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Distance Two Labeling of Quadrilateral Snake Families

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Abstract

An $L(2,1)$ labeling (or) distance two labeling of a graph G is a function f from the vertex set $V(G)$ to the set of all non-negative integers such that $|f(x) - f(y)| \geq 2$ if $d(x,y)=1$ and $|f(x) - f(y)| \geq 1$ if $d(x,y)=2$. The $L(2, 1)$ labeling number $\lambda(G)$ of G is the smallest number k such that G has an $L(2, 1)$ labeling with $\max\{f(v), v \in V(G)\} = k$. In this paper we determine the $L(2, 1)$ labeling number $\lambda(G)$ for quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs.

Keywords: $L(2, 1)$ labeling, λ-number, Cycle, Path

1. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain. The concept of graph labeling was introduced by Rosa in 1967 [10]. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 2100 papers.

Hale [6] introduced the graph theory model of assignment of channels in 1980. A channel assignment problem was designed in such a way that the vertices of distance two are considered to be close and vertices which are adjacent, are considered to be very close which paved way for distance two labeling of graphs. Labeling with a condition of distance two was introduced by J.R. Griggs and R.K.Yeh[5] who proved that every graph with maximum degree k has an $L(2,1)$-labeling with span at most k^2+2k and proved the conjecture for 2-regular graphs. G.J. Chang and D. Kuo [1] improved this upper bound to k^2+k. Chang et al. [2] generalized this to obtain $k^2+(d-1)k$ as an upper bound on the minimum span of an $L(d,1)$-labeling. Z. Fredi and J.H. Kang [3] proved it for 3-regular Hamiltonian graphs and for the incidence graphs of projective planes.

Tight bounds on the maximum span have been obtained for special classes of graphs like paths, cycles, wheels, complete k-partite graphs and graphs with diameter 2 , trees [1,2], etc. Bounds have also been obtained for various other graph families like chordal graphs and unit interval graphs and planar graphs [8] and hyper cubes [5, 11, 12]. R. Ponraj et al. [9] determined the difference cordial labeling of triangular and quadrilateral snake graphs.

2. PRELIMINARIES

In this section we give the basic notations relevant to this paper. In this paper, the graphs considered are all finite, undirected and simple. $V(G)$ and $E(G)$ denote the vertex set and the edge set of G.

Definition 2.1.

An L(2,1) labeling (or) distance two labeling of a graph G is a function f from the vertex set V(G) to the set of all non-negative integers such that $|f(x) - f(y)| \geq 2$ if d(x,y)=1 and |f(x) − f(y)| ≥ 1 if d(x,y) =2. The L(2, 1) labeling number λ(G) of G is the smallest number k such that G has an L(2, 1) labeling with max{f(v), v∈ V(G)} = k.

Definition 2.2.

A quadrilateral snake Q_n is obtained from a path $a_1, a_2 \ldots a_n$ by joining a_i and a_{i+1} to new vertices b_i and c_i respectively and joining the vertices b_i *and* c_i for i=1,2,….. *n-1*. That is every edge of a path is replaced by a cycle C_4.

Definition 2.3.

An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $a_1, a_2 \ldots a_n$ by joining a_i and a_{i+1} to new vertices b_i and c_i respectively for i ≡ 1 (mod 2) and I ≤ n-1 and then joining b_i and c_i. That is every alternative edge of a path is replaced by a cycle C_4.

Definition 2.4.

A double quadrilateral snake $D(Q_n)$ is obtained from two quadrilateral snakes that have a common path.

Definition 2.5.

An alternative double quadrilateral snake $A(D(Q_n))$ is obtained from two alternative quadrilateral snakes that have a common path.

Definition 2.6.

A function $f : V(G) \rightarrow N \cup \{0\}$is said to be a valid $L(2,1)$ labeling if and only if it satisfies the condition $d(u, v) + |f(u) - f(v)| \geq 3$.

3. MAIN RESULTS

Theorem 3.1.

The L(2,1) labeling number $\lambda(G)$ of a of a quadrilateral snake Q_n is 8, for $n \geq 3$.

Proof.

