Distance Two Labeling of Quadrilateral Snake Families

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Abstract

An $L(2,1)$ labeling (or) distance two labeling of a graph $G$ is a function $f$ from the vertex set $V(G)$ to the set of all non-negative integers such that $|f(x) - f(y)| \geq 2$ if $d(x,y)=1$ and $|f(x) - f(y)| \geq 1$ if $d(x,y)=2$. The $L(2, 1)$ labeling number $\lambda(G)$ of $G$ is the smallest number $k$ such that $G$ has an $L(2, 1)$ labeling with $\max \{f(v), v \in V(G)\} = k$. In this paper we determine the $L(2, 1)$ labeling number $\lambda(G)$ for quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs.

Keywords: $L(2, 1)$ labeling, $\lambda$-number, Cycle, Path

1. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of
applications such as coding theory, X-ray crystallography, radar, astronomy, circuit
design, communication networks and data base management and models for
constraint programming over finite domain. The concept of graph labeling was
introduced by Rosa in 1967 [10]. Hence in the intervening years various labeling of
graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic
labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling, k-
graceful labeling and odd graceful labeling etc., have been studied in over 2100
papers.

c channel assignment problem was designed in such a way that the vertices of distance
two are considered to be close and vertices which are adjacent, are considered to be
very close which paved way for distance two labeling of graphs. Labeling with a
condition of distance two was introduced by J.R. Griggs and R.K.Yeh[5] who proved
that every graph with maximum degree k has an L(2,1)-labeling with span at most
k^2+2k and proved the conjecture for 2-regular graphs. G.J. Chang and D. Kuo [1]
improved this upper bound to k^2+k. Chang et al. [2] generalized this to obtain
k^2+(d−1)k as an upper bound on the minimum span of an L(d,1)-labeling. Z. Fredi and
J.H. Kang [3] proved it for 3-regular Hamiltonian graphs and for the incidence graphs
of projective planes.

Tight bounds on the maximum span have been obtained for special classes of graphs
like paths, cycles, wheels, complete k-partite graphs and graphs with diameter 2 ,
trees [1,2], etc. Bounds have also been obtained for various other graph families like
chordal graphs and unit interval graphs and planar graphs [8] and hyper cubes [5, 11,
12]. R. Ponraj et al. [9] determined the difference cordial labeling of triangular and
quadrilateral snake graphs.

2. PRELIMINARIES

In this section we give the basic notations relevant to this paper. In this paper, the
graphs considered are all finite, undirected and simple. V(G) and E(G) denote the
vertex set and the edge set of G.

**Definition 2.1.**

An L(2,1) labeling (or) distance two labeling of a graph G is a function f from the
vertex set V(G) to the set of all non-negative integers such that |f(x) − f(y)| ≥ 2 if
distance two labeling of quadrilateral snake families

\[ d(x,y) = 1 \text{ and } |f(x) - f(y)| \geq 1 \text{ if } d(x,y) = 2. \]

The \( L(2, 1) \) labeling number \( \lambda(G) \) of \( G \) is the smallest number \( k \) such that \( G \) has an \( L(2, 1) \) labeling with \( \max \{ f(v), v \in V(G) \} = k. \)

**Definition 2.2.**

A quadrilateral snake \( Q_n \) is obtained from a path \( a_1a_2a_3...a_n \) by joining \( a_i \) and \( a_{i+1} \) to new vertices \( b_i \) and \( c_i \) respectively and joining the vertices \( b_i \) and \( c_i \) for \( i=1,2,\ldots, n-1. \) That is every edge of a path is replaced by a cycle \( C_4. \)

**Definition 2.3.**

An alternate quadrilateral snake \( A(Q_n) \) is obtained from a path \( a_1a_2a_3...a_n \) by joining \( a_i \) and \( a_{i+1} \) to new vertices \( b_i \) and \( c_i \) respectively for \( i \equiv 1 \mod 2 \) and \( 1 \leq n-1 \) and then joining \( b_i \) and \( c_i \). That is every alternative edge of a path is replaced by a cycle \( C_4. \)

**Definition 2.4.**

A double quadrilateral snake \( D(Q_n) \) is obtained from two quadrilateral snakes that have a common path.

