International Journal of Pure and Applied Mathematical Sciences.
ISSN 0972-9828 Volume 12, Number 1 (2019), pp. 5-16
©Research India Publications
http://www.ripublication.com/ijpams.htm

About the Solution of the Generalized Kundu's Equation

BOUKARY Béyi[*], **Joseph BONAZEBI-YINDOULA**[**],
Longin SOME[*] **and Gabriel BISSANGA**[**]
[*]*Université de Ouaga-1 Professeur Joseph KI ZERBO Burkina Faso*
[**]*Université Marien NGOUABI Brazzaville, Congo*

Abstract

In this paper, we use the SBA method to construct the solution of Kundu-Eckhaus's equation in a space dimension m.

AMS Subject Classification: 47H14; 34G20; 47J25; 65J15.
Keywords: SOME Blaise Abbo (SBA) method, Kundu-Eckhaus equation.

1. INTRODUCTION

In [1], using the SBA method, has been constructed the solution of a particular case of the Kundu-Eckhaus equation in the case of space dimension 1, here, using the same method, we firstly examine the generalized kundu's equation in the case of space dimension 1 and space dimension 3, secondly, we deduct the case of space dimension m.

2. THE KUNDU - ECKHAUS EQUATION IN THE CASE OF SPACE DIMENSION 1

The general theory of SBA method can be found in [2] , [3]

The generalized Kundu-Eckhaus equation Cauchy problem's in the case of space dimension 1 can be written as follows:

$$\begin{cases} i\dfrac{\partial u(x,t)}{\partial t} + \dfrac{\partial^2 u(x,t)}{\partial x^2} + \beta \frac{\partial |u(x,t)|^\lambda}{\partial x} u(x,t) + \gamma |u(x,t)|^d u(x,t) = 0 \ \forall \quad (\beta,\gamma,d,\lambda) \in \mathbb{R}^4, \quad t>0 \\ u(x,0) = a e^{i\alpha x} \quad \forall \quad a,\alpha \in \mathbb{R},\ x \in \Omega \subset \mathbb{R} \end{cases} \tag{1}$$

where $u(x,t)$is the unknown complex-valued function of two real variables x and t.

From (1), we have:

$$\frac{\partial u(x,t)}{\partial t} = i\frac{\partial^2 u(x,t)}{\partial x^2} + iNu(x,t) \tag{2}$$

where

$$Nu(x,t) = \beta\frac{\partial\,|u(x,t)|^{\lambda}}{\partial x}u(x,t) + \gamma|u(x,t)|^{d}u(x,t) \tag{3}$$

From (2), we obtain:

$$u(x,t) = u(x,0) + i\int_0^t \frac{\partial^2 u(x,s)}{\partial x^2}ds + i\int_0^t N(u(x,s))ds \tag{4}$$

According to the SBA method, we suppose that the solution of (1) has the following form:

$$u(x,t) = \lim_{k\to+\infty} u^k(x,t) \tag{5}$$

where

$$u^k(x,t) = \sum_{n=0}^{+\infty} u_n^k(x,t) \quad ; \quad k \geq 1 \tag{6}$$

and, for every $k \geq 1$, we get $u_n^k(x,t)$ for $n \geq 0$, through the following SBA algorithm:

$$\left\{\begin{array}{l} u_0^k(x,t) = u^k(x,0) + i\int_0^t N(u^{k-1}(x,s))ds \ ; \ \ k \geq 1 \\ u_{n+1}^k(x,t) = \int_0^t i\frac{\partial^2 u_n^k}{\partial x^2}ds \ ; \ \ n \geq 0 \end{array}\right. \tag{7}$$

For $k = 1$, we have the following SBA algorithm:

$$\left\{\begin{array}{l} u_0^1(x,t) = u^1(x,0) + i\int_0^t N(u^0(x,s))ds \\ u_{n+1}^1(x,t) = i\int_0^t \frac{\partial^2 u_n^1}{\partial x^2}ds \ \ n \geq 0 \end{array}\right. \tag{8}$$

let's suppose that one can find u^0 as $N\left(u^0(x,t)\right) = 0$, we obtain the following SBA algorithm:

$$\left\{\begin{array}{l} u_0^1(x,t) = ae^{i\alpha x} \\ u_{n+1}^1(x,t) = i\int_0^t \frac{\partial^2 u_n^1(x,s)}{\partial x^2}ds \end{array}\right. \tag{9}$$

