Properties of \( k \) - CentroSymmetric and \( k \) – Skew CentroSymmetric Matrices

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Abstract

The basic concepts and theorems of \( k \) - Centrosymmetric, \( k \) – Skew Centrosymmetric matrices are introduced with examples.

Keywords: Symmetric matrix, Centrosymmetric , \( k \)- Centrosymmetric matrix, Skewsymmetric matrix Skew Centrosymmetric matrix and \( k \)-Skew centrosymmetric matrix.

AMS CLASSIFICATIONS: 15B05, 15A09

I. INTRODUCTION

The concept of \( k \)-symmetric matrices and was introduced in [1], [2] and [3] Some properties of symmetric matrices given in [5],[6] ,[7]. In this paper, our intention is to define \( k \)- Centrosymmetric matrix, \( k \)-Skew Centrosymmetric matrix and also we discussed some results on Centrosymmetric matrices.

II. PRELIMINARIES AND NOTATIONS

\( C \) is centrosymmetric matrix, \( C^T \) is called Transpose of \( C \). Let \( k \) be a fixed product of disjoint transposition in \( S_n \) and \( K \) be the permutation matrix associated with \( K \). Clearly \( K \) satisfies the following properties. \( K^2 = I \), \( K^T = K \).
III. DEFINITIONS AND THEOREMS

DEFINITION: 1
A Square matrix $A = [a_{ij}]_{n \times n}$ is said to be symmetric if $A = A^T$ (i.e.) $a_{ij} = a_{ji}$ $\forall$ $i, j$

DEFINITION: 2
A Square matrix which is symmetric about the centre of its array of elements is called centrosymmetric thus $C = [a_{ij}]_{n \times n}$ centrosymmetric if $a_{ij} = a_{n-i+1,n-j+1}$.

DEFINITION: 3
A centrosymmetric matrix $C \in \mathbb{R}^{n \times n}$ is called a k-centrosymmetric matrix if $C = K C^T K$.

THEOREM: 1
Let $C \in \mathbb{R}^{n \times n}$ be k-centrosymmetric matrix then $C^T = K C K$.
Proof:

\[
K C K = K C^T K \quad \text{where } C = C^T \\
= C^T K K \quad \text{where } K C^T = C^T K \\
= C^T K^2 = C^T
\]

THEOREM: 2
If $C_1$ and $C_2$ are k-centrosymmetric matrices then $C_1 C_2$ is also k-centrosymmetric matrix
Proof:
Let $C_1$ and $C_2$ are k-centrosymmetric matrices if $C_1 = K C_1^T K$ and $C_2 = K C_2^T K$.
Since $C_1^T$ and $C_2^T$ are also k-centrosymmetric matrices then $C_1^T = K C_1 K$ and $C_2^T = K C_2 K$.
To prove $C_1 C_2$ is k-centrosymmetric matrix

We will show that $C_1 C_2 = K (C_1 C_2)^T K$
Now $K (C_1 C_2)^T K = K C_2^T C_1^T K$
\[
= K [(K C_2 K)(K C_1 K)]K \quad \text{where } C_1^T = K C_1 K \text{ and } C_2^T = K C_2 K.
\]
\[
= K^2 C_2 K^2 C_1 K^2
\]
\[
= C_2 C_1
\]
\[
= C_1 C_2
\]

where \( C_2 C_1 = C_1 C_2 \)

**THEOREM : 3**

If \( C \) is \( k \)-centro symmetric matrices and \( K \) is the permutation matrix, \( k = (1 \ 2) \) then \( KC \) is also \( k \)-centro symmetric matrix.

**Proof :**

A matrix \( C \in \mathbb{R}^{n \times n} \) is said to be \( k \)-centrosymmetric matrix if \( C = K C^T K \)

Since \( C^T \) is also \( k \)-centrosymmetric matrices then \( C^T = K C K \)

To prove \( K C \) is \( k \)-centrosymmetric matrix

We will show that, \( KC = (KC)^T K \)

Now \( K (KC)^T K = K (C^T K^T) K \) where \( (KC)^T = C^T K^T \)

\[
= KC^T
\]

where \( K^T K = I \)

\[
= KC
\]

where \( KC^T = KC \)

**THEOREM: 4**

If \( C \in \mathbb{R}^{n \times n} \) is \( k \)-centrosymmetric matrix then \( C C^T \) is also \( k \)-centrosymmetric matrix

