

Tsunami Propagation Speeds: Dependence on the Direction of Propagation?

Arjun Tan* & Samaiyah Farid**

Department of Physics, Alabama A & M University
Normal, AL 35762, U.S.A.

*arjun.tan@mail.com **samaiyah.farid@aamu.edu

Abstract

Tsunamis are ocean waves generated by the displacement of a large volume of water due to earthquakes, volcanic eruptions, landslides or other causes above or below the ocean floor. Tsunamis have variously been termed ‘canal waves,’ ‘solitary waves’ and ‘long waves in shallow water.’ For such waves, the group velocity is equal to the wave velocity as a result of which it travels dispersionless for long distances with minimum loss of energy. Over the open ocean having depth of 4 km, tsunami waves typically have wavelengths of about 200 km and velocity of around 700 km/hr. Accounts of significant historical tsunami events can be found in National Oceanic and Atmospheric Administration (NOAA) website. In this paper, we analyze two historically significant tsunami events which propagated in directly opposite directions to determine if their propagation speeds were direction-dependent, viz., the Great Japan Tsunami of 2011 and the Great Chilean Tsunami of 1960. The landfall locations are chosen such that the two propagation paths are nearly identical but in the opposite directions. The propagation speeds are calculated from the observed propagation times recorded in the NOAA website. The results indicate that the propagation speeds of the Chilean Tsunami were greater than those of the Japan Tsunami for both the north-western and south-eastern segments and over the entire trajectory. The effects of the extensive equatorial current systems in the Pacific Ocean helps to explain these findings qualitatively. The existence of other factors needs to be investigated.

Introduction

Tsunamis are ocean waves generated by the displacement of a large volume of water due to earthquakes, volcanic eruptions, landslides or other causes above or below the ocean floor. Tsunamis exhibit the characteristics of *solitary waves* or *solitons* like those of *canal waves*, first observed by John Russell in Scotland in 1834. These disturbances travel like a single wave over long distances and maintain their shapes with little loss of energy. Advanced tsunami models consist of solving *hydrodynamic equations* which involve tedious *numerical methods*[1]. Recently, simplified tsunami models based upon *first*

principles have made appearances in the literature [2 – 6]. Most of these models are based on the traditional **one-dimensional treatment of water waves under gravity and surface tension forces** [2 – 6] and extended to **two-dimensional spherical surface of the world's oceans** [5, 6]. Accounts of significant historical tsunami events can be found in **National Oceanic and Atmospheric Administration (NOAA)** website [7]. In this paper, we analyze two historically significant tsunami events which propagated in directly opposite directions to determine if their propagation speeds were direction-dependent and find possible factors responsible for any difference.

THEORY OF WATER WAVES

The theory of **water waves** is well-documented in the literature [8–11]. The **wave velocity** of waves on water of density ρ under the action of gravity and **surface tension** T is obtained from a **linear wave equation** by ignoring the non-linear term in **Bernoulli's equation** [8, 11]:

$$v = \sqrt{\left(\frac{g}{k} + \frac{Tk}{\rho}\right) \tanh kh} \quad (1)$$

Here g is the **acceleration due to gravity**, h is the **depth of water** and k the **wave number**. In terms of the **wavelength** $\lambda = 2\pi/k$, we have:

$$v = \sqrt{\left(\frac{g\lambda}{2\pi} + \frac{2\pi T}{\rho\lambda}\right) \tanh \frac{2\pi h}{\lambda}} \quad (2)$$

For long wavelengths, the **gravitational term** predominates whence one has:

$$v = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh kh} \quad (3)$$

and

$$\omega = \sqrt{gk \tanh kh} \quad (4)$$

Here ω represents the **angular frequency** or **angular velocity** of the wave. Such waves are called **gravity waves**. Further, if $\lambda > h$, $\tanh kh \approx kh$ and:

$$v = \sqrt{gh} \quad (5)$$

whence, the **group velocity** u is:

$$u = \frac{d\omega}{dk} = \sqrt{gh} = v \quad (6)$$

The **group velocity is equal to the wave velocity**, which means that the wave maintains its form in a dispersion-less propagation for long distances with little loss of energy! Such waves are called **long waves in shallow water** which display the salient features of

tsunami waves. Tsunami waves in deep ocean waters typically have wavelengths of about 200 km and velocity of around 700 km/hr.

