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Free-Convection Flow Past an Accelerated Vertical Plate with Thermal Radiation in a Rotating Fluid

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Abstract

An analysis is performed to study the effects of thermal radiation on unsteady free convective flow past a uniformly accelerated vertical plate in a rotating fluid. An exact solution is obtained for the axial and transverse components of the velocity by defining a complex velocity. The effects of rotation, radiation, free-convection parameters and the skin friction components on the plate are discussed.

Keywords: Radiation, rotation, accelerated vertical plate, isothermal, heat transfer.

Nomenclature:

a^*	absorption coefficient
C_p	specific heat at constant pressure
g	acceleration due to gravity
Gr	thermal Grashof number
k	thermal conductivity of the fluid
Pr	Prandtl number
q_r	radiative heat flux in the y-direction
R	radiation parameter
T'_∞	temperature of the fluid far away from the plate
T'_w	temperature of the plate
T'	temperature of the fluid near the plate
t'	time
t	dimensionless time

u'	velocity of the fluid in the x'-direction
u	dimensionless velocity
v'	velocity of the fluid in the y'-direction
v	dimensionless velocity
y'	coordinate axis normal to x'-axis
z'	coordinate axis normal to the plate
z	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion
μ	coefficient of viscosity
ν	kinematic viscosity
Ω'	rotation parameter
Ω	dimensionless rotation parameter
ρ	density of the fluid
τ	dimensionless skin-friction
θ	dimensionless temperature
erfc	complementary error function

Subscripts

w	conditions at the wall
∞	conditions in the free stream

Introduction

Free-convection flow is encountered in cooling of nuclear reactor and electronic devices. The effect of radiation is quite significant at high temperature. Radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. Radiative convective flows are encountered in countless industrial and environmental processes, particularly in astrophysical studies, Nuclear power plants, gas turbines, ship dynamics and various propulsion devices for aircraft, missiles, satellites and space vehicles. The effect of coriolis force has wide applications in science and technology.

Arpaci (1968) studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. In all the above studies, the stationary plate was considered. Singh (1984) studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Bestman and Adjepong (1988) studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous and optically transparent medium, particularly for the opaque galaxies. Again Bestman(1989) had analyzed the case when the radiative flux satisfies the exact integral equation, which bridges the opacity and transparency.

Das *et al* (1996) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Raptis and Perdikis (1999) considered the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically.

However the effect of thermal radiation on an accelerated infinite vertical plate in a rotating fluid is not studied in the literature. It is proposed to study thermal radiation effects on flow past an accelerated infinite isothermal vertical plate in a rotating fluid. The dimensionless governing equations are solved by Laplace transform technique.

Basic Equations and Analysis

Consider the three dimensional flow of a viscous incompressible fluid induced by uniformly accelerated motion of an infinite vertical isothermal plate in a rotating fluid. On this plate, the x' -axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to x'-axis in the plane of the plate and z'-axis is normal to it. Both the fluid and the plate are in a state of rigid rotation with uniform angular velocity Ω' about the z'-axis. Initially, the plate and fluid were at rest and with the same temperature. At time t'>0, the plate starts moving with a velocity ct' in its own plane in the vertical direction against gravitational field, in the presence of thermal radiation. At the same time the plate temperature is raised or lowered to T'_w which is there after maintained constant. Since the plate occupying the plane z'=0 is of infinite extent, all the physical quantities depend only on z' and t'. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial z'^2} \tag{1}$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2} \tag{2}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z'} \tag{3}$$

The term $\frac{\partial q_r}{\partial z'}$ represents the change in the radiative flux with distance normal to the plate with the following initial and boundary conditions:

$$\begin{aligned}
&t' \le 0: \quad u' = 0, \quad v' = 0, \quad T' = T'_\infty, \quad \text{for all } z' \\
&t' > 0: \quad u' = ct', \quad v' = 0, \quad T' = T'_w, \quad \text{at} \quad z' = 0 \\
&\qquad u \to 0, \quad v' \to 0 \quad T \to T_\infty, \quad \text{as} \quad z' \to \infty.
\end{aligned} \tag{4}$$

By Rosseland approximation, radiative heat flux of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z'} = -4a^*\sigma(T_\infty'^4 - T'^4) \qquad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \qquad (6)$$

By using equations (5) and (6), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + 16a^*\sigma\, T_\infty'^3 (T'_\infty - T') \qquad (7)$$

