Point Spread Function of Symmetrical Optical System Apodised with Gaussian Filter

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Abstract

In this study, symmetrical optical system is considered. Point spread function is important parameter in the performance of optical system. Point spread function in the case of optical system apodised with Gaussian filter is derived. Analytical and numerical results are computed for various cases of apodised parameter and are presented graphically.

Key words: Optical system, Point spread function, Gaussians filter, Defocusing parameter.

1.INTRODUCTION

Optical system is equipped with lenses, mirrors, and prisms that constitute the optical part of instruments such as microscope and telescope. Optical imaging is a technique that depends on illumination of light in ultraviolet, visible, and infrared regions of the electromagnetic spectrum. Optical imaging plays important role as Non Destructive Evaluation (NDE) tool in the health care domain. In optical imaging systems, the Point Spread Function (PSF) describes the response of imaging system to a point source and it plays an important role in the resolution studies of an optical system. There are two important metrics for performance evaluation of optical systems, one is PSF and other is Optical Transfer Function (OTF). The PSF and OTF are mathematically interchangeable in the sense, one is Fourier transform of other. The diffraction theory of PSF was first studied by Airy. Airy developed expression for PSF amplitude and intensity of a perfect system, which is free from aberrations. An apodisation function is used to purposely change the input intensity profile of an optical system. Overviews of PSF analysis is available in the review paper [1 and references therein]. The Leaver and
Smith [1] have studied radially symmetric pupil function having properties of producing a point spread function which decreases monotonically with the increase of radius. The PSF obtained with the above pupil is from the secondary maxima and minima but is wider than that of a clear aperture. Magiera [2] has studied the filters which minimize the second moment of the PSF, Magiera, and Pluta [3] have used second moment as a measure of the energy scatter and evaluated the effects of optimal appraising filters on the distribution of energy in the PSF produced by axial object point. Bareket [4] considered the second moment of the PSF to study the image quality criterion. Tschunko [5] has obtained an expression for the PSF by considering geometrical relations of the incoming and diffracted wave fronts and established the diffraction integral. Rayalu and Mondal [6] have evaluated the defocusing effect on the PSF of an optical system apodised with parabolic filters and shaded apertures. Tschunko [7] has studied the PSF of optical system with annular apertures. Venkat et.al. [8] have given a general expression for the PSF with an object having a non-central phase annulus coating. Murthy and Mondal [9] have obtained the PSF of optical system in the sinusoidal amplitude filters. Linsen and Yaghanng [10] have studied the restoration of blurred images by convolution of arbitrary PSF. Chung et. al [11] have investigated the influence of apodisation on aberrated point spread function. Nakomura and Toyoda [12] have proposed an apodised annular pupil to suppress the side lobes in PSF. To the best of our knowledge, no one has analyzed the PSF of symmetrical optical system apodised with Gaussian filter. In this paper, the same is analyzed.

The rest of the paper is organized as follows. In the section II, mathematical expression for PSF of optical system apodised with Gaussian filter is derived. In the section III, numerical results are presented. Finally, conclusion is given in section IV.

2. POINT SPREAD FUNCTION (PSF) WITH GAUSSIAN AMPLITUDE FILTER

The exact nature of the modifications produced in the amplitude spectrum depends on the type of filter used. The apodisation filter considered here is Gaussian type. The schematic representation for diffraction at an apodised circular aperture of an optical system is shown in Fig. I. Consider a spherical wave front with surface area 'W' with
the radius $f$ emerging from a circular aperture and converging towards to the axial focal point $F$. Let $G(P)$ be the optical disturbance at a point $P(\xi, \eta, \zeta)$ in the vicinity of the focal plane $F$. Position vector of $P$ is $R$. Magnitude of $R$ is assumed to be small when compared to that of the radius of the wave front $CF = f$. Our aim is to study the diffracted image at the point $P$. Let $x$ be the distance of the point $P$ from an arbitrary point $Q(u, v, w)$ on the wave front at a moment when it is incident on the aperture. Let $\frac{A}{f}$ be the amplitude of the incident wave front at a point $Q$. It is assumed that the incident light is quasi monochromatic light and the wavelength ($\lambda$) is very small compared to the radius of the aperture i.e., $a \gg \lambda$. A general expression for complex amplitude at the point $P$ is [13]

$$G(P) = \frac{-i}{\lambda} \left( \frac{A}{f} \right) \exp \left( -ikf \right) \iint \frac{f(r) \exp(ikx)}{x} \, dw.$$  \hspace{1cm} (1)