From (9), we get:

$$\begin{cases} u_0^1(x,t) = ae^{i\alpha x} \\ u_1^1(x,t) = ai^3\alpha^2 te^{i\alpha x} \\ u_2^1(x,t) = ai^6\alpha^4\dfrac{t^2}{2!}e^{i\alpha x} \\ u_3^1(x,t) = ai^9\alpha^6\dfrac{t^3}{3!}e^{i\alpha x} \\ \ldots \\ u_n^1(x,t) = a\dfrac{(i^3\alpha^2 t)^k}{n!}e^{i\alpha x} \end{cases} \tag{10}$$

Thus the exact solution to the first step is

$$u^1(x,t) = ae^{i\alpha x}\sum_{k=0}^{\infty}\frac{(i^3\alpha^2 t)^k}{k!} = ae^{i\alpha x}e^{i^3\alpha^2 t} = ae^{-i\alpha^2 t + i\alpha x} \tag{11}$$

For $k = 2$, we have the following SBA algorithm:

$$\begin{cases} u_0^2(x,t) = ae^{i\alpha x} + i\int_0^t N(u^1(x,s))ds\ ;\ \ k \geq 1 \\ u_{n+1}^2(x,t) = \int_0^t i\dfrac{\partial^2 u_n^2}{\partial x^2}ds\ ;\ \ n \geq 0 \end{cases} \tag{12}$$

We remark that

$$\begin{aligned} Nu^1(x,t) &= \beta\tfrac{\partial|u^1(x,t)|^\lambda}{\partial x}u^1(x,t) + \gamma|u^1(x,t)|^d u^1(x,t) \\ &= \gamma|a|^d ae^{-i\alpha^2 t + i\alpha x} \\ &= \gamma|a|^d u^1(x,t) \neq 0 \end{aligned} \tag{13}$$

According the SBA method, we should have

$$Nu^1(x,t) = 0 \tag{14}$$

We rewrite the problem (1) in the following form:

$$\begin{cases} i\dfrac{\partial u(x,t)}{\partial t} + \dfrac{\partial^2 u(x,t)}{\partial x^2} + \beta\tfrac{\partial|u(x,t)|^\lambda}{\partial x}u(x,t) + \gamma|u(x,t)|^d u(x,t) - \\ \gamma|a|^d u(x,t) + \gamma|a|^d u(x,t) = 0\ \ \forall\ \ (\beta,\gamma,d,\lambda) \in \mathbb{R}^4, \quad t > 0 \\ u(x,0) = ae^{i\alpha x}\ \ \ \forall\ \ \ a,\alpha \in \mathbb{R},\ x \in \Omega \subset \mathbb{R} \end{cases} \tag{15}$$

From (15), we have:

$$\frac{\partial u}{\partial t} = i\frac{\partial^2 u}{\partial x^2} + iNu(x,t) + i\gamma|a|^d u(x,t) \tag{16}$$

where

$$Nu(x,t) = \beta\frac{\partial\,|u(x,t)|^\lambda}{\partial x}u(x,t) + \gamma|u(x,t)|^d u(x,t) - \gamma|a|^d u(x,t) \tag{17}$$

From (16), we obtain

$$u(x,t) = u(x,0) + i\int_0^t \frac{\partial^2 u(x,s)}{\partial x^2}ds + \int_0^t iNu(x,s)ds + \int_0^t i\gamma|a|^d u(x,s)ds. \tag{18}$$

and we have the following SBA algorithm:

$$\begin{cases} u_0^k(x,t) = ae^{i\alpha x} + \int_0^t iNu^{k-1}(x,s)ds\ ;\ \ k \geq 1 \\ u_{n+1}^k(x,t) = i\int_0^t \dfrac{\partial^2 u_n^k(x,s)}{\partial x^2}ds + i\gamma|a|^d\int_0^t u_n^k(x,s)ds\ ;\ \ n \geq 0 \end{cases} \tag{19}$$

