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Analysis Of Rotating Elastic-Plastic Annular Disc With Exponentially Variable Thickness And Exponentially Variable Density

Professor Anukul De and Doyal Debnath

(Retd.), Department of mathematics,
Tripura University, Suryamaninagar-799022.

Research Scholar, Department of mathematics,
Tripura University, Suryamaninagar-799022.

Abstract

The object of this work is to theoretically investigate the elasto-plastic problem of a thin annular rotating disc with exponentially varying thickness(h) and exponentially varying density(ρ). The equation of equilibrium for this annular plate is obtained in both elastic and plastic range. Exact solutions of both the equations are obtained. Stresses are obtained for both elastic and plastic range. The corresponding angular velocity for which the rotating annular plate becomes wholly plastic has been obtained here. Variation of that angular velocity with radii ratio is shown graphically for some fixed values of parameters.

Keywords: elasto-plastic, variable thickness, variable density, annular disc, exponential variation.

Major Field: Theory of Elasticity.

AMS Subject Classification: 73C.

Introduction:

Theoretical investigation of rotating discs has been receiving wide spread attention due to their application in engineering. High speed gears, turbine motors, sink fits, turbo jet engines are some examples. Recent studies show that the stresses in the plates with variable material property are lower than that of a plate with constant material property. Hence for the better utilization of the material, plates with variable thickness and density should be considered. Various researchers like Chakravorty and Choudhuri[1], You, Tang, Zhang and Zheng[9], Jahed and Shirazi[4], Eraslan[2] have studied different problems on rotating discs.

Previous literature shows that most of the researches have concentrated on the analytical solution of circular disc with variable thickness but constant density. Recently authors like Lal[5], Martain[6] and Gou, Wu, Sun and Ma[3] have solved different problems of plates with exponentially variable thickness. You, Tang, Zhang and Zheng[9] has obtained the stresses of a rotating plate with variable thickness and variable density using Runge- Kutta method.

The object of this work is to theoretically investigate the elasto-plastic problem of a thin annular rotating disc with exponentially varying thickness(h) and density(ρ). The stresses and the corresponding angular velocity for which the rotating annular plate becomes wholly plastic has been obtained. Variation of that angular velocity with radii ratio is shown graphically.

Formulation and Solution of the problem: A rotating non homogeneous annular disc with outer radius b and inner radius a is considered. The differential equation of motion of a rotating disc with variable thickness and variable density in the elastic range is given by (vide, You, Tang, Zhang and Zheng[9]),

$$r^2\frac{d^2\varphi}{dr^2}+(1-\frac{r}{h(r)}\frac{dh(r)}{dr})r\frac{d\varphi}{dr}+(1-\frac{\upsilon r}{h(r)}\frac{dh(r)}{dr})\varphi$$
$$=-(3+\upsilon)h(r)\rho(r)\omega^2r^3-h(r)\omega^2r^4\frac{d\rho(r)}{dr} \tag{1}$$

where, φ is the stress function, υ is the Poisson's ratio, ω is the angular velocity of the rotating disc. The thickness and the density of the plate material with exponential variation is considered as,

$$h(r)=h_0e^{m(1-\frac{r}{b})},\qquad \rho(r)=\rho_0e^{n(1-\frac{r}{b})} \tag{2}$$

where *m, n* are real numbers, h_0, ρ_0 are real constants.

From equation (1) using (2) and taking $\frac{r}{b}=R$, we get,

$$R^2\frac{d^2\varphi}{dR^2}+(1+mR)R\frac{d\varphi}{dR}-(1+m\upsilon R)\varphi=(k_1\omega^2R^3+k_2\omega^2R^4)e^{-R(m+n)} \tag{3}$$

where, $\lambda_1=-e^{(m+n)}(3+\upsilon)h_0\rho_0b^3,\qquad \lambda_2=e^{(m+n)}nh_0\rho_0b^3$

For complementary function let us consider,

$$R^2\frac{d^2\varphi}{dR^2}+(1+mR)R\frac{d\varphi}{dR}-(1+m\upsilon R)\varphi=0 \tag{4}$$

Substituting $\varphi = Ry$, equation (4) transforms in to,

$$R\frac{d^2y}{dR^2} + (a_1R + b_1)\frac{dy}{dR} + a_0\varphi = 0 \tag{5}$$

where, $a_0 = m(1-\upsilon)$, $a_1 = m$, $b_1 = 3$.

This is a generalized hypergeometric differential equation of order two.

Hence the solution of the equation (5) is given by,

$$y = \{A_1F_1(R) + A_2F_2(R)]\}e^{-mR} \tag{6}$$

where, $F_1(R) = U[2+\upsilon, 3, mR]$ is the confluent hypergeometric function and $F_2(R) = L[-2-\upsilon, 2, mR]$ is the generalized Laguerre polynomial.