In the above expression $k$ stands for propagation constant, $\frac{2\pi}{\lambda}$ and $f(r)$ is the pupil function, where $r$ is radial co-ordinate of $P$. The usual inclination factor has been omitted here, since only small angles are involved. If $\vec{q}$ denotes a unit vector in the direction $QF$, then from the figure, we have

$$x - f = -\vec{q} \cdot \vec{R}$$ \hspace{1cm} (2)

The surface element $dw$ can be expressed as

$$dw = f^2 \, d\Omega,$$ \hspace{1cm} (3)

where $dw$ is the surface element which subtends solid angle $d\Omega$ at the point $F$, then we have

$$d\Omega = \frac{dw}{f^2} = \frac{a^2 r dr d\theta}{f^2}.$$ \hspace{1cm} (4)

Here $x$ can be replaced by $f$ as it does not amount any considerable error in Eq. (1). Thus Eq. (1) is reduced to

$$G(P) = \frac{-iA}{\lambda} \iint \frac{\exp(-ikf)}{f} \exp(ikx) f(r) \, dw.$$  \hspace{1cm} (5)

Further using Eqs. (2) and (3), above equation can be written as

$$G(P) = \frac{-iA}{f} \int \exp \left( -ik \vec{q} \cdot \vec{R} \right) d\Omega.$$  \hspace{1cm} (5)

The integration extends over the solid angle subtended by the aperture at point $P$. For a clear aperture $f(r) = 1$, and the Eq. (5) reduces to the Debye integral of an Airy case.
\[ G(P) = -\frac{iA}{\lambda} \iiint_{\Omega} \exp(-ikqR) d\Omega. \]  

In the aperture plane, let \((r, \theta)\) and \((\rho, \phi)\) be the polar coordinates of P and Q, respectively, then we have

\[
\begin{align*}
  u &= a r \sin \theta, \quad v = a r \cos \theta, \\
  \xi &= \rho \sin \phi, \quad \eta = \rho \cos \phi.
\end{align*}
\]

Since Q is on the spherical wave front \(W\), we have

\[
\left( f^2 - a^2 r^2 \right)^{\frac{1}{2}},
\]

\[
= -f \left( 1 - \frac{a^2 r^2}{2 f^2} \right)^{\frac{1}{2}},
\]

\[
= -f \left( 1 - \frac{a^2 r^2}{2 f^2} + .... \right). \tag{9}
\]

Neglecting the terms of higher powers of \(r\), the above equation becomes

\[
= -f \left( 1 - \frac{a^2 r^2}{2 f^2} \right). \tag{10}
\]

Then, we have

\[
\vec{q} \cdot \vec{R} = \frac{\xi u + \eta v + \zeta w}{f}.
\]

From Eqs. (7), (8), and (10), we have

\[
\vec{q} \cdot \vec{R} = \left( \frac{\rho \sin \phi \arcsin \theta + \rho \cos \phi \arccos \theta}{f} \right) - f \left( 1 - \frac{a^2 r^2}{2 f^2} \right). \tag{11}
\]

Let us now introduce two dimensionless variables \(y\) and \(z\) to specify the position of the \(P\). That is

\[
y = \left( \frac{2\pi}{\lambda} \right) \left( \frac{a}{f} \right)^2 \xi, \tag{12}
\]

and

\[
z = \left( \frac{2\pi}{\lambda} \right) \left( \frac{a}{f} \right) \rho. \tag{13}
\]

In Eqs. (12) and (13), \(y\) is defocusing parameter. If \(y = 0\), it represents the Gaussian focal plane. If \(z / y < 1\), the point \(P\) lies in the direct light beam, and if \(z / y > 1\), it lies
in the geometrical shadow.

From Eqs. (12) and (13), and using \( k = \frac{2\pi}{\lambda} \), we can have

\[
k(q \cdot \vec{R}) = zr \cos(\theta - \phi) - \left( \frac{f}{a} \right)^2 y + \frac{1}{2} yr^2. \quad (14)
\]

Using Eqs. (4) and (14) in Eq. (5), we get

\[
G(P) = -iA \left( \frac{2\pi}{\lambda} \right) \int_0^{2\pi} \int_0^\infty f(r) \exp \left( -izr \cos(\theta - \phi) + i \left( \frac{f}{a} \right)^2 y - \frac{1}{2} iyr^2 \right) \frac{a}{f} d\theta dr. \quad (15)
\]

where \( J_0(zr) \) is the Bessel function of first kind and zero order. From the above equation, we get

\[
G(P) = -i \left( A f \right) \frac{2\pi}{\lambda} \int_0^\infty f(r) \exp \left( i \left( \frac{f}{a} \right)^2 y \right) \frac{a}{f} \int_0^{2\pi} \exp \left( -iyr^2 \right) J_0(zr) rdr. \quad (16)
\]