**Proof :**

A matrix \( C \in \mathbb{R}^{n \times n} \) is said to be \( k \)-centrosymmetric matrix if \( C = K C^T K \)

Since \( C^T \) is also \( k \)-centrosymmetric matrices then \( C^T = K CK \)

We will show that, \( C C^T = K (C C^T)^T K \)

For that, \( K (C C^T)^T K = K [ (C^T)^T C^T ] K \) where \( (KC)^T = C^T K^T \)

\[
= K (C C^T) K \quad \text{where} \quad (C^T)^T = C
\]

\[
= (C C^T) KK \quad \text{where} \quad KC = CK
\]

\[
= (C C^T) K^2 \quad \text{where} \quad KK = K^2
\]

\[
= (C C^T)
\]

**THEOREM: 5**

If \( C \in \mathbb{R}^{n \times n} \) is \( k \)-centrosymmetric matrix then \( C \pm C^T \) is also \( k \)-centrosymmetric matrix
Proof:
A matrix $C \in \mathbb{R}^{n \times n}$ is said to be k-centrosymmetric matrix if $C = K C^T K$

Since $C^T$ is also k-centrosymmetric matrices then $C^T = K CK$

We will show that, $C + C^T = K (C + C^T)^T K$

For that, $K (C + C^T)^T K = K [ (C^T)^T + C^T ] K$ where $(C_1 + C_2)^T = (C_1^T + C_2^T)$

$= K (C + C^T) K$ where $(C^T)^T = C$

$= (C + C^T) K K$ where $KC = CK$

$= (C + C^T) K^2$

$= (C + C^T)$

THEOREM:6

If $C_1$ and $C_2$ are k-centrosymmetric matrices then $C_1 \pm C_2$ is also k-centrosymmetric matrix

Proof:

Let $C_1$, and $C_2$ are k-centrosymmetric matrices if $C_1 = K C_1^T K$ and $C_2 = K C_2^T K$.

Since $C_1^T$ and $C_2^T$ are also k-centrosymmetric matrices then $C_1^T = K C_1 K$ and $C_2^T = K C_2 K$.

To prove $C_1 + C_2$ is k-centrosymmetric matrix

We will show that $C_1 + C_2 = K (C_1 + C_2)^T K$

Now $K (C_1 + C_2)^T K = K (C_1^T + C_2^T) K$

$= K C_1 K + K C_2 K$ where $C_1^T = K C_1 K$ and $C_2^T = K C_2 K$.

$= C_1 + C_2$

RESULT:

Let $C_1$ and $C_2$ are k-centrosymmetric matrices for the following conditions are holds

[i] $C_1 C_2 = C_2 C_1$

[ii] $(C_1^TC_2 C_1)$ and $(C_2^TC_1 C_2)$ are also k-centrosymmetric matrices.

[iii] $\text{Adj} C_1$ also k-centrosymmetric matrix.

[iv] $C_1(\text{Adj} C_1)$ is also k-centrosymmetric matrix.
EXAMPLE: 1

Let \( \mathbf{C}_1 = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \) and \( \mathbf{C}_2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \); \( \mathbf{K} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \)

(i) \( \mathbf{K} (\mathbf{C}_1 \mathbf{C}_2)^T \mathbf{K} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix} = \mathbf{C}_1 \mathbf{C}_2 \)

(ii) \( \mathbf{K} (\mathbf{C}_1^T \mathbf{C}_2 \mathbf{C}_1)^T \mathbf{K} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 37 & 38 \\ 38 & 37 \end{pmatrix} = \mathbf{C}_1^T \mathbf{C}_2 \mathbf{C}_1 \)

DEFINITION: 4

A Square matrix \( \mathbf{A} = [a_{ij}]_{n \times n} \) is said to be skew symmetric matrix if \( \mathbf{A} = -\mathbf{A}^T \) (ie) \( a_{ij} = -a_{ji} \) \( \forall \ i, j \)

DEFINITION: 5

A Square matrix \( \mathbf{C} = [a_{ij}]_{n \times n} \) is called skew centrosymmetric matrix if \( \mathbf{C} = -\mathbf{C}^T \)