For $k = 1$, we have the following SBA algorithm:

$$\begin{cases} u_0^1(x,t) = ae^{i\alpha x} + \int_0^t iNu^0(x,s)ds \\ u_{n+1}^1(x,t) = i\int_0^t \dfrac{\partial^2 u_n^1(x,s)}{\partial x^2}ds + i\gamma|a|^d\int_0^t u_n^1(x,s)ds\ ;\ \ n \geq 0 \end{cases} \tag{20}$$

let's suppose that one can find u^0 as $N\left(u^0(x,t)\right) = 0$ (we can take $u^0 = ae^{-i\alpha^2 t + i\alpha x}$), we obtain the following SBA algorithm:

$$\begin{cases} u_0^1(x,t) = ae^{i\alpha x} \\ u_{n+1}^1(x,t) = i\int_0^t \dfrac{\partial^2 u_n^1(x,s)}{\partial x^2}ds + i\gamma|a|^d\int_0^t u_n^1(x,s)ds\ ;\ \ n \geq 0 \end{cases} \tag{21}$$

From (21), we obtain:

$$\begin{cases} u_0^1(x,t) = ae^{i\alpha x} \\ u_1^1(x,t) = (-\alpha^2 + \gamma|a|^d)iate^{i\alpha x} \\ u_2^1(x,t) = (-\alpha^2 + \gamma|a|^d)^2 ai^2 \dfrac{t^2}{2!} e^{i\alpha x} \\ u_3^1(x,t) = (-\alpha^2 + \gamma|a|^d)^3 ai^3 \dfrac{t^3}{3!} e^{i\alpha x} \\ \dots \\ u_n^1(x,t) = ae^{i\alpha x} \dfrac{[i(-\alpha^2 + \gamma|a|^d)t]^n}{n!} \end{cases} \tag{22}$$

Thus the exact solution to the first step is

$$u^1(x,t) = ae^{i\alpha x} \sum_{n=0}^{\infty} \frac{[i(-\alpha^2 + \gamma|a|^d)t]^n}{n!} = ae^{i[(\gamma|a|^d - \alpha^2)t + \alpha x]} \tag{23}$$

For $k = 2$, we have the following SBA algorithm:

$$\begin{cases} u_0^2(x,t) = ae^{i\alpha x} + \int_0^t iNu^1(x,s)ds \\ u_{n+1}^2(x,t) = i\int_0^t \dfrac{\partial^2 u_n^2(x,s)}{\partial x^2} ds + i\gamma|a|^d \int_0^t u_n^2(x,s)ds \; ; \;\; n \geq 0 \end{cases} \tag{24}$$

We remark that:

$$\begin{aligned} Nu^1(x,t) &= \beta \tfrac{\partial|u^1(x,t)|^\lambda}{\partial x} u^1(x,t) + \gamma|u^1(x,t)|^d u^1(x,t) - \gamma a^d u^1(x,t) \\ &= 0 + \gamma|a|^d u^1(x,t) - \gamma|a|^d u^1(x,t) \\ &= 0 \end{aligned} \tag{25}$$

and (24) becomes:

$$\begin{cases} u_0^2(x,t) = ae^{i\alpha x} \\ u_{n+1}^2(x,t) = i\int_0^t \dfrac{\partial^2 u_n^2(x,s)}{\partial x^2} ds + i\gamma|a|^d \int_0^t u_n^2(x,s)ds \; ; \;\; n \geq 0 \end{cases} \tag{26}$$

We remark that (26) is the same algorithm that (21), thus

$$u^2(x,t) = ae^{i[(\gamma|a|^d - \alpha^2)t + \alpha x]} \tag{27}$$

Using the same the same procedure, we get:

$$u^1(x,t) = u^2(x,t) = ... = u^k(x,t) = ae^{i[(\gamma|a|^d - \alpha^2)t + \alpha x]} \tag{28}$$

and the exact solution of (1) is:

$$u(x,t) = \lim_{k\to+\infty} u^k(x,t)t) = ae^{i\left[\left(\gamma|a|^d - \alpha^2\right)t + \alpha x\right]} \tag{29}$$