Denote the vertices of a quadrilateral snake graph as follows:

$V(G) = V_1 \cup V_2$ where $V_1 = \{v_i \,/\, 1 \leq i \leq 2n\text{-}2\}$ and $V_2 = \{w_i \,/\, 1 \leq i \leq n\}$

Define a mapping $f : V(G) \to N \cup \{0\}$by

$f(v_{2i-1}) = 6; \quad 1 \leq i \leq n-1$

$f(v_{2i}) = 8; \quad 1 \leq i \leq n-1$

$f(\mathrm{w}_{3i-2}) = 0; \quad 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil$ $f(\mathrm{w}_{3i-1}) = 2; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$

$f(\mathrm{w}_{3i}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$

Now let us find the $L(2,1)$ labeling number of this graph.

Case(i):

Let x,y be any two vertices in $V_1(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $f(x) = 6$, $f(y) = 8$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |6-8| = 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(ii):

Let x,y be any two vertices in $V_2(G)$.

Subcase (i).

Let x,y be any two adjacent vertices on $V_2(G)$, such that $x = w_{3i-2}$ and $y = w_{3i-1}$.

Then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$.

Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$.

Subcase (ii).

Let x,y be any two non–adjacent vertices on $V_2(G)$., such that $x = \mathrm{w}_{3i-2}$ and $y = w_{3i}$.Then $f(x) = 0$, $f(y) = 4$ and $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$.

Case(iii):

Let x,y be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let x be a vertex on $V_1(G)$and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y = w_i$ .Then $f(x) = 6$ and $f(y) = 0$ and $d(x, y) = 1$.Therefore $d(x, y) + |f(x) - f(y)| = 1 + |6-0| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.

Let x be a vertex on $V_1(G)$ and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y = w_{3i}$ .Then $f(x) = 6$ and $f(y) = 4$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |6-4| \geq 3$.

Similarly, for all the other possibilities of x and y we find that $\mathrm{d}(x, y) + |f(x) - f(y)| \geq 3$. Therefore the $L(2, 1)$ labeling number of Q_n is $\lambda(G) = 8$.

Example 3.3.

The $L(2,1)$ labeling of a quadrilateral snake Q_6 is shown in Figure 1.

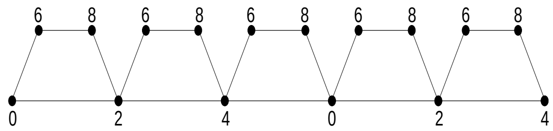

Figure 1

Hence the $L(2,1)$ labeling of a quadrilateral snake Q_n is 8.

Theorem 3.2.

The $L(2, 1)$ labeling number $\lambda(G)$ of an alternate quadrilateral snake graph A(Q_ni is 5, for n ≥ 2 and n ≡ 0 (mod 2)

Proof.

Denote the vertices of an alternate quadrilateral snake graph as follows:

$V(G) = V_1 \cup V_2$ where $V_1 = \{v_i / 1 \leq i \leq n\}$ and $V_2 = \{w_i / 1 \leq i \leq n\}$

Define a mapping $f : V(G) \rightarrow N \cup \{0\}$ by

$$f(v_{3i-2}) = 3; \quad 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil$$

$$f(v_{3i-1}) = 5; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$$

$$f(v_{3i}) = 1; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

$$f(\mathrm{w}_{3i-2}) = 0; \quad 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil \; f(\mathrm{w}_{3i-1}) = 2; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$$

$$f(\mathrm{w}_{3i}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

Now let us find the $L(2,1)$ labeling number of this graph.

Case(i):

Let x,y be any two vertices in $V_1(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = v_{3i-2}$ and $y = v_{3i-1}$ where i ≡ 1 (mod 2) .Then $f(x) = 3$, $f(y) = 5$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |3-5| = 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(ii):

Let x,y be any two vertices in $V_2(G)$.

Subcase (i).

Let x,y be any two adjacent vertices on $V_2(G)$, such that $x = w_{3i-2}$ and $y = w_{3i-1}$, then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$.Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$.

Subcase (ii).

Let x,y be any two non–adjacent vertices on $V_2(G)$., such that $x = w_{3i-2}$ and $y = w_{3i}$ Then $f(x) = 0$, $f(y) = 4$ and $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$.