**Definition 2.5.**

An alternative double quadrilateral snake \( A(D(Q_n)) \) is obtained from two alternative quadrilateral snakes that have a common path.

**Definition 2.6.**

A function \( f : V(G) \to \mathbb{N} \cup \{0\} \) is said to be a valid \( L(2,1) \) labeling if and only if it satisfies the condition \( d(u,v) + |f(u) - f(v)| \geq 3. \)

**3. MAIN RESULTS**

**Theorem 3.1.**

The \( L(2,1) \) labeling number \( \lambda(G) \) of a quadrilateral snake \( Q_n \) is 8, for \( n \geq 3. \)
Proof.

Denote the vertices of a quadrilateral snake graph as follows:

\[ V(G) = V_1 \cup V_2 \text{ where } V_1 = \{ v_i / 1 \leq i \leq 2n-2 \} \text{ and } V_2 = \{ w_i / 1 \leq i \leq n \} \]

Define a mapping \( f : V(G) \rightarrow N \cup \{0\} \) by

\[
\begin{align*}
  f(v_{2i}) &= 6; \quad 1 \leq i \leq n-1 \\
  f(v_{2i-1}) &= 8; \quad 1 \leq i \leq n-1 \\
  f(w_{3i-2}) &= 0; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor \\
  f(w_{3i-1}) &= 2; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor \\
  f(w_{3i}) &= 4; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor 
\end{align*}
\]

Now let us find the \( L(2,1) \) labeling number of this graph.

Case (i):

Let \( x, y \) be any two vertices in \( V_1(G) \).

Subcase (i).

Let \( x \) and \( y \) be adjacent vertices on \( V_1(G) \), such that \( x = v_{2i-1} \) and \( y = v_{2i} \). Then \( f(x) = 6 \), \( f(y) = 8 \) and \( d(x, y) = 1 \). Therefore \( d(x, y) + |f(x) - f(y)| = 1 + |6 - 8| = 3 \).

Subcase (ii).

Let \( x \) and \( y \) be non-adjacent vertices on \( V_1(G) \), then \( d(x, y) \geq 3 \). Therefore the \( L(2,1) \) labeling condition is satisfied.

Case (ii):

Let \( x, y \) be any two vertices in \( V_2(G) \).
Subcase (i).

Let \( x, y \) be any two adjacent vertices on \( V_2(G) \), such that \( x = w_{3i-2} \) and \( y = w_{3i-1} \).

Then \( f(x) = 0, f(y) = 2 \) and \( d(x, y) = 1 \).

Therefore \( d(x, y) + |f(x) - f(y)| = 1 + |0 - 2| = 3 \).

Subcase (ii).

Let \( x, y \) be any two non-adjacent vertices on \( V_2(G) \), such that \( x = w_{3i-2} \) and \( y = w_{3i} \). Then \( f(x) = 0, f(y) = 4 \) and \( d(x, y) \geq 2 \). Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + |0 - 4| \geq 3 \).

Case (iii):

Let \( x, y \) be any two vertices, on the \( V_1(G) \) and \( V_2(G) \) respectively.

Subcase (i).

Let \( x, y \) be any two adjacent vertices, on \( V_1(G) \) and \( V_2(G) \) respectively. Let \( x \) be a vertex on \( V_1(G) \) and \( y \) be a vertex on \( V_2(G) \) such that \( x = v_{2i-1} \) and \( y = w_i \). Then \( f(x) = 6 \) and \( f(y) = 0 \) and \( d(x, y) = 1 \). Therefore \( d(x, y) + |f(x) - f(y)| = 1 + |6 - 0| \geq 3 \).

Subcase (ii).

Let \( x, y \) be any two non-adjacent vertices, on \( V_1(G) \) and \( V_2(G) \) respectively.

Let \( x \) be a vertex on \( V_1(G) \) and \( y \) be a vertex on \( V_2(G) \) such that \( x = v_{2i-1} \) and \( y = w_{3i} \). Then \( f(x) = 6 \) and \( f(y) = 4 \), then \( d(x, y) \geq 2 \). Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + |6 - 4| \geq 3 \).