2.1 The Kundu-Eckhaus equation Cauchy problem's in the case of space dimension 3

In the case of space dimension space 3, the Kundu-Eckhaus equation Cauchy problem's is:

$$\begin{cases} i\frac{\partial u(x,y,z,t)}{\partial t} + \mu_1\frac{\partial^2 u(x,y,z,t)}{\partial x^2} + \mu_2\frac{\partial^2 u(x,y,z,t)}{\partial y^2} + \mu_3\frac{\partial^2 u(x,y,z,t)}{\partial z^2} + \beta\frac{\partial|u(x,y,z,t)|^\lambda}{\partial x}u(x,y,z,t) + \\ \qquad \sigma\frac{\partial|u(x,y,z,t)|^\lambda}{\partial y}u(x,y,z,t) + \delta\frac{\partial|u(x,y,z,t)|^\lambda}{\partial z}u(x,y,z,t) + \gamma|u(x,y,z,t)|^d u(x,y,z,t) = 0 \\ u(x,y,z,0) = ae^{i\alpha(x+y+z)}\ ;\ \forall \quad a,\alpha \in \mathbb{R},\ (x,y,z) \in \mathbb{R}^3 \end{cases} \tag{30}$$

From (30), we have:

$$\frac{\partial u}{\partial t} = i\mu_1\frac{\partial^2 u}{\partial x^2} + i\mu_2\frac{\partial^2 u}{\partial y^2} + i\mu_3\frac{\partial^2 u}{\partial z^2} + iNu \tag{31}$$

where

$$Nu = \beta\frac{\partial\,|u|^\lambda}{\partial x}u + \sigma\frac{\partial\,|u|^\lambda}{\partial y}u + \delta\frac{\partial\,|u|^\lambda}{\partial z}u + \gamma|u|^d u \tag{32}$$

From (31), we obtain:

$$u(x,y,z,t) = u(x,y,z,0) + \int_0^t (i\mu_1\frac{\partial^2 u}{\partial x^2} + i\mu_2\frac{\partial^2 u}{\partial y^2} + i\mu_3\frac{\partial^2 u}{\partial z^2} + iNu)ds \tag{33}$$

According to the SBA method, we suppose that the solution of (30) has the following form:

$$u(x,y,z,t) = \lim_{k\to+\infty} u^k(x,y,z,t) \tag{34}$$

where

$$u^k(x,y,z,t) = \sum_{n=0}^{+\infty} u_n^k(x,y,z,t) \quad ; \quad k \geq 1$$

and, for every $k \geq 1$, we get $u_n^k(x,y,z,t)$ for $n \geq 0$, through the following SBA algorithm:

$$\left\{ \begin{array}{l} u_0^k(x,y,z,t) = u^k(x,y,z,0) + i\int_0^t N(u^{k-1}(x,y,z,s))ds \; ; \;\; k \geq 1 \\ u_{n+1}^k(x,y,z,t) = \int_0^t i\mu_1 \dfrac{\partial^2 u_n^k}{\partial x^2} ds + \int_0^t i\mu_2 \dfrac{\partial^2 u_n^k}{\partial y^2} ds + i\mu_3 \int_0^t \dfrac{\partial^2 u_n^k}{\partial z^2} ds \; ; \;\; n \geq 0 \end{array} \right. \tag{35}$$

For $k = 1$, we have the following SBA algorithm:

$$\left\{ \begin{array}{l} u_0^1(x,y,z,t) = u^1(x,y,z,0) + i\int_0^t N(u^0(x,y,z,s))ds \\ u_{n+1}^1(x,y,z,t) = \int_0^t i\mu_1 \dfrac{\partial^2 u_n^1}{\partial x^2} ds + \int_0^t i\mu_2 \dfrac{\partial^2 u_n^1}{\partial y^2} ds + i\mu_3 \int_0^t \dfrac{\partial^2 u_n^1}{\partial z^2} ds \; ; \;\; n \geq 0 \end{array} \right. \tag{36}$$

let's suppose that one can find u^0 as $N\left(u^0(x,y,z,t)\right) = 0$, we obtain the following SBA algorithm:

$$\left\{ \begin{array}{l} u_0^1(x,y,z,t) = a e^{i\alpha(x+y+z)} \\ u_{n+1}^1(x,t) = \int_0^t i\mu_1 \dfrac{\partial^2 u_n^1}{\partial x^2} ds + \int_0^t i\mu_2 \dfrac{\partial^2 u_n^1}{\partial y^2} ds + i\mu_3 \int_0^t \dfrac{\partial^2 u_n^1}{\partial z^2} ds \; ; \;\; n \geq 0 \end{array} \right. \tag{37}$$