Case(iii):

Let x,y be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let x be a vertex on $V_1(G)$and y be a vertex on $V_2(G)$ such that $x = v_{3i-2}$ and $y = w_{3i-2}$. Then $f(x) = 3$ and $f(y) = 0$ and $d(x, y) = 1$.Therefore $d(x, y) + |f(x) - f(y)| = 1 + |3-0| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.

Let x be a vertex on $V_1(G)$ and y be a vertex on $V_2(G)$ such that $x = v_{3i-2}$ and $y = w_{3i-1}$ .Then $f(x) = 3$ and $f(y) =2$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |3-2| \geq 3$.

Similarly, for all the other possibilities of x and y we find that $d(x, y) + |f(x) - f(y)| \geq 3$. Therefore the $L(2, 1)$ labeling number of $A(Q_n)$ is $\lambda(G) = 5$.

Example 3.2.

$L(2,1)$ labeling of an alternate quadrilateral snake graph $A(Q_n)$ is shown in Figure 2.

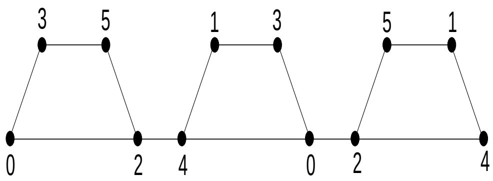

Figure 2

Hence the $L(2,1)$ labeling number $\lambda(G)$ of an alternate quadrilateral snake graph $A(Q_n)$ is 5.

Theorem 3.3.

The L(2,1) labeling number $\lambda(G)$ of a of a double quadrilateral snake $D(Q_n)$. is 9, for $n \geq 3$.

Proof.

Denote the vertices of a double quadrilateral snake graph as follows:

$V(G) = V_1 \cup V_2 \cup V_3$ where $V_1 = \{v_i / 1 \leq i \leq 2n-2\}$, $V_2 = \{w_i / 1 \leq i \leq n\}$, $V_3 = \{u_i / 1 \leq i \leq 2n-2\}$

Define a mapping $f : V(G) \rightarrow N \cup \{0\}$ by

$$f(v_{2i-1}) = 0; \quad 1 \leq i \leq n-1$$

$$f(v_{2i}) = 2; \quad 1 \leq i \leq n-1$$

$f(w_{3i-2}) = 5; \quad 1 \le i \le \left\lceil \frac{n}{3} \right\rceil$ $f(w_{3i-1}) = 7; \quad 1 \le i \le \left\lceil \frac{n-1}{3} \right\rceil$

$f(w_{3i}) = 9; \quad 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor$

$f(u_{2i-1}) = 1; \quad 1 \le i \le n-1$

$f(u_{2i}) = 3; \quad 1 \le i \le n-1$

Now let us find the $L(2,1)$ labeling number of this graph.

Case(i):

Let x,y be any two vertices in $V_1(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| \ge 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \ge 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(ii):

Let x,y be any two vertices in $V_2(G)$.

Subcase (i).

Let x,y be any two adjacent vertices on $V_2(G)$, such that $x = w_{3i-2}$ and $y = w_{3i-1}$.

Then $f(x) = 5$, $f(y) = 7$ and $d(x, y) = 1$.

Therefore $d(x, y) + |f(x) - f(y)| = 1 + |5-7| = 3$.

Subcase (ii).

Let x,y be any two non–adjacent vertices on $V_2(G)$., such that x = w_{3i-2} and y = w_{3i}.Then $f(x) = 5$, $f(y) = 9$ and $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |5-9| \geq 3$.

Case(iii):

Let x,y be any two vertices in $V_3(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = u_{2i-1}$ and $y = u_{2i}$. Then $f(x) = 1$, $f(y) = 3$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |1-3| \geq 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(iv):

Let x,y be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let x be a vertex on $V_1(G)$and y be a vertex on $V_2(G)$ such that $x = v_{6i-5}$ and $y = w_{3i-2}$ Then $f(x) = 0$ and $f(y) = 5$ and $d(x, y) = 1$.Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-5| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.