Putting \( -A f \frac{2\pi}{\lambda} \exp \left( i \left( \frac{f}{a} \right)^2 y \right) = \alpha \), and after simplification, we get

\[
G(P) = 2\pi i\alpha a^2 \int_0^\infty f(r) \exp \left( -iyr^2 \right) J_0(zr) rdr. \quad (17)
\]

The term \( i\alpha a^2 \) outside the sign of integration does not have any effect on the diffraction pattern, hence it can be omitted. Now the diffracted light amplitude at a point in the Gaussian focal plane and away from the focusing point \( F \) is given by

\[
G(P) = 2\int_0^1 f(r) \exp \left( -iyr^2 \right) J_0(zr) rdr. \quad (18)
\]

Point spread function of the optical system can be evaluated by knowing the explicit expression of the pupil function \( f(r) \) and then taking the squared modulus of the Eq. (18) at the focused plane of observation corresponding to \( y = 0 \) (which represents Gaussian focal plane). In this case, we have

\[
G(0,Z) = 2\int_0^1 f(r) J_0(zr) rdr. \quad (19)
\]

In the case, \( r \) is normalization distance of Gaussian amplitude filter,

\[
f(r) = e^{-\frac{r^2}{\sigma^2}}. \quad (20)
\]
In the Eq. (20), $\sigma$ is the apodisation parameter, which determines the degree of uniform transmission with in the apodised region. Substituting Eq. (20) in Eq. (19), we get

$$G(0, z) = 2 \int_0^1 e^{-r^2/2\sigma^2} J_0(zr)rdr.$$  \hspace{1cm} (21)

The intensity $B(0, Z)$ of point spread function is given by

$$B(0, z) = |G(0, z)|^2.$$  \hspace{1cm} (22)

3. NUMERICAL RESULTS

In this section, the intensity of point spread function is computed for a Gaussian amplitude filter using Eqs. (21) and (22) against $z$ in the range 0 to ±15 in the stepping 0.5, and the anodization parameter $\sigma$ is taken to be arbitrarily in the range 0.1 to 1.0 in the step of 0.1, and 3.086, 10, 20, 30. The results are depicted in Fig. II and Fig. III. From the Fig.II, we can infer that peak of the central maxima of diffraction pattern increases while first minima position decreases in the cases of $\sigma =0.1$ to 0.4. There is no second order maxima position in this case, that is negative amplitude is zero. For the cases $\sigma =0.5$ and 0.6, peak of the central maxima increases while first order minima position decreases as in the earlier cases. But the first order negative peak values increase. For the cases $\sigma =0.7$ to 1.0, and other higher values, central peak values increase and first and second order negative peak values also increase, while minima positions decrease. From these results, it is clear that aperture shading is affecting central maxima. As $\sigma$ increases central amplitude and intensity increase and also the radius of the first dark rings decreases. It can be stated that, the use of fully apodised circular aperture reduces the effects of the anodization. Thus, the aperture shading efficiently sharpens the central maxima as $\sigma$ increases. From the Fig. III., it is clearly evident that the pupil function $f(r)$ smoothly decreases as $r$ increases and approaches zero in the cases of $\sigma =0.1$ to 0.4. For other higher values of $\sigma$ from 0.5 to 1.0, the pupil function $f(r)$ decreases $r$ increases smoothly but does not touch zero. In the cases $\sigma =3.086, 10, 20, 30$, pupil function $f(r)$ decreases as $r$ increases gradually and latter three curves coincide with the line $f(r) = 1$. The value of $f(r)$ increases as $\sigma$ increases. Thus, the Gaussian pupil function typically suppresses the side lobes but broaden the main lobe of the point spread function (PSF). The Gaussian amplitude transmittance decreases exponentially for $\sigma =0.1$ to 0.4, and then after decreases monotonically from the center towards the edges of Gaussian filters.
4. CONCLUSION

First, complex amplitude in the case of Gaussian filter is derived thereby the point spread function is derived. The numerical values are computed for the various values of apodisation parameter \( \sigma \). This kind of analysis is useful in the design of optical systems in domains of health care, Astronomy, and Communication Engineering.

REFERENCES


Figure II Variation of amplitude of PSF for various values of \( \sigma \) [0.1 to 1.0, 3.08607, 30, 20, 10]

Figure III Pupil Transmission Curves for various values of \( \sigma \)

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