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Alternative Methods of Euler's Theorem on Second Degree Homogenous Functions

Introduce Multiple New Methods of Matrices

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Abstract

In this method to Explain the Euler's theorem of second degree homogeneous function. This method. It is alternative method of Euler's theorem on second degree function. This method is very short method of Euler's theorem. Euler's theorem explain this method is very long terms. But I explain that this method is very short terms. I use only the differentiation and Trignometric functions. I don't derivative every step. I derivative only nu. n – is constant u is a function.

Keywords: Differentiation, Trigometric functions, Homogenous functions, Degree, Trignometric fomula.

Introduction – 1

Euler's Theorem on Homogenous Functions

Previous Result

Statement

If u is a homogenous function of degree n in x & y then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

Proof

since u is a homogenous functions of degree n in x & y

$$u = x^n f\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial x} = nx^{n-1} f\left(\frac{y}{x}\right) + f'\left(\frac{y}{x}\right)\left(-\frac{y}{x^2}\right)x^n$$

$$x\frac{\partial u}{\partial x} = nx^n f\left(\frac{y}{x}\right) - yf'\left(\frac{y}{x}\right)x^{n-1} \quad \text{--} \quad (1)$$

$$\frac{\partial u}{\partial y} = x^n f'\left(\frac{y}{x}\right)\frac{1}{x}$$

$$y\frac{\partial u}{\partial y} = x^{n-1} f'\left(\frac{y}{x}\right)y \quad \text{--} \quad (2)$$

(1) + (2)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nx^n f\left(\frac{y}{x}\right)$$

$$= nu$$

Previous Proof

Euler's theorem on homogenous functions of second degree (or) deduction form of homogenous functions.

Statement Previous Proof

If u is a homogenous function of degree n in $x \,\&\, y$ S.T $x^2\frac{\partial^2 u}{\partial x^2} +$ $+ 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$

Proof

By Euler's thereon $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$ -- (1)

Differentiating (1) partially w.r. to x we get

$$x\frac{\partial^2 u}{\partial x^2} + 1 - \frac{\partial u}{\partial x} + \text{y}\frac{\partial^2 u}{\partial x \partial y} = n.\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + \text{y}\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x} \quad \text{--} \quad (2)$$

Differentiating (1) partially w.r. to y we get

$$x\frac{\partial^2 u}{\partial y \partial x} + \text{y}\frac{\partial^2 u}{\partial y^2} + 1.\ \frac{\partial u}{\partial y} = n\frac{\partial u}{\partial y}$$

$$x^2\frac{\partial^2 u}{\partial y \partial x} + \text{y}\frac{\partial^2 u}{\partial y^2} = (n-1)\frac{\partial u}{\partial y} \quad \text{--} \quad (2)$$

Equ (2) x $\boldsymbol{x}$ + (3) x $\boldsymbol{y}$

$$x\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + xy\frac{\partial^2 u}{\partial x \partial y}$$

$$= (n-1)x\frac{\partial u}{\partial x} + (n-1)y\frac{\partial u}{\partial y}$$

$$= (n-1)(nu)$$

$$= n\,(n-1)u$$

Euler's theorem on homogenous function prove by New method

Statement

If u is a homogenous functions of degree n in x & y S.T $x\,\frac{\partial u}{\partial x} + \,y\,\frac{\partial u}{\partial y} = nu$

Proof

Leg $\quad u \quad = \quad x^n\, f\left(\frac{y}{x}\right) = f\,(x, y)$

Put $\quad x = xt, \quad y = yt$

$$u \quad = \quad (xt)^{\,n}\, f\left(\frac{yt}{xt}\right)$$

$$u \quad = \quad x^{\,n}\, t^{\,n} f\left(\frac{y}{x}\right)$$

$$u \quad = \quad t^{\,n}\left(x^{\,n} f\left(\frac{y}{x}\right)\right) = \; t^{\,n} f\,(x, y)$$

u is a homogenous of functions of degree in x & y

$$\therefore\; x\,\frac{\partial t}{\partial x} + \,y\,\frac{\partial u}{\partial y} = nf \qquad\qquad f = u$$

$$\boxed{x\,\frac{\partial u}{\partial x} + \,y\,\frac{\partial u}{\partial y} = nu}$$

Euler's theorem of second degree homogenous functions (or) Deduction form of Euler's theorem

Prove by New Method

Statement

If u is a homogenous functions of degree in x & y then

$$x^2\,\frac{\partial^2 u}{\partial x^2} + + 2xy\,\frac{\partial^2 u}{\partial x \partial y} + \,y^2\,\frac{\partial^2 u}{\partial y^2} = \emptyset\,(u)[\,\emptyset'(u) - 1]$$

Proof

By Euler's theorem

$$x\,\frac{\partial t}{\partial x} + \,y\,\frac{\partial u}{\partial y} = nu$$

Put $\quad u = F\,(u)$

$$x\,\frac{\partial}{\partial x} F(u) + \,y\,\frac{\partial}{\partial y}\,F(u) = n\,F(u) \qquad -- \qquad (1)$$

Differentiating (1) Partially w.r. to x & y

$$x\,F'(u)\frac{\partial}{\partial x}(u) + y\;F'(u)\frac{\partial}{\partial y} = n\,F(u) \qquad -- \qquad (1)$$

$$x\,\frac{\partial u}{\partial x} + y\;\frac{\partial u}{\partial x} = n\;\frac{F(u)}{F'(u)}$$