From (37), we obtain:

$$\left\{ \begin{array}{l} u_1^1(x,y,z,t) = \frac{(\mu_1+\mu_2+\mu_3)\alpha^2 i^3 t}{1!} a e^{i\alpha(x+y+z)} \\ u_2^1(x,y,z,t) = \dfrac{[(\mu_1+\mu_2+\mu_3)i^3\alpha^2 t]^2}{2!} a e^{i\alpha(x+y+z)} \\ \ldots\ldots \\ u_n^1(x,y,z,t) = \dfrac{[(\mu_1+\mu_2+\mu_3)i^3\alpha^2 t]^n}{n!} a e^{i\alpha(x+y+z)} \end{array} \right. \tag{38}$$

Thus the exact solution to the first step is

$$\left\{\begin{aligned} u^1(x,y,z,t) &= ae^{i\alpha(x+y+z)}\sum_{n=0}^{\infty}\frac{[(\mu_1+\mu_2+\mu_3)i^3\alpha^2t]^n}{n!}\\ &= ae^{i\alpha(x+y+z)}e^{(\mu_1+\mu_2+\mu_3)i^3\alpha^2t}\\ &= ae^{i\left(\alpha(x+y+z)-(\mu_1+\mu_2+\mu_3)\alpha^2t\right)} \end{aligned}\right. \tag{39}$$

For $k=2$, we have the following SBA algorithm:

$$\left\{\begin{aligned} u_0^2(x,y,z,t) &= ae^{i\alpha(x+y+z)} + i\int_0^t N(u^1(x,y,z,s))ds\\ u_{n+1}^2(x,y,z,t) &= \int_0^t i\mu_1\frac{\partial^2 u_n^2}{\partial x^2}ds + \int_0^t i\mu_2\frac{\partial^2 u_n^2}{\partial y^2}ds + i\mu_3\int_0^t \frac{\partial^2 u_n^2}{\partial z^2}ds;\ \ n\geq 0 \end{aligned}\right. \tag{40}$$

We remark that

$$\begin{aligned} Nu^1(x,y,z,t) &= \beta\frac{\partial|u^1|^\lambda}{\partial x}u^1 + \sigma\frac{\partial|u|^\lambda}{\partial y}u + \delta\frac{\partial|u|^\lambda}{\partial z}u + \gamma|u^1|^d u^1\\ &= \gamma|a|^d ae^{-i\alpha^2t+i\alpha x} = \gamma|a|^d u^1 \neq 0 \end{aligned} \tag{41}$$

According the SBA method, we should have

$$Nu^1(x,y,z,t) = 0 \tag{42}$$

Let’s rewrite the problem (30) in the following form:

$$\left\{\begin{array}{l} i\frac{\partial u}{\partial t} + \mu_1\frac{\partial^2 u}{\partial x^2} + \mu_2\frac{\partial^2 u}{\partial y^2} + \mu_3\frac{\partial^2 u}{\partial z^2} + \beta\frac{\partial|u|^\lambda}{\partial x}u + \sigma\frac{\partial|u|^\lambda}{\partial y}u \not+ \delta\frac{\partial|u|^\lambda}{\partial z}u + \gamma|u|^d u + \gamma|a|^d u - \gamma|a|^d u = 0\\ u(x,y,z,0) = ae^{i\alpha(x+y+z)} \end{array}\right. \tag{43}$$

From (43), we have:

$$i\frac{\partial u}{\partial t} + \mu_1\frac{\partial^2 u}{\partial x^2} + \mu_2\frac{\partial^2 u}{\partial y^2} + \mu_3\frac{\partial^2 u}{\partial z^2} + Nu + \gamma|a|^d u = 0 \tag{44}$$

where

$$Nu = \beta\frac{\partial\,|u|^\lambda}{\partial x}u + \sigma\frac{\partial\,|u|^\lambda}{\partial y}u \not+ \delta\frac{\partial\,|u|^\lambda}{\partial z}u + \gamma|u|^d u - \gamma|a|^d u \tag{45}$$

