing the radial variation of various properties (ρ and E), the values of gradation indices n_1 and n_2 are estimated by DATA-FIT software.

Table 1 Description of FGM discs with varying thickness gradient (TG)

Disc	h_a *mm*	h_b *mm*	TG *mm*	SiC_p Content (*vol %*)		
				V_i	V_{av}	V_o
(D1) K= -0.25	5.78	4.60	1.18	26.45	20	16.73
(D2) K= -0.50	6.66	4.21	2.45	26.25	20	16.60
(D3) K= -0.75	7.65	3.84	3.80	26.02	20	16.46
(D4) K= -1	8.75	3.50	5.25	25.80	20	16.32

Table 2 Values of material properties and constants

Disc	ρ_o	n_1	E_o	n_2
(D1) k= -0.25	2782.98	-0.01915	133.04	-0.2668
(D2) k= -0.50	2782.33	-0.01900	132.55	-0.2658
(D3) k= -0.75	2781.63	-0.01885	132.02	-0.2647
(D4) k= -1	2780.92	-0.01870	131.49	-0.2637

A. Effect of varying disc thickness gradient on elastic stresses and strains

We have made an attempt to investigate the effect of varying this kind of radially decreasing disc thickness gradient by varying *k* from -0.25 to -1. It is revealed from Table 1 that with the decrease of *k* from -0.25 to -1 the disc thickness gradient (TG) increases. With the increase in disc thickness gradient (TG) of the FGM disc, the radial as well as tangential stresses (Figs. 1a and 1b) are observed to decrease over the entire disc. The decrease observed in radial stress is more somewhere in the middle of the disc. The tangential stress on the other hand exhibits relatively higher decrease near the inner radius than that observed towards the outer radius.

The effect of increasing TG on the radial and tangential strains in the FGM disc is shown in Figs. 2a-2b. It is revealed that the radial as well as tangential strain decreases with the increase in TG. It is also evident that with increasing TG, the distributions of strains become relatively uniform.

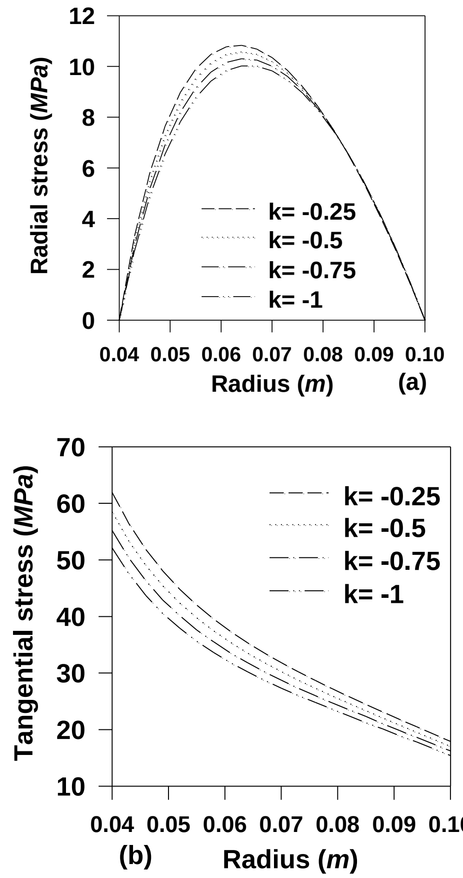
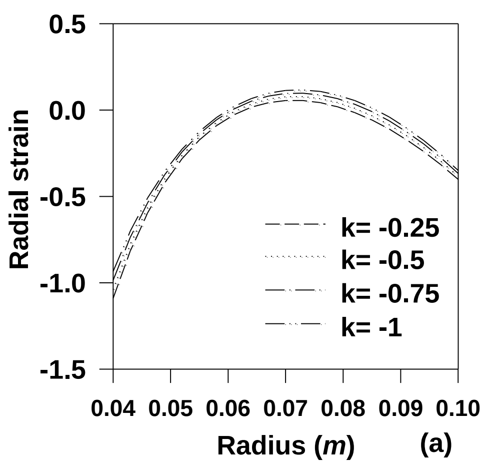


Fig. 1: Effect of increasing disc thickness gradient on (a) Radial and (b) Tangential stresses.



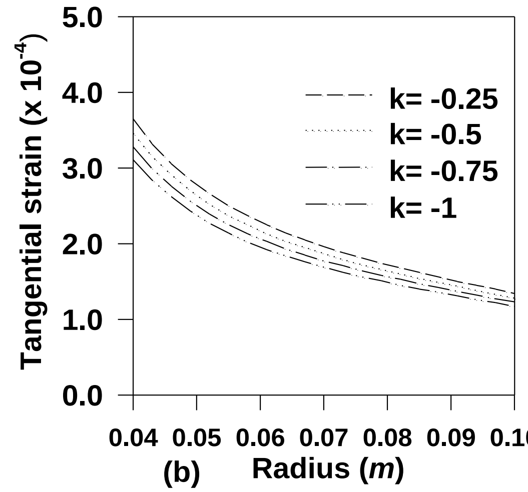


Fig. 2: Effect of thickness gradient on (a) Radial and (b) Tangential strains

V. Conclusion

In this paper, effect of variation of thickness profile in FGM disc has been analyzed on the stress strain analysis of rotating FGM disc. For the varying material properties, we have obtained the exact solutions by solving the governing equation. It is concluded that, radial and tangential stresses in the FGM disc is significantly reduced for FGM disc having variable thickness as compared to similar FGM disc having constant thickness. It is also observed that radial and tangential strain also decreases for FGM disc with variable thickness disc as compare constant thickness disc. Therefore, FGM disc with decreasing thickness profile have relatively lesser chances of distortion.

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