Let x be a vertex on $V_1(G)$ and y be a vertex on $V_2(G)$ such that $x = v_{6i-5}$ and $y=w_{3i-1}$ .Then $f(x) = 0$ and $f(y) = 7$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-7| \geq 3$.

Case(iv):

Let x,y be any two vertices, on the $V_2(G)$ and $V_3(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively. Let x be a vertex on $V_2(G)$and y be a vertex on $V_3(G)$ such that $x = w_{3i-2}$ and $y = u_{6i-5}$ Then $f(x) = 5$ and $f(y) = 1$ and $d(x, y) = 1$.Therefore $d(x, y) + |f(x) - f(y)| = 1 + |5-1| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively. Let x be a vertex on $V_2(G)$ and y be a vertex on $V_3(G)$ such that $x = w_{3i-1}$ and $y = u_{6i-5}$ .Then $f(x)=7$ and $f(y) =1$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |7-1| \geq 3$.

Case(v):

Let x,y be any two vertices in $V_1(G)$ and $V_3(G)$ respectively. Clearly x and y are non-adjacent vertices. such that $x = v_{2i-1}$ and $y = u_{2i-1}$.Then $f(x) = 0$, $f(y) = 1$ and $d(x, y) \geq 2$.

Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-1| \geq 3$.

Similarly, for all the other possibilities of x and y we find that $d(x, y) + |f(x) - f(y)| \geq 3$. Therefore the $L(2,1)$ labeling number of $D(Q_n)$ is $\lambda(G) = 9$.

Example 3.3.

The $L(2,1)$ labeling of a double triangular snake $D(Q_7)$ is shown in Figure 3.

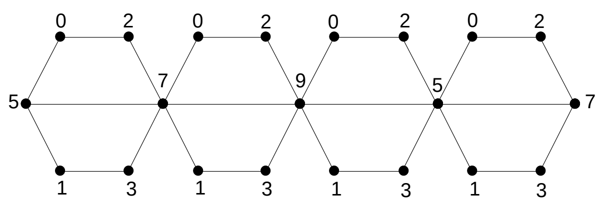

Figure 3

Hence the $L(2,1)$ labeling of a double quadrilateral snake $D(Q_n)$ is $\lambda(G) = 9$.

Theorem 3.4.

The $L(2,1)$ labeling number $\lambda(G)$ of an alternate double quadrilateral snake graph A(D(Q_n)) is 8, for $n \geq 4$ and $n \equiv 0 \pmod 2$

Proof.

Denote the vertices of an alternate double quadrilateral snake graph A(D(Q_n)) as follows:

$V(G) = V_1 \cup V_2 \cup V_3$ where $V_1 = \{v_i \,/\, 1 \leq i \leq n\}$, $V_2 = \{w_i \,/\, 1 \leq i \leq n\}$, $V_3 = \{u_i \,/\, 1 \leq i \leq n\}$

Denote the vertices of an alternate double quadrilateral snake graph A(D(Q_n)) as follows:

$V(G) = V_1 \cup V_2 \cup V_3$ where $V_1 = \{v_i \,/\, 1 \leq i \leq n\}$, $V_2 = \{w_i \,/\, 1 \leq i \leq n\}$, $V_3 = \{u_i \,/\, 1 \leq i \leq n\}$

Define a mapping $f : V(G) \rightarrow N \cup \{0\}$ by

$$f(v_{2i-1}) = 0; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(v_{2i}) = 2; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(\mathrm{w}_{4i-3}) = 4; \quad 1 \leq i \leq \left\lceil \frac{n}{4} \right\rceil \quad f(\mathrm{w}_{4i-2}) = 7; \quad 1 \leq i \leq \left\lceil \frac{n-1}{4} \right\rceil$$

$$f(\mathrm{w}_{4i-1}) = 5; \quad 1 \leq i \leq \left\lceil \frac{n-2}{4} \right\rceil$$

$$f(\mathrm{w}_{4i}) = 8; \quad 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$f(\mathrm{u}_{2i-1}) = 1; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(\mathrm{u}_{2i}) = 3; \quad 1 \leq i \leq \frac{n}{2}$$

Claim:

The $L(2,1)$ labeling number of an alternate double quadrilateral snake graph A(D(Q_n)) = 8.

