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On nano $gb^\star$-closed sets

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Abstract

The purpose of this paper is to define and study a new class of sets called nano $gb^\star$-closed in nano topological spaces is introduced and its properties are investigated.

AMS subject classification: 54A05, 54C10, 54B05.
Keywords: nano closed, nano b-closed set, nano gb-closed set, nano $gb^\star$-closed.

1. Introduction

Lellis Thivagar et al. [7] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space.

Bhuvaneswari et al. [5] introduced and investigated nano g-closed sets in nano topological spaces. In this paper is to define and study a new class of sets called nano $gb^\star$-closed in nano topological spaces is introduced and its properties are investigated.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. [7] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. [7] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. [7] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano semi-open [7] if $H \subseteq Ncl(Nint(H))$.
2. nano regular-open [7] if $H = Nint(Ncl(H))$.
3. nano pre-open [7] if $H \subseteq Nint(Ncl(H))$.
4. nano α-open [9] if $H \subseteq Nint(Ncl(Nint(H)))$.
5. nano b-open [10] if $H \subseteq Nint(Ncl(H)) \cup Ncl(Nint(H))$.
6. nano β-open [12] if $H \subseteq Ncl(Nint(Ncl(H))$.
7. nano π-open [1] if the finite union of nano regular-open sets.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano g-closed [4] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
2. nano $g^{\star}$-closed [15] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano g-open.
3. nano $\hat{g}$-closed [6] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.
4. nano sg-closed [3] if $Nscl(H)' \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.
5. nano wg-closed [8] if $Ncl(Nint(H))' \subseteq G$, whenever $H \subseteq G$ and G is nano open.
6. nano $g\alpha$-closed [16] if $N\alpha cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano α-open.

7. nano gp-closed [5] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
8. nano $g\beta$-closed [12] if $N\beta cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
9. nano gb-closed [2] if $Nbcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

The complements of the above mentioned sets is called their respective open sets.

3. On nano $gb^\star$-closed sets

Definition 3.1. A subset H of space $(U, \tau_R(X))$ is called nano $gb^\star$-closed if $Nint(Ncl(H)) \subseteq G$, whenever $H \subseteq G$ and G is nano gb-open.

The complement of nano $gb^\star$-open if $H^c = U - H$ is nano $gb^\star$-closed.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$.

1. then $\{c, d\}$ is nano $gb^\star$-closed.
2. then $\{a, b, d\}$ is nano $gb^\star$-open.

Theorem 3.3. In a space $(U, \tau_R(X))$, every nano closed is nano $gb^\star$-closed.

Proof. Let H be a nano closed in U, then $Ncl(H) = H$ and G is any nano gb-open where $H \subseteq G$. Since $Nint(H) \subseteq H$. implies that $Nint(Ncl(H) \subseteq G$. Therefore H is nano $gb^\star$-closed. ■

Remark 3.4. The converse of the Theorem 3.3 need not be true as seen by the following example.

Example 3.5. In Example 3.2, then $\{d\}$ is nano $gb^\star$-closed but not nano closed.

Theorem 3.6. In a space $(U, \tau_R(X))$, a set H is nano $gb^\star$-closed $\iff$ $Nint(Ncl(H)) - H$ contains no non-empty nano gb-closed.

Proof. Necessity: Suppose that F is a non-empty nano gb-closed subset of $Nint(Ncl(H))$ such that $F \subseteq Nint(Ncl(H)) - H$. Then $F \subseteq Nint(Ncl(H)) \cap H^c$. Hence $F \subseteq Nint(Ncl(H))$ and $F' \subseteq H^c$. Since F^c is nano gb-closed and H is nano $gb^\star$-closed $Nint(Ncl(H))' \subseteq F^C$. Therefore $F \subseteq (Nint(Ncl(H)))^c$. Hence $F \subseteq (Nint(Ncl(H)))\cap (Nint(Ncl(H)))^c = \phi$. Thus $F = \phi$ and this implies that $Nint(Ncl(H)) - H$ contains no non-empty nano gb-closed.

Sufficiency: Assume that $Nint(Ncl(H)) - H$ contains no non-empty nano gb-closed. Let $H' \subseteq G$, G is nano gb-open. Suppose that $Nint(Ncl(H))$ is not contained in G, then $Nint(Ncl(H)) \cap G^c$ is a nonempty nano gb-closed of $Nint(Ncl(H)) - H$ which is a contradiction. Hence $Nint(Ncl(H)) \subseteq G$ and hence H is nano $gb^\star$-closed. ■

Theorem 3.7. In a space $(U, \tau_R(X))$, let $K \subseteq Y' \subseteq U$ if K is nano $gb^\star$-closed relative to Y and that Y is both nano gb-open and nano $gb^\star$'closed in U. Then K is nano $gb^\star$'closed.