IDEAL VENUE FOR TSUNAMI PROPAGATION

The *greater Pacific Ocean* is the largest continental or oceanic entity on the surface of the Earth, covering over a full one-third of the globe. It is comfortably larger than the rest of the *world ocean* or the entire *landmasses* put together, for that matter. It provides an ideal venue for tsunami propagation studies for several reasons. Besides its enormous size, it is bounded by *active tectonic plate junctions*, studded with volcanoes called the *Ring of Fire*. Tsunamis produced at these hotspots can traverse the length and breadth of the ocean with relative ease. There are no landmasses or large islands to block or interfere with the propagation of tsunamis formed in the ocean. To the contrary, the ocean itself is dotted with small islands which pose little interference with tsunami propagation, but provide valuable platforms for recording tsunami wave amplitudes. Many of these islands are volcanic in origin and are sources of tsunamis themselves. The longest stretch of ocean water is found between the *Japan archipelago* in the west-north-west and *south-central Chile* on the east-south-east covering a distance of around 17,000 km or 85% of the distance between the North pole and the South Pole. At both the ends of this diameter lie some of the most *active plate tectonic regions* and tsunamis from either ends have traversed this favorite racetrack. Finally, an astonishing *80% of all tsunamis are recorded in the Pacific ocean*.

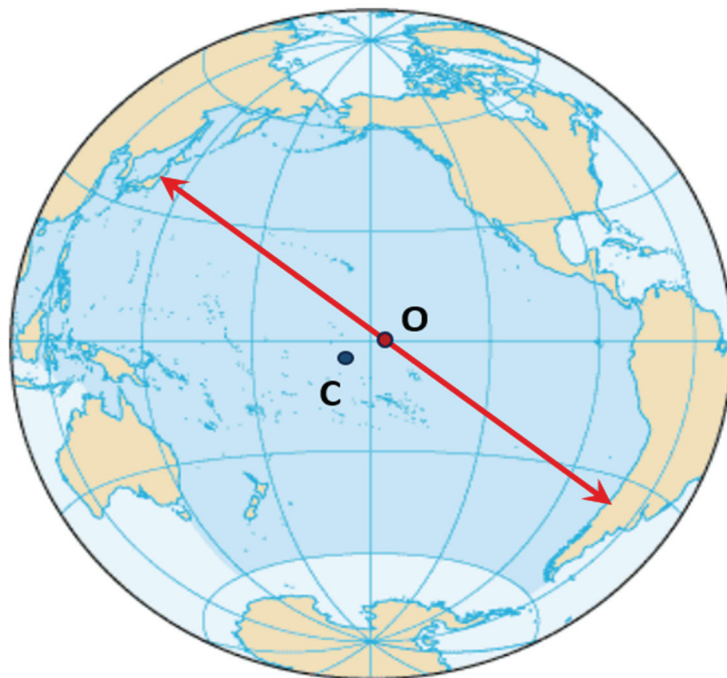


Fig. 1

A TALE OF TWO TSUNAMIS

Accounts of several *historical tsunami events* can be found in the NOAA website [7]. Amongst them, two particular tsunami events running on the great corridor between Japan's Honshu Island coast and south-central Chilean coast in opposite directions are utilized in this study: one, the *Great Japan Tsunami of 2011*; and two, the *Great Chilean Tsunami of 1960* [7]. The latitudes of the sources of these two tsunamis are nearly the same albeit in opposite hemispheres (Table I). Their landfall locations are chosen such that the two propagation paths are nearly the same but run in the opposite directions (Table I). On the surface of the Earth (oceans), this common path lies on a *great circle*. Figure 1 (from [12]) is an *equal-area global map centered on the Pacific Ocean*. On this map, the common tsunami path appears as a straight line. This line crosses the equator at the location **O** (0° , 145.644°W) on Fig.1. The location **C** (4.97°S , 158.75°W) on the map marks the *geographical center of the greater Pacific Ocean* (from [13]).

On common *cylindrical projection maps*, the great circles are no longer straight lines in general and the *angular coordinates (latitude and longitude)* become *Cartesian coordinates*. Figure 2 (from [14]) is an example of such a map relevant to this study. Here the general equation of a great circle in latitude λ and longitude ϕ is given by (cf. [15]):

$$\lambda = \tan^{-1}[A \sin(\phi - \alpha)] \quad (7)$$

where the *constants* A and α are determined from the *boundary conditions* at the cross-over point **O** of Fig. 1: (1) $\lambda = 0$, and (2) $\sin(\phi - \alpha) = 0$; giving $A = -0.843769$, and $\alpha = 214.356^\circ$. The propagation path, which is a great circles on the Pacific Ocean and appears as a straight line in Fig. 1 acquires a *sinusoidal shape* in Fig. 2.