From (44), we obtain:

$$\begin{aligned} u(x,y,z,t) = u(x,y,z,0) + i\mu_1 \int_0^t \frac{\partial^2 u(x,y,z,s)}{\partial x^2} ds + i\mu_2 \int_0^t \frac{\partial^2 u(x,y,z,s)}{\partial y^2} ds + \\ i\mu_3 \int_0^t \frac{\partial^2 u(x,y,z,s)}{+\partial z^2} ds \int_0^t iNu(x,y,z,s) ds + \int_0^t i\gamma|a|^d u(x,y,z,s) ds \end{aligned} \tag{46}$$

According to the SBA method, we suppose that the solution of (43) has the following form:

$$u(x,y,z,t) = \lim_{k\to+\infty} u^k(x,y,z,t) \tag{47}$$

where

$$u^k(x,y,z,t) = \sum_{n=0}^{+\infty} u_n^k(x,y,z,t); \quad k \geq 1 \tag{48}$$

and, for every $k \geq 1$, we get $u_n^k(x,y,z,t)$ for $n \geq 0$, through the following SBA algorithm:

$$\begin{cases} u_0^k(x,y,z,t) = ae^{i\alpha(x+y+z)} + i\int_0^t N(u^{k-1}(x,y,z,s))ds \,; \;\; k \geq 1 \\ u_{n+1}^k(x,y,z,t) = \int_0^t i\mu_1 \frac{\partial^2 u_n^k}{\partial x^2} ds + \int_0^t i\mu_2 \frac{\partial^2 u_n^k}{\partial y^2} ds + i\mu_3 \int_0^t \frac{\partial^2 u_n^k}{\partial z^2} ds \; + \int_0^t i\gamma|a|^d u^k(x,y,z,s)ds; \;\; n \geq 0 \end{cases} \tag{49}$$

Thus, for $k = 1$, we have :

$$\begin{cases} u_0^1(x,y,z,t) = ae^{i\alpha(x+y+z)} + i\int_0^t N(u^0(x,y,z,s))ds \,; \;\; k \geq 1 \\ u_{n+1}^1(x,y,z,t) = \int_0^t i\mu_1 \frac{\partial^2 u_n^1}{\partial x^2} ds + \int_0^t i\mu_2 \frac{\partial^2 u_n^1}{\partial y^2} ds + i\mu_3 \int_0^t \frac{\partial^2 u_n^1}{\partial z^2} ds \; + \int_0^t i\gamma|a|^d u^1(x,y,z,s)ds; \;\; n \geq 0 \end{cases} \tag{50}$$

Let's suppose that one can find u^0 as $N\left(u^0(x,y,z,t)\right) = 0$, we obtain the following SBA algorithm:

$$\begin{cases} u_0^1(x,y,z,t) = ae^{i\alpha(x+y+z)} \\ u_{n+1}^1(x,y,z,t) = \int_0^t i\mu_1 \frac{\partial^2 u_n^1}{\partial x^2} ds + \int_0^t i\mu_2 \frac{\partial^2 u_n^1}{\partial y^2} ds + i\mu_3 \int_0^t \frac{\partial^2 u_n^1}{\partial z^2} ds \; + \int_0^t i\gamma|a|^d u^1 ds; \;\; n \geq 0 \end{cases} \tag{51}$$

From (51), we obtain:

$$\begin{cases} u_1^1(x,y,z,t) = (-(\mu_1+\mu_2+\mu_3)\alpha^2 + \gamma|a|^d) i a t e^{i\alpha(x+y+z)} \\ u_2^1(x,y,z,t) = \dfrac{\left[\gamma|a|^d - \alpha^2(\mu_1+\mu_2+\mu_3)\right]^2}{2!} a e^{i\alpha(x+y+z)} \\ \quad \ldots\ldots \\ u_n^1(x,y,z,t) = \dfrac{\left[(\gamma|a|^d - (\mu_1+\mu_2+\mu_3) i\alpha^2) it\right]^n}{n!} a e^{i\alpha(x+y+z)} \end{cases} \tag{52}$$