Proof. Let $G \subseteq K$ and G be a nano gb-open in U. But $K \subseteq Y \subseteq U$. Hence $K' \subseteq Y$ and $G \subseteq K$. This implies that $Y \cap G \subseteq K$. Since K is nano $gb^\star$-closed relative to Y, Then $Y \cap G' \subseteq Nint(Ncl(Y))$. That is $Y \cap G \subseteq Y \cap Nint(Ncl(Y)) \Rightarrow G \subseteq Y \cap Nint(Ncl(Y))$ thus $G' \cup (Nint(Ncl(K)))^c \subseteq Y \cap (Nint(Ncl(K)) \cup (Nint(Ncl(K)))^c$. This implies that $G \cup (Nint(Ncl(K)))^c \subseteq Nint(Ncl(Y))' \subseteq Nint(Ncl(K))$. Hence $G' \subseteq Nint(Ncl(K))$. Since $Nint(Ncl(K))$ is not contained in $Nint(Ncl(K))^c$. Thus K is nano $gb^\star$-closed relative to U. ■

Theorem 3.8. In a space $(U, \tau_R(X))$, let $H \subseteq Y \subseteq U$ and suppose that H is nano $gb^\star$-closed in U then H is nano $gb^\star$-closed relative to Y.

Proof. Assume that $H \subseteq Y \subseteq U$ and H is nano $gb^\star$-closed in U . Then H is nano $gb^\star$-closed relative to Y, let $H \subseteq Y' \cap G$ where G is nano gb-open in U. Since H is nano $gb^\star$ -closed in U, $H \subseteq G \Rightarrow Nint(Ncl(H)) \subseteq G$. That is $Y \cap Nint(Ncl(H)) \subseteq Y \cap G$. Where $Y' \cap Nint(Ncl(H))$ is $Nint(Ncl(H)) \in Y$. Hence H is nano $gb^\star$-closed relative to Y. ■

Theorem 3.9. In a space $(U, \tau_R(X))$, if H is a nano $gb^\star$-closed and $H \subseteq K \subseteq Nint(Ncl(H))$ then K is a nano $gb^\star$-closed.

Proof. Let G be a nano gb-open set of U, such that $K \subseteq G$. Then $H \subseteq G$. Since H is nano $gb^\star$-closed. Then $Nint(Ncl(H)) \subseteq G$. Now $Nint(Ncl(K))' \subseteq Nint(Ncl(H)) \subseteq G$. Thus K is nano $gb^\star$-closed in U. ■

Theorem 3.10. In a space $(U, \tau_R(X))$, the intersection of a nano $gb^\star$-closed and nano closed is a nano $gb^\star$-closed.

Proof. Let H be a nano $gb^\star$-closed and F be a nano closed. Since H is nano $gb^\star$-closed, $Nint(Ncl(H)) \subseteq G$ whenever $H \subseteq G$, where G is a nano gb-open. To show that $H \cap F$ is nano $gb^\star$-closed, it is enough to show that $Nint(Ncl(H)) \cap F \subseteq G$ whenever $H' \cap F' \subseteq G$, where G is nano gb-open. Let $M = U.\ F$ then $H' \subseteq G \cup M$. Since M is nano open, $G \cup M$ is nano gb-open and H is nano $gb^\star$-closed, $Nint(Ncl(H)) \subseteq G' \cup M$. Now $Nint(Ncl(H)) \cap F \subseteq Nint(Ncl(H))' \cap Nint(Ncl(F)) \subseteq N(int(Ncl(H)))' \cap F \subseteq (G' \cup M)' \cap F \subseteq (G \cap F) \cup (M \cap F) \subseteq (G \cap F) \cup \phi \subseteq G$. This $\Rightarrow H \cap F$ is nano $gb^\star$-closed. ■

Theorem 3.11. If H and K are two nano $gb^\star$-closed defined for a non-empty set U, then their intersection $H \cap K$ is nano $gb^\star$-closed in U.

Proof. Let H and K are two nano $gb^\star$-closed in U. Let $H \cap K \subseteq G$, G is nano gp-open in U. Since H is nano $gb^\star$-closed, $Nint(Ncl(H)) \subseteq G$, whenever $H' \subseteq G$, G is nano g-open in U. Since K is nano $gb^\star$-closed, $Nint(Ncl(K)) \subseteq G$, whenever $K' \subseteq G$, G is nano g-open in U. Thus $H' \cap K$ is nano $gb^\star$-closed. ■

Remark 3.12. In a space $(U, \tau_R(X))$, the Union of two nano $gb^\star$-closed need not to be nano $gb^\star$-closed.

Example 3.13. In Example 3.2, we have $E = \{d\}$ and $F = \{a, b\}$. Clearly $E \cup F = \{a, b, d\}$ is not nano $gb^\star$-closed.

Theorem 3.14. In a space $(U, \tau_R(X))$, every nano $gb^\star$-closed is nano b-closed.

Proof. Assume that H is a nano $gb^\star$-closed in U and let G be a nano open such that $H \subseteq G$. Since every nano open is nano b-open and H is nano $gb^\star$-closed, then $Nint(Ncl(H)) \subseteq Nint(Ncl(H)) \cup Ncl(Nint(H)) \subseteq G$. Thus H is nano b-closed in U. ■

Remark 3.15. The converse of the Theorem 3.14 need not be true as seen by the following example.

Example 3.16. In Example 3.2, then $\{b\}$ is nano b-closed but not nano $gb^\star$-closed.