Similarly, for all the other possibilities of \( x \) and \( y \) we find that \( d(x, y) + |f(x) - f(y)| \geq 3 \). Therefore the \( L(2, 1) \) labeling number of \( Q_n \) is \( \lambda(G) = 8 \).
Example 3.3.

The $L(2,1)$ labeling of a quadrilateral snake $Q_6$ is shown in Figure 1.

![Figure 1](image)

Hence the $L(2,1)$ labeling of a quadrilateral snake $Q_n$ is 8.

Theorem 3.2.

The $L(2,1)$ labeling number $\lambda(G)$ of an alternate quadrilateral snake graph $A(Q_n)$ is 5, for $n \geq 2$ and $n \equiv 0 \pmod{2}$

Proof.

Denote the vertices of an alternate quadrilateral snake graph as follows:

$V(G) = V_1 \cup V_2$ where $V_1 = \{v_i \mid 1 \leq i \leq n\}$ and $V_2 = \{w_i \mid 1 \leq i \leq n\}$

Define a mapping $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$ by

$f(v_{n,1}) = 3; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$

$f(v_{n,2}) = 5; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$

$f(v_{n,3}) = 1; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$

$f(w_{n,2}) = 0; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$  \quad $f(w_{n,1}) = 2; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$

$f(w_{n,3}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$

Now let us find the $L(2,1)$ labeling number of this graph.
Case(i):
Let $x, y$ be any two vertices in $V_1(G)$.

Subcase (i).
Let $x$ and $y$ be adjacent vertices on $V_1(G)$, such that $x = v_{3i-2}$ and $y = v_{3i-1}$ where $i \equiv 1 \pmod{2}$. Then $f(x) = 3, f(y) = 5$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |3-5| = 3$.

Subcase (ii).
Let $x$ and $y$ be non-adjacent vertices on $V_1(G)$, Then $d(x, y) \geq 3$. Therefore the L(2,1) labeling condition is satisfied.

Case(ii):
Let $x, y$ be any two vertices in $V_2(G)$.

Subcase (i).
Let $x, y$ be any two adjacent vertices on $V_2(G)$, such that $x = w_{3i-2}$ and $y = w_{3i-1}$. Then $f(x) = 0, f(y) = 2$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$.

Subcase (ii).
Let $x, y$ be any two non-adjacent vertices on $V_2(G)$, such that $x = w_{3i-2}$ and $y = w_{3i}$. Then $f(x) = 0, f(y) = 4$ and $d(x, y) \geq 2$. Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$.

Case(iii):
Let $x, y$ be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

Subcase (i).
Let $x, y$ be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let $x$ be a vertex on $V_1(G)$ and $y$ be a vertex on $V_2(G)$ such that $x = v_{3i-2}$ and $y = w_{3i-2}$. Then $f(x) = 3$ and $f(y) = 0$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |3-0| \geq 3$.

Subcase (ii).
Let $x, y$ be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.
Let \( x \) be a vertex on \( V_1(G) \) and \( y \) be a vertex on \( V_2(G) \) such that \( x = v_{3i-2} \) and \( y = w_{3i} \). Then \( f(x) = 3 \) and \( f(y) = 2 \), then \( d(x, y) \geq 2 \). Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + [3-2] \geq 3 \).

Similarly, for all the other possibilities of \( x \) and \( y \) we find that \( d(x, y) + |f(x) - f(y)| \geq 3 \). Therefore the \( L(2, 1) \) labeling number of \( A(Q_n) \) is \( \lambda(G) = 5 \).

**Example 3.2.**

\( L(2,1) \) labeling of an alternate quadrilateral snake graph \( A(Q_n) \) is shown in Figure 2.

![Figure 2](image)

Hence the \( L(2,1) \) labeling number \( \lambda(G) \) of an alternate quadrilateral snake graph \( A(Q_n) \) is 5.

**Theorem 3.3.**

The \( L(2,1) \) labeling number \( \lambda(G) \) of a double quadrilateral snake \( D(Q_n) \) is 9, for \( n \geq 3 \).