Thus the exact solution to the first step is

$$\begin{aligned} u^1(x,y,z,t) &= ae^{i\alpha(x+y+z)} \sum_{n=0}^{\infty} \frac{\left[(\gamma|a|^d-(\mu_1+\mu_2+\mu_3)\alpha^2)it\right]^n}{n!} \\ &= ae^{i\alpha(x+y+z)} e^{(\gamma|a|^d-(\mu_1+\mu_2+\mu_3)\alpha^2)it} \\ &= ae^{i\left(\alpha(x+y+z)-(\mu_1+\mu_2+\mu_3)\alpha^2 t\right)} \end{aligned} \tag{53}$$

For $k=2$, we have the following SBA algorithm:

$$\begin{cases} u_0^2(x,y,z,t) = ae^{i\alpha(x+y+z)} + i\int_0^t N(u^1(x,y,z,s))ds \\ u_{n+1}^2(x,y,z,t) = \int_0^t i\mu_1 \dfrac{\partial^2 u_n^2}{\partial x^2} ds + \int_0^t i\mu_2 \dfrac{\partial^2 u_n^2}{\partial y^2} ds + i\mu_3 \int_0^t \dfrac{\partial^2 u_n^2}{\partial z^2} ds + \int_0^t i\gamma|a|^d u^2 ds\ ;\ \ n \geq 0 \end{cases} \tag{54}$$

We remark that

$$Nu^1(x,y,z,t) = \beta\frac{\partial\left|u^1\right|^\lambda}{\partial x}u^1+\sigma\frac{\partial\left|u^1\right|^\lambda}{\partial y}u^1+\delta\frac{\partial\left|u^1\right|^\lambda}{\partial z}u^1+\gamma|u^1|^d u^1-\gamma|a|^d u^1 = \gamma|a|^d u^1-\gamma|a|^d u^1 = 0 \tag{55}$$

(54) becomes:

$$\begin{cases} u_0^2(x,y,z,t) = ae^{i\alpha(x+y+z)} \\ u_{n+1}^2(x,y,z,t) = \int_0^t i\mu_1 \dfrac{\partial^2 u_n^2}{\partial x^2} ds + \int_0^t i\mu_2 \dfrac{\partial^2 u_n^2}{\partial y^2} ds + i\mu_3 \int_0^t \dfrac{\partial^2 u_n^2}{\partial z^2} ds\ + \int_0^t i\gamma|a|^d u^2 ds;\ \ n \geq 0 \end{cases} \tag{56}$$

We remark that (56) is the same algorithm that (51), thus

$$u^2(x,y,z,t) = ae^{i\left(\alpha(x+y+z)-(\mu_1+\mu_2+\mu_3)\alpha^2 t\right)} \tag{57}$$

Using the same procedure, we get:

$$u^1(x,y,z,t) = u^2(x,y,z,t) = ... = u^k(x,y,z,t) = ae^{i\left(\alpha(x+y+z)-(\mu_1+\mu_2+\mu_3)\alpha^2 t\right)} \tag{58}$$

and the exact solution of (30) is:

$$u(x,y,z,t) = \lim_{k\to+\infty} u^k(x,y,z,t) = ae^{i\left(\alpha(x+y+z)-(\mu_1+\mu_2+\mu_3)\alpha^2 t\right)} \tag{59}$$

3. MAIN RESULT

Proposition $\forall\ \gamma, \alpha, a, \beta_j, \mu_j, \lambda\ \in\ \mathbb{R}^*\ \ (j\ =\ 1,2,...,m)$, the exact solution of the following Kundu-ECKHAUS equation in the case of space dimension $m \in \mathbb{N}$:

$$\begin{gathered} i\frac{\partial u(x_1,...,x_m,t)}{\partial t} + \sum_{j=1}^m \mu_j \frac{\partial^2 u(x_1,...,x_m,t)}{\partial x_j^2} + \sum_{j=1}^m \beta_j \frac{\partial |u(x_1,...,x_m,t)|^\lambda}{\partial x_j} + \\ \gamma\left|u(x_1,...,x_m,t)\right|^d u(x_1,...,x_m,t) = 0 \end{gathered} \tag{60}$$

with the following initial conditions

$$u(x_1, ..., x_m, 0) = ae^{\sum_{j=1}^{m} i\alpha x_j} \tag{61}$$

is:

$$u(x_1, ..., x_m, t) = a\prod_{j=1}^{m} e^{i\left[\left(\frac{\gamma|a|^d}{m} - \alpha^2\mu_j\right)t + \alpha x_j\right]} \quad \forall \quad m \in \mathbb{N}^* \tag{62}$$