Theorem 3.17. In a space $(U, \tau_R(X))$, every nano $\hat{g}$-closed is nano $gb^\star$-closed.

Proof. Assume that H is nano $\hat{g}$-closed in U and G is nano semi-open such that $H \subseteq G$, every nano semi-open is nano gb-open then $Ncl(H) \subseteq Nint(Ncl(H))$. Thus H is nano $gb^\star$-closed. ■

Remark 3.18. The converse of the Theorem 3.17 need nt be true as seen by the following example.

Example 3.19. Example 3.2, then $\{a, b\}$ is nano $gb^\star$-closed but not nano $\hat{g}$-closed.

Theorem 3.20. In a space $(U, \tau_R(X))$, every nano $gb^\star$-closed is nano sg-closed.

Proof. Assume that H is nano $gb^\star$-closed in U and G is nano open such that $H \subseteq G$ every nano open is nano semi-open, H is nano $gb^\star$-closed and G is nano gb-closed then $Nint(Ncl(H)) \subseteq H \cup Nscl(H) \subseteq G$. Thus H is nano sg-closed. ■

Remark 3.21. The converse of the Theorem 3.20 need not be true as seen by the following example.

Example 3.22. In Example 3.2, then $\{a\}$ is nano sg-closed but not nano $gb^\star$-closed.

Theorem 3.23. In a space $(U, \tau_R(X))$, every nano $gb^\star$-closed is nano $g\beta$-closed.

Proof. Assume that H is nano $gb^\star$-closed in U and G is nano open such that $H \subseteq G$, every nano open is nano gb-open then $Nint(Ncl(H)) \subseteq H \cup \beta$-closed $\subseteq G$. Thus H is nano $gb^\star$-closed. ■

Remark 3.24. The converse of the Theorem 3.23 need not be true as seen by the following example.

Example 3.25. In Example 3.2, then $\{b, c, d\}$ is nano $g\beta$-closed but not nano $gb^\star$-closed.

Remark 3.26. We obtain Definitions, Theorems, Results and Examples follows from the implications.

$$\begin{array}{ccccc}
\textbf{nano closed} & & \textbf{nano } gb\textbf{-closed} & & \textbf{nano } \hat{g}\textbf{-closed} \\
\searrow & & \downarrow & & \swarrow \\
\textbf{nano } sg\textbf{-closed} & \leftarrow & \textbf{nano } gb^\star\textbf{-closed} & \rightarrow & \textbf{nano } g\beta\textbf{-closed} \\
\nearrow & & \downarrow & & \\
\textbf{nano } b^\star\textbf{-closed} & & \textbf{nano } b\textbf{-closed} & &
\end{array}$$

None the above implications are reversible.

Remark 3.27. In a space $(U, \tau_R(X))$, the following Example then nano g-closed and nano $gb^\star$-closed sets are independent.

Example 3.28. In Example 3.2, then

1. we have $\{a, c\}$ is nano g-closed but not nano $gb^\star$-closed.
2. we have $\{d\}$ is nano $gb^\star$-closed but not nano g-closed.

Remark 3.29. In a space $(U, \tau_R(X))$, the following Example then nano $g^\star$-closed and nano $gb^\star$-closed sets are independent.

Example 3.30. In Example 3.2, then

1. we have $\{b, c\}$ is nano $g^\star$-closed but not nano $gb^\star$-closed.
2. we have $\{a, b\}$ is nano $gb^\star$-closed but not nano $g^\star$-closed.

Remark 3.31. In a space $(U, \tau_R(X))$, the following Example then nano $g\alpha$-closed and nano $gb^\star$-closed sets are independent.

Example 3.32. In Example 3.2, then

1. we have $\{a, c\}$ is nano $g\alpha$-closed but not nano $gb^\star$-closed.
2. we have $\{a, b\}$ is nano $gb^\star$-closed but not nano $g\alpha$-closed.

Remark 3.33. In a space $(U, \tau_R(X))$, the following Example then nano gp-closed and nano $gb^\star$-closed sets are independent.

Example 3.34. In Example 3.2, then

1. we have $\{a\}$ is nano gp-closed but not $gb^\star$-closed.
2. we have $\{a, b\}$ is nano $gb^\star$-closed but not nano gp-closed.

Remark 3.35. In a space $(U, \tau_R(X))$, the following Example then nano wg-closed and nano $gb^\star$-closed sets are independent.

Example 3.36. In Example 3.2, then

1. we have $\{b\}$ is nano wg-closed but not $gb^\star$-closed.
2. we have $\{d\}$ is nano $gb^\star$-closed but not nano wg-closed.

Remark 3.37. We obtain Definitions, Theorems, Results and Examples follows from the implications.

nano $g\alpha$**-closed** $\nleftrightarrow$	**nano** $gb^\star$**-closed**	$\nleftrightarrow$ **nano** g**-closed**
nano gp**-closed** $\nleftrightarrow$	**nano** $gb^\star$**-closed**	$\nleftrightarrow$ **nano** wg**-closed**
nano $g^\star$**-closed** $\nleftrightarrow$	**nano** $gb^\star$**-closed**	

Above implications are independent.

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