On introducing the following dimensionless quantities

$$(\boldsymbol{u},\boldsymbol{v}) = \frac{(\boldsymbol{u}',\boldsymbol{v}')}{(\nu\,\boldsymbol{c})^{\frac{1}{3}}}, \quad \boldsymbol{t} = \boldsymbol{t}'\left(\frac{\boldsymbol{c}^2}{\nu}\right)^{\frac{1}{3}}, \quad \boldsymbol{z} = \boldsymbol{z}'\left(\frac{\boldsymbol{c}}{\nu^2}\right)^{\frac{1}{3}}, \quad \theta = \frac{\boldsymbol{T}' - \boldsymbol{T}'_\infty}{\boldsymbol{T}'_w - \boldsymbol{T}'_\infty},$$

$$Gr = \frac{g\beta(T'_w - T'_\infty)}{c}, \qquad (8)$$

$$Pr = \frac{\mu C_p}{k}, \quad \Omega = \Omega'\left(\frac{\nu}{c^2}\right)^{\frac{1}{3}}, \quad R = \frac{16a^*\nu\ \sigma\, T_\infty'^3}{k}\left(\frac{\nu}{c^2}\right)^{\frac{1}{3}}.$$

and the complex velocity $q = u + iv$, $i = \sqrt{-1}$ in equations (1) to (4), the equations relevant to the problem reduces to

$$\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{t}} + 2\boldsymbol{i}\Omega\boldsymbol{q} = \boldsymbol{Gr}\ \theta + \frac{\partial^2 \boldsymbol{q}}{\partial \boldsymbol{z}^2}, \qquad (9)$$

$$\frac{\partial \theta}{\partial \boldsymbol{t}} = \frac{1}{\boldsymbol{Pr}}\frac{\partial^2 \theta}{\partial \boldsymbol{z}^2} - \frac{\boldsymbol{R}}{\boldsymbol{Pr}}\theta \qquad (10)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llll} & q = 0, \quad \theta = 0, & \text{for all} & z \le 0 \,\&\, t \le 0 \\ t > 0: & q = t, \quad \theta = 1, & at & z = 0 \\ & q = 0, \quad \theta \to 0, & as & z \to \infty. \end{array} \qquad (11)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for the equations (9) to (10), subject to the boundary conditions (11), by Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(-2\eta\sqrt{Rt})\,erfc(\eta\sqrt{\Pr}-\sqrt{at}) + \exp(2\eta\sqrt{Rt})\,erfc(\eta\sqrt{\Pr}+\sqrt{at})\right] \qquad (12)$$

$$\begin{aligned}
q &= \frac{1}{2}\left(t+\frac{Gr}{b(1-Pr)}\right)\left[\exp(2\eta\sqrt{mt})\,erfc(d1) + \exp(-2\eta\sqrt{mt})\,erfc(d2)\right] \\
&- \frac{\eta\sqrt{t}}{2\sqrt{m}}\left[\exp(-2\eta\sqrt{mt})\,erfc(d2) - \exp(2\eta\sqrt{mt})\,erfc(d1)\right] \\
&- \frac{Gr\exp(bt)}{2b(1-\Pr)}\begin{pmatrix}\left[\exp(2\eta\sqrt{(b+m)t})\,erfc(d3) + \exp(-2\eta\sqrt{(b+m)t})\,erfc(d4)\right] \\ -\exp(2\eta\sqrt{\Pr(b+a)t})\,erfc(d7) - \exp(-2\eta\sqrt{\Pr(b+a)t})\,erfc(d8)\end{pmatrix} \qquad (13) \\
&- \frac{Gr}{2b(1-\Pr)}\left[\exp(2\eta\sqrt{Rt})\,erfc(d5) + \exp(-2\eta\sqrt{Rt})\,erfc(d6)\right]
\end{aligned}$$

where $d1, d2 = \left[\eta \pm \sqrt{mt}\right]$, $d3, d4 = \left[\eta \pm \sqrt{(b+m)t}\right]$

$d5, d6 = \left[\eta\sqrt{\Pr} \pm \sqrt{at}\right]$, $d7, d8 = \left[\eta\sqrt{\Pr} \pm \sqrt{(a+b)t}\right]$

$a = \frac{R}{\Pr}$, $b = \frac{R-m}{1-\Pr}$, $m = 2i\,\Omega$ *and* $\eta = \frac{z}{2\sqrt{t}}$

Using equation (13) we get the following expression for skin- friction components τ_x and τ_y.