**Proof.**

Denote the vertices of a double quadrilateral snake graph as follows:

\[
V(G) = V_1 \cup V_2 \cup V_3 \text{ where } V_1 = \{ v_i / 1 \leq i \leq 2n-2 \} , \; V_2 = \{ w_i / 1 \leq i \leq n \} , \; V_3 = \{ u_i / 1 \leq i \leq 2n-2 \}
\]

Define a mapping \( f : V(G) \rightarrow \mathbb{N} \cup \{0\} \) by

\[
f(v_{2i}) = 0; \quad 1 \leq i \leq n-1
\]

\[
f(v_{2i+1}) = 2; \quad 1 \leq i \leq n-1
\]
Now let us find the $L(2,1)$ labeling number of this graph.

**Case (i):**

Let $x, y$ be any two vertices in $V_1(G)$.

**Subcase (i).**

Let $x$ and $y$ be adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0 - 2| \geq 3$.

**Subcase (ii).**

Let $x$ and $y$ be non-adjacent vertices on $V_1(G)$, then $d(x, y) \geq 3$. Therefore the $L(2,1)$ labeling condition is satisfied.

**Case (ii):**

Let $x, y$ be any two vertices in $V_2(G)$.

**Subcase (i).**

Let $x, y$ be any two adjacent vertices on $V_2(G)$, such that $x = w_{3j-2}$ and $y = w_{3j-1}$. Then $f(x) = 5$, $f(y) = 7$ and $d(x, y) = 1$.

Therefore $d(x, y) + |f(x) - f(y)| = 1 + |5 - 7| = 3$.

**Subcase (ii).**
Let \( x, y \) be any two non–adjacent vertices on \( V_2(G) \), such that \( x = w_{3i-2} \) and \( y = w_{3i} \). Then \( f(x) = 5, f(y) = 9 \) and \( d(x, y) \geq 2 \). Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + |5 - 9| \geq 3 \).

**Case (iii):**

Let \( x, y \) be any two vertices in \( V_3(G) \).

**Subcase (i).**

Let \( x \) and \( y \) be adjacent vertices on \( V_1(G) \), such that \( x = u_{2i-1} \) and \( y = u_{2i} \). Then \( f(x) = 1, f(y) = 3 \) and \( d(x, y) = 1 \). Therefore \( d(x, y) + |f(x) - f(y)| = 1 + |1 - 3| \geq 3 \).

**Subcase (ii).**

Let \( x \) and \( y \) be non-adjacent vertices on \( V_1(G) \). Then \( d(x, y) \geq 3 \). Therefore the \( L(2,1) \) labeling condition is satisfied.

**Case (iv):**

Let \( x, y \) be any two vertices, on the \( V_1(G) \) and \( V_2(G) \) respectively

**Subcase (i).**

Let \( x, y \) be any two adjacent vertices, on \( V_1(G) \) and \( V_2(G) \) respectively. Let \( x \) be a vertex on \( V_1(G) \) and \( y \) be a vertex on \( V_2(G) \) such that \( x = v_{6i-5} \) and \( y = w_{3i-2} \). Then \( f(x) = 0 \) and \( f(y) = 5 \) and \( d(x, y) = 1 \). Therefore \( d(x, y) + |f(x) - f(y)| = 1 + |0 - 5| \geq 3 \).

**Subcase (ii).**

Let \( x, y \) be any two non-adjacent vertices, on \( V_1(G) \) and \( V_2(G) \) respectively.

Let \( x \) be a vertex on \( V_1(G) \) and \( y \) be a vertex on \( V_2(G) \) such that \( x = v_{6i-5} \) and \( y = w_{3i} \). Then \( f(x) = 0 \) and \( f(y) = 7 \), then \( d(x, y) \geq 2 \). Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + |0 - 7| \geq 3 \).
Case (iv):

Let \(x, y\) be any two vertices, on the \(V_2(G)\) and \(V_3(G)\) respectively

Subcase (i).

Let \(x, y\) be any two adjacent vertices, on \(V_2(G)\) and \(V_3(G)\) respectively. Let \(x\) be a vertex on \(V_2(G)\) and \(y\) is a vertex on \(V_3(G)\) such that \(x = w_{3i-2}\) and \(y = u_{6i-5}\). Then \(f(x) = 5\) and \(f(y) = 1\) and \(d(x, y) = 1\). Therefore \(d(x, y) + |f(x) - f(y)| = 1 + |5 - 1| \geq 3\).