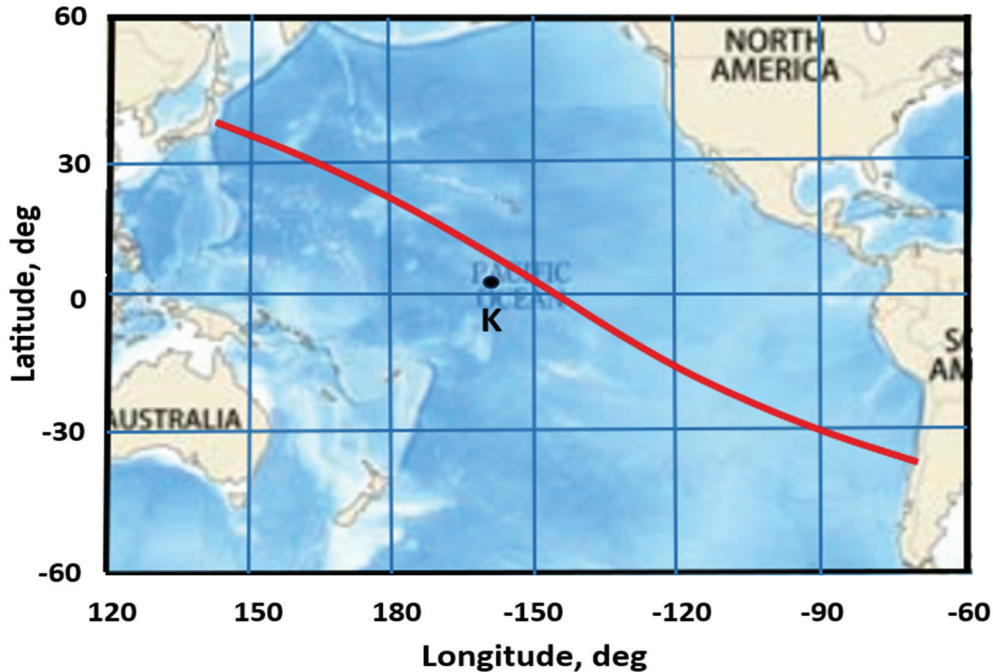


Fig. 2

RESULTS

The data in Table I are readily available in Ref. [7]. The *propagation time* between the source and landfall locations are based on actual observations. The *propagation distance* can be calculated from spherical trigonometry. If (λ_1, φ_1) and (λ_2, φ_2) are the coordinates of the two locations, the propagation distance is given by:

$$\ell = r_E [\sin\varphi_1 \sin\varphi_2 + \cos\varphi_1 \cos\varphi_2 \cos(\lambda_2 - \lambda_1)] \quad (8)$$

where r_E is the *reference radius of the Earth* = 6,378.137 km. It can be verified that the calculated values agree closely with those given in Ref. [7]. The Japan Tsunami traverses its trajectory in 22 hr 54 min at an average speed of $v_{J \rightarrow C} = 740.000$ km/hr, whereas the Chilean Tsunami traverses the same distance in 22 hr 24 min at the average speed of $v_{C \rightarrow J} = 755.089$ km/hr (Table I). The Chilean Tsunami takes a full 30 minutes fewer than the Japan Tsunami in covering the same distance! *The average speed of the Chilean Tsunami is 15 km/hr higher than the Japan Tsunami!*. This is definitely a significant result.

Table I. Tsunami Propagation Speeds in Opposite Directions		
Tsunami	2011 Japan Tsunami	1960 Chilean Tsunami
Source Location	Honshu Island	South Central Chile
Source Coordinates	38.297°N, 142.373°E	38.143°S, 73.407°W
Date	11 March 2011	22 May 1960
Time	05:46:24 UTC	19:11:17 GMT
Selected Landfall Location	Corral, Los Rios, Chile	Kamaishi, Iwate, Japan
Location Coordinates	39.878°S, 73.423°W	39.267°N, 141.883°E
Distance from Source	16,946 km	16,914 km
Travel Time	22 hr 54 min	22 hr 24 min
Average Travel Speed	740.000 km/hr	755.089 km/hr