Let $n\,\frac{F(u)}{F'(u)} = \emptyset\,(u)$

$$\therefore x\;\frac{\partial u}{\partial x} + y\;\frac{\partial u}{\partial x} = \emptyset\,(u) \qquad -- \qquad (2)$$

Differentiating (1) Partially w.r. to x

$$x\;\frac{\partial^2 u}{\partial y \partial x} + \mathrm{y}\,\frac{\partial^2 u}{\partial y^2} + 1.\;\frac{\partial u}{\partial y} = \emptyset'(u)\;\frac{\partial u}{\partial y}$$

$$x^2\;\frac{\partial^2 u}{\partial y \partial x} + \mathrm{y}\,\frac{\partial^2 u}{\partial y^2} = (\emptyset'(u) - 1)\,\frac{\partial u}{\partial y} \qquad -- \qquad (3)$$

Differentiating (1) partially w.r. to y we set

$$x\;\frac{\partial^2 u}{\partial y \partial x} + \mathrm{y}\,\frac{\partial^2 u}{\partial y^2} + 1.\;\frac{\partial u}{\partial y} = \emptyset'(u)\;\frac{\partial u}{\partial y}$$

$$x^2\;\frac{\partial^2 u}{\partial x \partial y} + \mathrm{y}\,\frac{\partial^2 u}{\partial y^2} = (\emptyset'(u) - 1)\,\frac{\partial u}{\partial y} \qquad -- \qquad (4)$$

Equ (2) x $\boldsymbol{x}$ + (3) x $\boldsymbol{y}$

$$x\;\frac{\partial^2 u}{\partial x^2} + xy\;\frac{\partial^2 u}{\partial x \partial y} + xy\;\frac{\partial^2 u}{\partial x \partial y} + y^2\;\frac{\partial^2 u}{\partial y^2}$$

$$= \quad (\emptyset'(u) - 1)\;\left(x\;\frac{\partial u}{\partial x} + y\;\frac{\partial u}{\partial y}\right)$$

$$= \quad (\emptyset'(u) - 1)\big(\emptyset\,(u)\big)$$

$$= \quad (\emptyset'(u) - 1)[\emptyset\,(u)\,]$$

Introduction -2

Problem :1 Previous Method

If $u = sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$ **P.T**

$$x\;\frac{\partial^2 u}{\partial x^2} + 2xy\;\frac{\partial^2 u}{\partial x \partial y} + y^2\;\frac{\partial^2 u}{\partial y^2} = tan^3 u$$

Sol

$$sinu = \left[\frac{x^2+y^2}{x+y}\right] = f(x, y)$$

$f(x, y)$ is homogenous function of degree 1 with x & y

$$= \qquad x\;\frac{\partial}{\partial x}\,(sinu) + y\;\frac{\partial}{\partial y}\,(sinu) = 1.sinu$$

$$x\, cosu\, \frac{\partial u}{\partial x} + y\, cosu\, \frac{\partial u}{\partial y} = sinu$$

$$= \quad x\, \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{sinu}{cosu}$$

$$= \quad x\, \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = tanu \qquad \text{--} \qquad (1)$$

$$x^2\, \frac{\partial^2 u}{\partial y^2} + 2xy\, \frac{\partial^2 u}{\partial x \partial y} + y^2\, \frac{\partial^2 u}{\partial y^2} = n\,(n-1)u$$

$$= \quad n(n-1)\, u \qquad n = 1,\ u = tan\text{u}$$

$$= \quad 1\,(1-1)\, tanu$$

$$= \quad 0$$

This method is not satisfy the above results

Proof by New Method

$$u = sin^{-1}\left[\frac{x^2+y^2}{x+y}\right] \quad \textbf{P.T}$$

$$x\, \frac{\partial^2 u}{\partial x^2} + 2xy\, \frac{\partial^2 u}{\partial x \partial y} + y^2\, \frac{\partial^2 u}{\partial y^2} = tan^3 u$$

Proof

$$sinu = \left[\frac{x^2+y^2}{x+y}\right] = f(x,y)$$

u is homogenous function of degree 1 with x & y

$$= \quad x\, \frac{\partial}{\partial x}\,(sinu) + y\, \frac{\partial}{\partial y}\,(sinu) = 1.sinu$$

$$x\, cosu\, \frac{\partial u}{\partial x} + y\, cosu\, \frac{\partial u}{\partial y} = sinu$$

$$= \quad x\, \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{sinu}{cosu}$$

$$= \quad x\, \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = tanu \qquad \text{--} \qquad (1)$$

$$x^2\, \frac{\partial^2 u}{\partial y^2} + 2xy\, \frac{\partial^2 u}{\partial x \partial y} + y^2\, \frac{\partial^2 u}{\partial y^2} = \emptyset\,(u)\,(\emptyset'(u) - 1)$$

$$= \quad \emptyset\,(u) = tanu$$

$$= \quad \emptyset\,(u) = sec^2 u$$

$$= \quad \tan u\,(sec^2 u\ 1)$$

$$= \quad tanu\,(tan^2 u)$$

$$= \quad tan^3 u$$

This method is satisfy by the above result.

CONCLUSION

This method is very useful to school students, Arts and Science students and Engineering students. It is very easily understanding methods. This method is useful to competitive exams, gate exams and any other entrance examinations.

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