After computing particular integral the complete solution of equation (3) is found to be,

$$\varphi = \{A_1RF_1(R) + A_2RF_2(R)\}e^{-mR} + \omega^2(A_3 + A_4R + A_5R^2 + A_6R^3)R^3 \tag{7}$$

A_3, A_4, A_5 and A_6 are known constants and A_1, A_2 are arbitrary constants.

In the elastic range, stresses are given by (vide, Timeshenko, Goodier [8]),

$$\sigma_r = \frac{\varphi}{rh(r)}, \qquad \sigma_\theta = \frac{1}{h(r)}\frac{d\varphi}{dr} + \rho\omega^2r^2 \tag{8}$$

Considering $\frac{r}{b} = R$ and substituting the values of h, ρ and φ in equation (8), the stresses are obtained as,

$$\begin{aligned}\sigma_{\mathrm{Re}} = \frac{e^{-m}}{bh_0}\{&A_1U[2+\upsilon,3,mR] + A_2L[-2-\upsilon,2,mR] \\ &+ \omega^2(A_3 + A_4R + A_5R^2 + A_6R^3)R^2e^{mR}\}\end{aligned} \tag{9}$$

$$\begin{aligned}\sigma_{\theta e} = b^2\rho_0\omega^2R^2e^{m(1-R)} + \frac{e^{-m}}{h_0}\{&\omega^2(3A_3 + 4A_4R + 5A_5R^2 + 6A_6R^3)R^2e^{mR} \\ &+ A_1(1-mR)U[2+\upsilon,3,mR] - A_1mR(2+\upsilon)U[3+\upsilon,4,mR] + \\ &A_2(1-mR)L[-2-\upsilon,2,mR] - A_2mRL[-3-\upsilon,3,mR]\}\end{aligned} \tag{10}$$

In the plastic range $\sigma_\theta = \sigma_0$ (vide, Nadai[7]) and the equation of equilibrium of rotating discs with variable thickness and variable density can be written as ,

$$\frac{d\varphi}{dr} - h(r)\sigma_\theta + h(r)\rho(r)\omega^2r^2 = 0 \tag{11}$$

Substituting the values of h, ρ, σ_θ and taking $\frac{r}{b} = R$, we get,

$$\frac{d\varphi}{dR} = b\{h_0\sigma_0 e^m - h_0\rho_0\omega^2 b^2 R^2 e^{m+n} e^{-nR}\} e^{-mR} \tag{12}$$

The complete solution of the above equation is given by,

$$\varphi = B_1 + \frac{bh_0 e^{m(1-R)}}{m(m+n)^3}\{-\sigma_0(m+n)^3 + b^2 m\rho_0 e^{n(1-R)}(2 + 2nR + m^2R^2 + n^2R^2 + 2mR + 2nmR^2)\omega^2\} \tag{13}$$

where, B_1 is an arbitrary constants and $m \neq -n, m \neq 0$.

Substituting (13) in equation (8) we get the redial stress in plastic region as,

$$\sigma_{Rp} = \frac{B_1 e^{-m(1-R)}}{h_0 Rb} + \frac{1}{m(m+n)^3 R}\{-\sigma_0(m+n)^3 + b^2 m\rho_0 e^{n(1-R)}(2 + 2nR + m^2R^2 + n^2R^2 + 2mR + 2nmR^2)\omega^2\}] \tag{14}$$

To find the angular velocity, that can convert the elastic disk wholly plastic, we consider the following set of boundary conditions (vide, Chakravorty and Choudhury [1]),

$$\begin{array}{lll} \sigma_{Rp} = 0 & \text{on} & R = 1 \text{ (or } r = b\text{)} \\ \sigma_{\mathrm{Re}} = 0 & \text{on} & R = \frac{a}{b} \text{ (or } r = a\text{)} \\ \sigma_{Rp} = \sigma_{\mathrm{Re}} & \text{on} & R = \eta \text{ (or } r = \eta b\text{)} \\ \sigma_{\theta p} = \sigma_{\theta e} & \text{on} & R = \eta \text{ (or } r = \eta b\text{)} \end{array} \tag{15}$$

where, $R = \eta$ is the elastic plastic inter face.