In order to substantiate this finding, we now investigate whether this result holds true at different parts of the trajectory, viz., the west-north-western and east-south-eastern parts of it. Unfortunately, there are no islands or observation platforms at the center of the trajectory (**O** in Fig. 1). Fortunately, nonetheless, there is an observation platform at *Kiritimati* (formerly *Christmas Island*) (1.85°N, 157.4°W), marked **K** in Fig. 2, which is not far from the central point (**O** in Fig. 1), 7,302 km away from the west-north-western end of the trajectory (from [7]). The travel time from the source of the Japan Tsunami was 9 hr 00 min [7]. The equivalent travel time and distance were subtracted from the common trajectory to give the time and distance of the remainder part of the trajectory to the east-south-eastern destination with the result: $v_{J \rightarrow K} = 811.333$ km/hr; and $v_{K \rightarrow C} =$

693.813 km/hr (Table II). Similar analysis of the Chilean Tsunami gives: $v_{C \rightarrow K} = 853.828$ km/hr; and $v_{K \rightarrow J} = 694.028$ km/hr (Table II). In both segments as well as over the entire trajectory, the Chilean Tsunami propagated faster than the Japan Tsunami. This must now be taken as a ***systematic result rather than an accidental one!*** Incidentally, these estimated speeds are quite consistent with those given by Eq. (5) [6]. In the south-east segment of the common trajectory, the average speed of the two tsunamis was 693.921 km/hr whereas in the north-west segment, the average speed was 832.581 km/hr, a full 138.66 km/hr higher! Figure 2 is concurs with this. ***The north-western Pacific Ocean is uniformly and significantly deeper than the south-eastern Pacific Ocean*** as is indicated by the color-code of that figure.

Table II. Tsunami Propagation Speeds in North-West and South-East Segments		
Tsunami	2011 Japan Tsunami	1960 Chilean Tsunami
Overall Distance Traversed	16,946 km	16,914 km
Overall Propagation Time	22 hr 54 min	22 hr 24 min
Overall Propagation Speed	740.000 km/hr	755.089 km/hr
North-West Distance Traversed	7,302 km	7,286 km
North-West Propagation Time	9 hr 00 min	8 hr 32 min
North-West Propagation Speed	811.333 km/hr	853.828 km/hr
South-East Distance Traversed	9,644 km	9,628 km
South-East Propagation Time	13 hr 54 min	13 hr 52 min
South-East Propagation Speed	693.813 km/hr	694.028 km/hr

Discussion

While it is not obvious why a tsunami from south-central Chile to Japan would travel faster than another in the opposite direction, one conspicuous cause for this directional dependence of tsunami speeds would be the ***equatorial current*** of the Pacific Ocean. Figure 3 is a detailed map of Pacific Ocean currents uploaded by Carleton College (from [16]). The major currents in the tropical regions include: (1) the very extensive ***east-to-west North Equatorial Current***; (2) another extensive ***east-to-west South Equatorial Current***; and (3) a narrow ***west-to-east Counter Equatorial Current*** sandwiched between the former two (Fig. 3).

The whole system acts as a slow but sure conveyor belt which will purportedly affect tsunami propagation speed. From the geometrical layout in Fig. 3, it is obvious that the equatorial current system will aid the propagation speeds of tsunamis from the east-south-east direction and hinder those from the west-north-west direction.