Proof

If $t = 0$, from (62), we obtain (61).
If $t \neq 0$, from (62), we get:

$$\frac{\partial u(x_1, ..., x_m, t)}{\partial t} = u(x_1, ..., x_m, t)i\left[\sum_{j=1}^{m} \frac{\gamma|a|^d}{m} - \sum_{j=1}^{m} \alpha^2\mu_j\right] \tag{63}$$

and from (63) we have

$$i\frac{\partial u(x_1, ..., x_m, t)}{\partial t} = -\gamma|a|^d u(x_1, ..., x_m, t) + \alpha^2 u(x_1, ..., x_m, t)\sum_{j=1}^{m} \mu_j \tag{64}$$

Besides

$$\sum_{j=1}^{m} \beta_j \frac{\partial\,|u(x_1, ..., x_m, t)|^\lambda}{\partial x_j} + \gamma\,|u(x_1, ..., x_m, t)|^d\, u(x_1, ..., x_m, t) = \gamma|a|^d u(x_1, ..., x_m, t) \tag{65}$$

$$\frac{\partial u(x_1, ..., x_m, t)}{\partial x_j} = u(x_1, ..., x_m, t)i\alpha \tag{66}$$

$$\frac{\partial^2 u(x_1, ..., x_m, t)}{\partial x_j^2} = -\alpha^2 u(x_1, ..., x_m, t) \tag{67}$$

$$\sum_{j=1}^{m} \mu_j \frac{\partial^2 u(x_m, t)}{\partial x_j^2} = -\alpha^2 u(x_m, t)\sum_{j=1}^{m} \mu_j \tag{68}$$

Thus

$$i\frac{\partial u}{\partial t} + \sum_{j=1}^{m} \mu_j \frac{\partial^2 u}{\partial x_j^2} + \sum_{j=1}^{m} \beta_j \frac{\partial\,|u|^\lambda}{\partial x_j} + \gamma\,|u|^d\, u = -\gamma|a|^d u + \alpha^2 u\sum_{j=1}^{m} \mu_j - \alpha^2 u(x_m, t)\sum_{j=1}^{m} \mu_j + \gamma|a|^d u = 0 \tag{69}$$

4. CONCLUSION

Through this example, we showed again the usefulness of the SBA method , in the search of an approximate or the exact solution of an nonlinear differential equation.

REFERENCES

[1] G.D.NKAYA, Travaux de thèse de doctorat,.Université Marien NGOUABI. Décembre 2017

[2] B. ABBO, Nouvel algorithme numérique de résolution des Equations Différentielles Ordinaires (EDO) et Equations aux Dérivées Partielles (EDP) non linéaires, Thèse de Doctorat Unique, Université de Ouagadougou, janvier 2007, UFR/SEA; Département de Mathématiques et Informatique (Burkina Faso).

[3] B. ABBO, N. N'GARASTA, B. SOME and L. SOME, A new Approach of the Adomian Algorithim for solving Nonlinear Partial or Ordinary Differential Equations, Far East J. Appl. Maths. 23(3) (2006), pp. 299-312.

[4] K. ABBAOUI and Y. CHERRUAULT, Convergence of Adomian method applied to non linear equations, Math. Comput. Modelling (20,9), pp60-73, 1994.

[5] K. ABBAOUI and Y. CHERRUAULT, The Decomposition method applied to the Cauchy problem, Kybernetes, (28,1), pp68-74, 1999.

[6] N. NGARHASTA, B. SOME, K.ABBAOUI and Y. CHERRUAULT, New numerical study of Adomian method applied to a diffusion model, Kybernetes, Vol.31, no1, pp61-75, 2002.