Using boundary conditions (15) arbitrary constants are determined. When the rotating disk becomes wholly plastic, we get, $\eta b = a$ (or $R = \frac{a}{b}$) then the corresponding angular velocity is given by,

$$w^2 = \frac{\sigma_0}{\rho_0}\frac{(e^{\frac{a(n+m)}{b}} - e^{m+\frac{an}{b}})(m+n)^3}{m[-2abe^{m+n}(m+n) - a^2e^{m+n}(m+n)^2 + b^2\{-2e^{(m+n)} + e^{\frac{a(n+m)}{b}}(2+m^2+2n+n^2+2m(1+n))\}]} \tag{16}$$

Numerical Results

The required angular velocity for which the annular disc become wholly plastic has been calculated numerically taking $b = 1$.

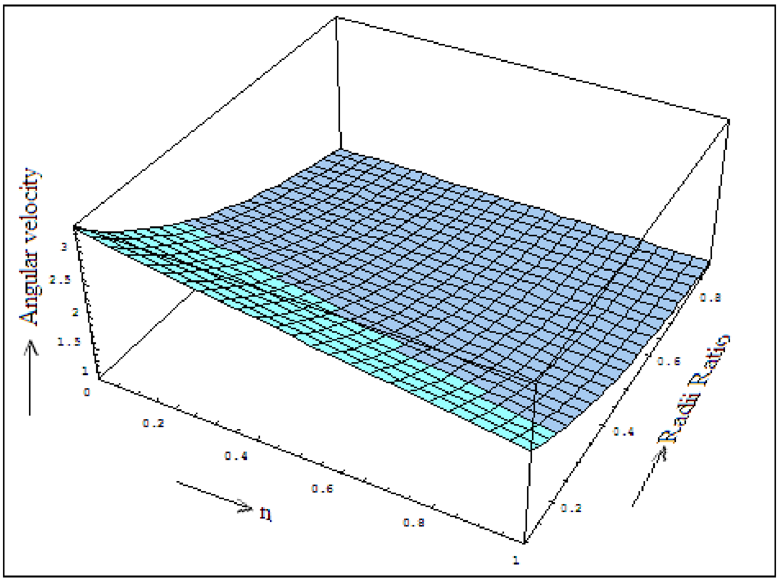

Figure 1: Variation of angular velocity $(\omega^2 \frac{\rho_0}{\sigma_0})$ with radii ration for different *n*.

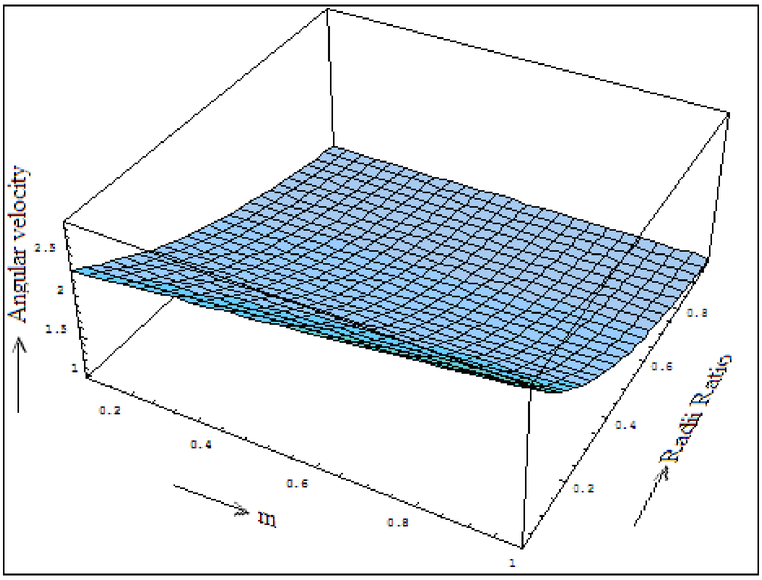

Figure 2: Variation of angular velocity $(\omega^2 \frac{\rho_0}{\sigma_0})$ with radii ration for different *m*.

Discussion and conclusions

Figure 1 shows the numerical values of angular velocity of the annular plate for different values of *n* and different radii ratio $(\frac{a}{b})$ taking $m = 1$. Figure 2 shows the variation of angular velocity with radii ratio for different *m* taking $n = 1$. From figure 1 we observe that the required angular velocity is very high near the inner surface and gradually decreases towards the outer surface. The same variation can be found from figure 2 also. From figure1 we also see that as *n* increases the required angular velocity decreases but figure 2 shows that as *m* increases the required angular velocity also increases. Hence we can say that when the density of the plate increases the required angular velocity decreases but when thickness increases the required angular velocity also decreases.

The work presented here is a theoretical study of elasto-plastic problem of plate with variable material property. In particular the influence of parameters on the stresses and angular velocity is discussed here. Accurate data has been obtained for some fixed values of taper constants. This can be useful for subsequent research carried out for these types of problems. The work presented here is a theoretical mathematical model and design engineers can make use of it with a practical approach.

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