DEFINITION: 6

A skew centrosymmetric matrix \( \mathbf{C} \in \mathbb{R}^{n \times n} \) is called k-skew centrosymmetric matrix if

\[ \mathbf{K} \mathbf{C} \mathbf{K} = -\mathbf{C}^T. \]

THEOREM: 7

Let \( \mathbf{C} \in \mathbb{R}^{n \times n} \) is k-skew centrosymmetric matrix then \( \mathbf{K} \mathbf{C}^T \mathbf{K} = -\mathbf{C} \)

Proof:

\[ \mathbf{K} \mathbf{C}^T \mathbf{K} = \mathbf{K} (-\mathbf{C}) \mathbf{K} = \text{where } \mathbf{C}^T = -\mathbf{C} \]

\[ = -\mathbf{C} \mathbf{K} \mathbf{K} \]

\[ = -\mathbf{C} \]

THEOREM: 8

If \( \mathbf{C}_1 \) and \( \mathbf{C}_2 \) are k-skew centrosymmetric matrices then \( \mathbf{C}_1 \mathbf{C}_2 \) is also k-skew centrosymmetric matrix

Proof:
Let $C_1$, and $C_2$ are k-skew centrosymmetric matrices if $K C_1^T K = -C_1$ and $K C_2^T K = -C_2$.

Since $C_1^T$ and $C_2^T$ are also k- skew centrosymmetric matrices then $K C_1 K = - C_1^T$ and $K C_2 K = - C_2^T$.

To prove $C_1 C_2$ is k- skew centrosymmetric matrix

We will show that $C_1 C_2 = K (C_1 C_2)^T K$

Now $K (C_1 C_2)^T K = K C_2^T C_1^T K$

\[ = K [(-K C_2 K)(-K C_1 K)] K \text{ where } K C_1 K = - C_1^T \text{ and } K C_2 K = - C_2^T. \]

\[ = K^2 C_2 K^2 C_1 K^2 \]

\[ = C_2 C_1 \text{ where } K^2 = I \]

\[ = C_1 C_2 \text{ where } C_2 C_1 = C_1 C_2 \]

**THEOREM : 9**

If $C$ is k-skew centrosymmetric matrix and $K$ is the permutation matrix, $k = (1 \ 2)$ then $KC$ is also k- skew centro symmetric matrix.

**Proof :**

A matrix $C \in \mathbb{R}^{n \times n}$ is said to be k- skew centrosymmetric matrix if $KC^T K = -C$

Since $C^T$ is also k-skew centrosymmetric matrices then $K C K = - C^T$

To prove $-KC$ is K- skew centrosymmetric matrix

We will show that $-KC = K (KC)^T K$

Now $K (KC)^T K = K (C^T K^T) K$ where $(KC)^T = C^T K^T$

\[ = KC^T \text{ where } K^T K = I \]

\[ = - KC \text{ where } KC^T = - KC \]

**THEOREM: 10**

If $C \in \mathbb{R}^{n \times n}$ is k-skew centrosymmetric matrix then $C C^T$ is also k- skew centrosymmetric matrix.

**Proof :**

A matrix $C \in \mathbb{R}^{n \times n}$ is said to be k-skew centrosymmetric matrix if $KC^T K = -C$
Since $C^T$ is also k- skew centro-symmetric matrices then $K CK = - C^T$

We will show that , $C C^T = K (C C^T)^T K$

For that, $K (C C^T)^T K = K [ (C^T)^T C^T ] K$ where $(KC)^T = C^T K^T$

$= K (C C^T) K$ where $(C^T)^T = C$

$= (C C^T) KK$ where $KC = CK$

$= (C C^T) K^2$ where $KK = K^2$

$= (C C^T)$

**RESULT:**

1. If $C \in \mathbb{R}^{n \times n}$ is k-skew centro-symmetric matrix then $C - C^T$ is also k- skew centro-symmetric matrix.

2. Let $C_1$ and $C_2$ are k- skewcentrosymmetric matrices for the following conditions are holds
   
   [i] $C_1 C_2 = C_2 C_1$

   [ii] $(C_1^T C_2 C_1)$ and $(C_2^T C_1 C_2)$ are also k- skew centro-symmetric matrices

   [iii] $\text{Adj} C_1$ also k- skew centro symmetric matrix

   [iv] $C_1 (\text{Adj} C_1)$ is also k- skew centro-symmetric matrix .

**REFERENCES**