Case(i):

Let x,y be any two vertices in $V_1(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| \geq 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied

Case(ii):

Let x,y be any two vertices in $V_2(G)$.

Subcase (i).

Let x,y be any two adjacent vertices on $V_2(G)$, such that $x = w_{4i-3}$ and $y = w_{4i-2}$.

Then $f(x) = 4$, $f(y) = 7$ and $d(x, y) = 1$.

Therefore $d(x, y) + |f(x) - f(y)| = 1 + |4-7| = 3$.

Subcase (ii).

Let x,y be any two non–adjacent vertices on $V_2(G)$., such that $x = w_{4i-3}$ and $y = w_{4i-1}$.Then $f(x) = 4$, $f(y) = 5$ and $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |4-5| \geq 3$.

Case(iii):

Let x,y be any two vertices in $V_3(G)$.

Subcase (i).

Let x and y be adjacent vertices on $V_1(G)$, such that $x = u_{2i-1}$ and $y = u_{2i}$. Then $f(x) = 1$,

$f(y) = 3$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |1-3| \geq 3$.

Subcase (ii).

Let x and y be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(iv):

Let x,y be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let x be a vertex on $V_1(G)$and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y = w_{4i-3}$ Then $f(x) = 0$ and $f(y) = 4$ and $d(x, y) \geq 1$.Therefore $d(x, y) + |f(x) - f(y)| \geq 1 + |0-4| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.

Let x be a vertex on $V_1(G)$ and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y=w_{4i-2}$ .Then $f(x) = 0$ and $f(y) =7$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-7| \geq 3$.

Case(iv):

Let x,y be any two vertices, on the $V_2(G)$ and $V_3(G)$ respectively

Subcase (i).

Let x,y be any two adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively. Let x be a vertex on $V_2(G)$and y be a vertex on $V_3(G)$ such that $x = w_{4i-3}$ and $y = u_{2i-1}$ Then $f(x) = 4$ and $f(y) = 1$ and $d(x, y) \geq 1$.Therefore $d(x, y) + |f(x) - f(y)| \geq 1 + |4-1| \geq 3$.

Subcase (ii).

Let x,y be any two non-adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively.

Let x be a vertex on $V_2(G)$ and y be a vertex on $V_3(G)$ such that $x = w_{4i-3}$ and $y = u_{2i}$ .Then $f(x)=4$ and $f(y) =3$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |4-3| \geq 3$.

Case(v):

Let x,y be any two vertices in $V_1(G)$ and $V_3(G)$ respectively. Clearly x and y are non-adjacent vertices. such that $x = v_{2i-1}$ and $y = u_{2i-1}$.Then $f(x) = 0$, $f(y) = 1$ and $d(x, y) \geq 2$.

Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-1| \geq 3$.

Similarly, for all the other possibilities of x and y we find that $d(x, y) + |f(x) - f(y)| \geq 3$. Therefore the $L(2,1)$ labeling number of A(D(Q_n)) is $\lambda(G) = 8$.

Example 3.2.

$L(2,1)$ labeling of alternate triangular snake graph A(D(Q_8)) is shown in Figure 4.

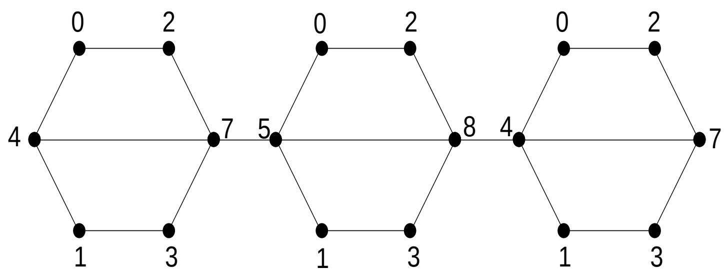

Figure 4

Hence the $L(2,1)$ labeling number $\lambda(G)$ of alternate double triangular snake graph

4. CONCLUSION

In this paper the $L(2,1)$ labeling number for quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs are determined.

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