Subcase (ii).

Let \(x, y\) be any two non-adjacent vertices, on \(V_2(G)\) and \(V_3(G)\) respectively. Let \(x\) be a vertex on \(V_2(G)\) and \(y\) is a vertex on \(V_3(G)\) such that \(x = w_{3i-1}\) and \(y = u_{6i-5}\). Then \(f(x) = 7\) and \(f(y) = 1\). Then \(d(x, y) \geq 2\). Therefore \(d(x, y) + |f(x) - f(y)| \geq 2 + |7 - 1| \geq 3\).

Case (v):

Let \(x, y\) be any two vertices in \(V_1(G)\) and \(V_3(G)\) respectively. Clearly \(x\) and \(y\) are non-adjacent vertices. such that \(x = v_{2i-1}\) and \(y = u_{2i-1}\). Then \(f(x) = 0\), \(f(y) = 1\) and \(d(x, y) \geq 2\). Therefore \(d(x, y) + |f(x) - f(y)| \geq 2 + |0 - 1| \geq 3\).

Similarly, for all the other possibilities of \(x\) and \(y\) we find that \(d(x, y) + |f(x) - f(y)| \geq 3\). Therefore the \(L(2,1)\) labeling number of \(D(Q_n)\) is \(\lambda(G) = 9\).

Example 3.3.

The \(L(2,1)\) labeling of a double triangular snake \(D(Q_7)\) is shown in Figure 3.

\[\text{Figure 3}\]

Hence the \(L(2,1)\) labeling of a double quadrilateral snake \(D(Q_n)\) is \(\lambda(G) = 9\).
Theorem 3.4.

The $L(2,1)$ labeling number $\lambda(G)$ of an alternate double quadrilateral snake graph $A(D(Q_n))$ is 8, for $n \geq 4$ and $n \equiv 0 \pmod{2}$.

Proof.

Denote the vertices of an alternate double quadrilateral snake graph $A(D(Q_n))$ as follows:

$$V(G) = V_1 \cup V_2 \cup V_3$$

where

- $V_1 = \{v_i / 1 \leq i \leq n\}$,
- $V_2 = \{w_i / 1 \leq i \leq n\}$,
- $V_3 = \{u_i / 1 \leq i \leq n\}$

Denote the vertices of an alternate double quadrilateral snake graph $A(D(Q_n))$ as follows:

$$V(G) = V_1 \cup V_2 \cup V_3$$

where

- $V_1 = \{v_i / 1 \leq i \leq n\}$,
- $V_2 = \{w_i / 1 \leq i \leq n\}$,
- $V_3 = \{u_i / 1 \leq i \leq n\}$

Define a mapping $f : V(G) \to \mathbb{N} \cup \{0\}$ by

$$f(v_{i,n}) = 0; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(v_{i,n}) = 2; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(w_{i,n}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$f(w_{i,n}) = 7; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{4} \right\rfloor$$

$$f(w_{i,n}) = 5; \quad 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor$$

$$f(w_{i,n}) = 8; \quad 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$f(u_{i,n}) = 1; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_{i,n}) = 3; \quad 1 \leq i \leq \frac{n}{2}$$
Claim:

The $L(2,1)$ labeling number of an alternate double quadrilateral snake graph $A(D(Q_n)) = 8$.

Case(i):

Let $x,y$ be any two vertices in $V_1(G)$.

Subcase (i).

Let $x$ and $y$ be adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $f(x) = 0$, $f(y) = 2$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |0-2| \geq 3$.

Subcase (ii).

Let $x$ and $y$ be non-adjacent vertices on $V_1(G)$, such that $x = v_{2i-1}$ and $y = v_{2i}$. Then $d(x, y) \geq 3$. Therefore the $L(2,1)$ labeling condition is satisfied.

Case(ii):

Let $x,y$ be any two vertices in $V_2(G)$.

Subcase (i).

Let $x,y$ be any two adjacent vertices on $V_2(G)$, such that $x = w_{4i-3}$ and $y = w_{4i-2}$. Then $f(x) = 4, f(y) = 7$ and $d(x, y) = 1$.