$$\begin{aligned}
\tau_x + i\tau_y &= -\left[\frac{\partial q}{\partial z}\right]_{z=0} = \frac{1}{\sqrt{\pi t}}\left\{\left(\frac{1}{2}t + \frac{Gr}{b\,(1-Pr)}\right)\left(1+\sqrt{m\pi t}\; erf(\sqrt{mt})\right)\right. \\
&+ \frac{\sqrt{\pi \boldsymbol{t}}}{2\sqrt{\boldsymbol{m}}}\,\boldsymbol{erf}\,(\sqrt{\boldsymbol{mt}}) \\
&- \frac{Gr\exp(bt)}{b(1-\Pr)}\left(1+\sqrt{(m+b)\pi t}\;\; erf(\sqrt{(b+m)t})\right) \\
&+ \frac{Gr\exp(bt)}{b(1-\Pr)}\left(\sqrt{\Pr} + \sqrt{\Pr(a+b)\pi t}\;\; erf(\sqrt{(b+a)t})\right) \qquad (14) \\
&\left.- \frac{\boldsymbol{Gr}}{\boldsymbol{a}(1-\boldsymbol{Pr})}\left(\sqrt{\Pr} + \sqrt{R\pi t}\;\; erf(\sqrt{at})\right)\right\}
\end{aligned}$$

In equations (13) and (14), the argument of the complementary error function and error function is complex. Hence in order to obtain the u and v components of the velocity and skin -friction, we have used the following formula due to Abramowitz and stegun (1964):

$$erf(a+ib) = erf(a) + \frac{\exp(-a^2)}{2a\pi}[1-\cos(2ab) + i\sin(2ab)]$$
$$+ \frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}[f_n(a,b) + ig_n(a,b)] + \varepsilon(a,b)$$

Where ,

$f_n = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab)$

$g_n = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab)$

$|\varepsilon(a,b)| \approx 10^{-16} |erf(a+ib)|$

Discussion of Results

Using the above formula, expressions for u, v are obtained but they are omitted here to save the space. In order to get a physical view of the problem, these expressions are used to obtain the numerical values of u, v, τ_x and τ_y for different values of the parameter like rotation, radiation and thermal Grashof number.

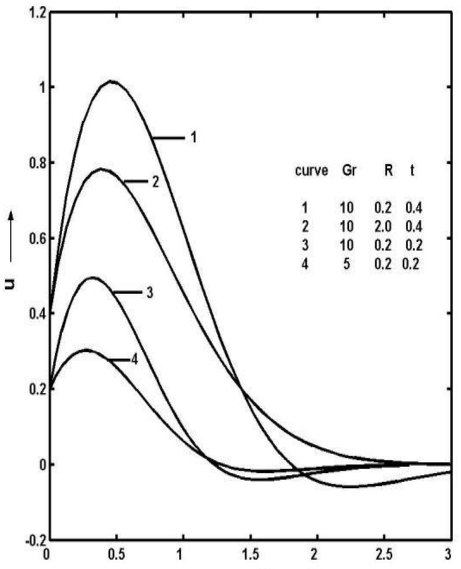

Fig.1. Primary velocity profile for different R

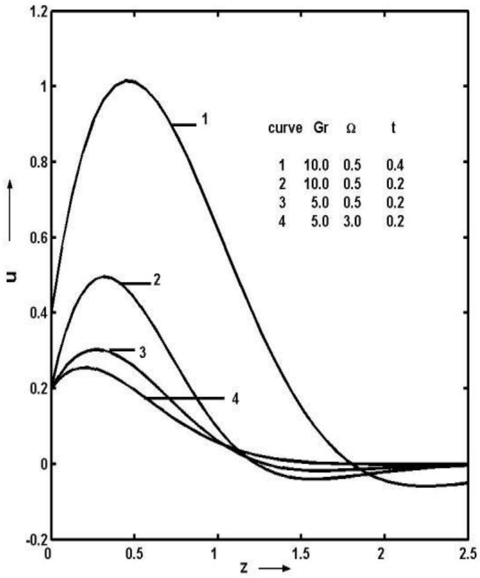

Fig.2. Primary velocity profile for different Ω

The primary velocity profiles of air for different values of the radiation parameter are shown in Fig. 1. It is observed that the primary velocity increases with decreasing radiation parameter R in cooling of the plate. This shows that primary velocity decreases in the presence of high thermal radiation. It is also observed that greater cooling of the plate, due to free convection currents, increases the primary velocity of the plate.

The primary velocity profiles of air for different values of the rotation parameter are shown in Fig. 2. It depicts that the primary velocity increases with decreasing rotation parameter **Ω**. It is also observed that greater cooling of the plate, due to free-convection currents, increases the primary velocity of the plate in this case too.