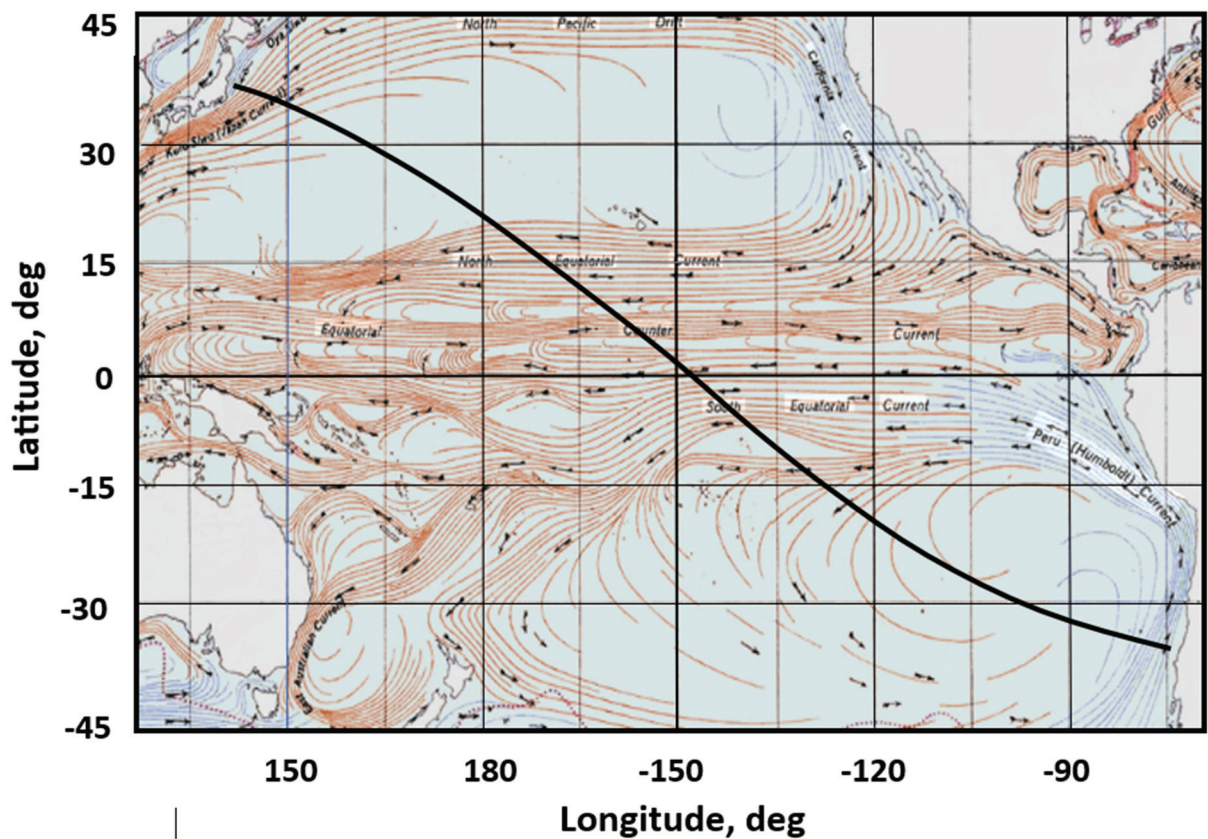


Fig. 3

However, the speed of the equatorial current reaches only up to 100 cm/s which can only partially explain the speed difference of the two tsunamis going in opposite directions. Perhaps, the possible roles of other factors such as *sub-surface currents* and *upwellings* needs to be investigated.

References

- [1] M.A. Imteaz, F. Imamura & J. Naser in *The tsunami threat – Research and Technology*, N.A. Mörner, ed., InTech (2011).
- [2] F. Behroozi & N. Podolefsky, Dispersion of capillary-gravity waves: a derivation based on conservation of energy. *Eur. J. Phys.* **22**, 225-231 (2001).
- [3] G. Margaritondo, G. Explaining the physics of tsunamis to undergraduate and non-physics students, *Eur. J. Phys.* **26**, 401-407 (2005).
- [4] O. Helene & M.T. Yamashita, Understanding the tsunami with a simple model, *Eur. J. Phys.* **27**, 855-863 (2006).
- [5] A. Tan & I. Lyatskaya, Alternative tsunami models. *Eur. J. Phys.* **30**, 157-162 (2009).
- [6] A. Tan, A. Chilvery, M. Dokhanian & S. Crutcher, Tsunami propagation models based on first principles, in *Tsunami – Analysis of a hazard*, G. Lopez, ed., InTech, 107-140 (2012).

- [7] <https://ncei.noaa.gov/products/natural-hazards/tsunamis-earthquakes-volcanoes/tsunamis/recent-significant-events>.
- [8] C.A. Coulson, *Waves*, Oliver & Boyd, Edinburgh (1955).
- [9] R.V. Sharman, *Vibrations and Waves*, Butterworths, London (1963).
- [10] D.H. Towne, *Wave Phenomena*, Dover, New York (1967).
- [11] W.C. Elmore & M.A. Head, *Physics of Waves*, Dover, New York (1969).
- [12] https://en.wikipedia.org/wiki/File:Pacific_Ocean.png.
- [13] Arjun Tan, Determining the areas and geographical centers of Pacific Ocean and its northern and southern halves, *Int. J. Oceanogr.*, **15**, 25-31 (2021).
- [14] <https://freeworldmaps.net/world/pacific-centered/>.
- [15] J.B. Marion & S.T. Thornton, *Classical Dynamics of Particles and Systems*, Saunders (1995).
- [16] https://serc.arleton.edu/integrate/teaching_materials/climate_change/student_materials/unit3_article.html.