Therefore $d(x, y) + |f(x) - f(y)| = 1 + |4-7| = 3$.

Subcase (ii).

Let $x,y$ be any two non-adjacent vertices on $V_2(G)$, such that $x = w_{4i-3}$ and $y = w_{4i-2}$. Then $f(x) = 4, f(y) = 5$ and $d(x, y) \geq 2$. Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |4-5| \geq 3$.

Case(iii):

Let $x,y$ be any two vertices in $V_3(G)$.

Subcase (i).

Let $x$ and $y$ be adjacent vertices on $V_1(G)$, such that $x = u_{2i-1}$ and $y = u_{2i}$. Then $f(x) = 1$,
$f(y) = 3$ and $d(x, y) = 1$. Therefore $d(x, y) + |f(x) - f(y)| = 1 + |1-3| \geq 3$.

**Subcase (ii).**

Let $x$ and $y$ be non-adjacent vertices on $V_1(G)$. Then $d(x, y) \geq 3$. Therefore the L(2,1)
labeling condition is satisfied.

**Case(iii):**

Let $x,y$ be any two vertices, on the $V_1(G)$ and $V_2(G)$ respectively

**Subcase (i).**

Let $x,y$ be any two adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively. Let $x$ be a
vertex on $V_1(G)$and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y = w_{4i-3}$ Then $f(x) = 0$ and $f(y) = 4$ and $d(x, y) \geq 1$.Therefore $d(x, y) + |f(x) - f(y)| \geq 1 + |0-4| \geq 3$.

**Subcase (ii).**

Let $x,y$ be any two non-adjacent vertices, on $V_1(G)$ and $V_2(G)$ respectively.

Let $x$ be a vertex on $V_1(G)$ and y be a vertex on $V_2(G)$ such that $x = v_{2i-1}$ and $y = w_{4i}$.
Then $f(x) = 0$ and $f(y) = 7$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |0-7| \geq 3$.

**Case(iv):**

Let $x,y$ be any two vertices, on the $V_2(G)$ and $V_3(G)$ respectively

**Subcase (i).**

Let $x,y$ be any two adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively. Let $x$ be a
vertex on $V_2(G)$and y be a vertex on $V_3(G)$ such that $x = w_{4i-3}$ and $y = u_{2i-1}$ Then $f(x) = 4$ and $f(y) = 1$ and $d(x, y) \geq 1$.Therefore $d(x, y) + |f(x) - f(y)| \geq 1 + |4-1| \geq 3$.

**Subcase (ii).**

Let $x,y$ be any two non-adjacent vertices, on $V_2(G)$ and $V_3(G)$ respectively.

Let $x$ be a vertex on $V_2(G)$ and y be a vertex on $V_3(G)$ such that $x = w_{4i-3}$ and $y = u_{2i}$.Then $f(x) = 4$ and $f(y) = 3$, then $d(x, y) \geq 2$.Therefore $d(x, y) + |f(x) - f(y)| \geq 2 + |4-3| \geq 3$. 
Case(v):

Let \( x, y \) be any two vertices in \( V_1(G) \) and \( V_3(G) \) respectively. Clearly \( x \) and \( y \) are non-adjacent vertices. such that \( x = v_{2i-1} \) and \( y = u_{2i-1} \). Then \( f(x) = 0, f(y) = 1 \) and \( d(x, y) \geq 2 \).

Therefore \( d(x, y) + |f(x) - f(y)| \geq 2 + |0-1| \geq 3 \).

Similarly, for all the other possibilities of \( x \) and \( y \) we find that \( d(x, y) + |f(x) - f(y)| \geq 3 \). Therefore the \( L(2,1) \) labeling number of \( A(D(Q_n)) \) is \( \lambda(G) = 8 \).

Example 3.2.

\( L(2,1) \) labeling of alternate triangular snake graph \( A(D(Q_8)) \) is shown in Figure 4.

![Figure 4](image)

Hence the \( L(2,1) \) labeling number \( \lambda(G) \) of alternate double triangular snake graph

4. CONCLUSION

In this paper the \( L(2,1) \) labeling number for quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs are determined.
REFERENCES