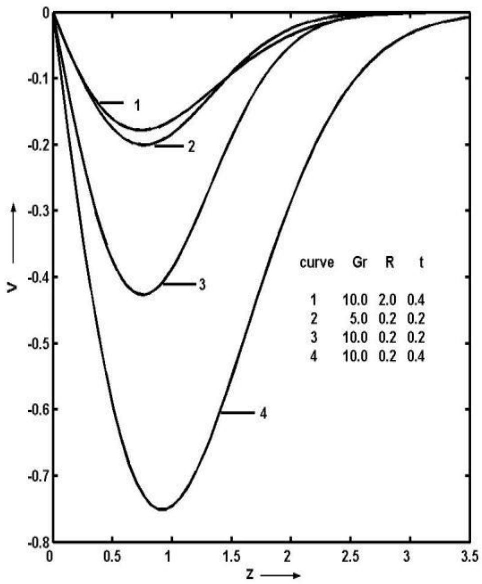

Fig.3. Secondary velocity profiles for different R

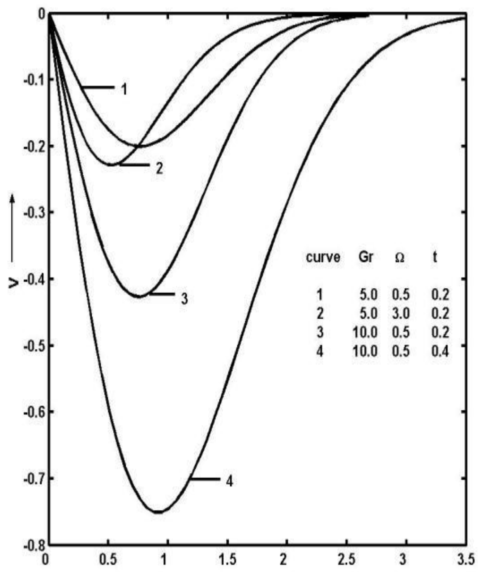

Fig.4.Secondary velocity profiles for different Ω

The secondary velocity profiles of air for different values of the radiation parameter are shown in Fig.3, the effect of radiation increases the secondary velocity v. Fig. 4. shows the effect of rotation on v which is just reverse to that of radiation parameter. Further, greater cooling of the plate, due to free- convection currents, decreases the secondary velocity of the plate.

The temperature profiles for air (Pr = 0.71) are calculated for different values of thermal radiation parameter from Equation (12) and these are shown in Fig. 5. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter

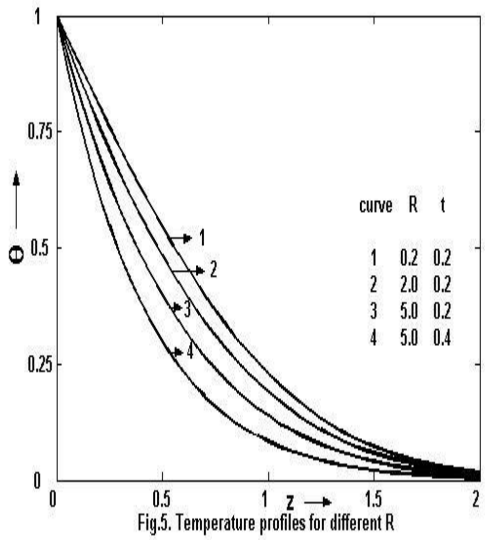

Fig.5. Temperature profiles for different R

Table 1. Skin-friction τ

Gr	Ω	t	R	τ_x	τ_y
5.0	0.5	0.2	0.2	3.5769	- 1.7345
	2.0			0.7166	- 1.2037
	3.0			0.1217	- 0.5145
10.0	0.5			6.7794	- 3.2479
		0.4	0.2	5.9823	- 4.2804
			2.0	6.9631	-11.5111
			5.0	103.4774	-253.9815

The skin-friction components for air (Pr=0.71) at the wall for different values of Gr, t, R and Ω are shown in Table 1. The effect of radiation increases the value of the components τ_x and decreases τ_y of skin-friction, but the effect of rotation on skin-friction decreases the component τ_x and increases τ_y as Ω increases. As time advances the component τ_x and τ_y decreases. Greater cooling of the plate, due to free-convection currents, raises τ_x and lowers τ_y

Conclusions

Theoretical analysis is performed to study flow past an accelerated infinite vertical plate with uniform temperature, in the presence of thermal radiation in a rotating fluid. The dimensionless governing equations are solved by Laplace-transform technique. The conclusions of the study are as follows:

- The influence of the radiation or rotation parameter on primary flow has a retarding effect for cooling of the plate.
- As time advances, the influence of the radiation or rotation parameter on primary flow increases velocity but the trend is reversed in secondary flow.
- The secondary velocity is enhanced with the raise in thermal radiation and opposite phenomenon occurs with the rotation parameter.
- The skin-friction components τ_x increases and τ_y decreases with increasing radiation parameter. But the effect of rotation on skin-friction components are reversed.

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