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The Earth Hemispheres and their Geoid Elevations

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Abstract

The orbital analysis of artificial Earth satellites have revealed the existence of a strong third harmonic in the gravitational potential of the Earth which gives its 'pear-shape' in the averaged meridional section. In a recent study, a novel approach to explain this pear-shape based upon statistical correlation between variables was reported. It was shown that the average distribution of water was directly correlated to the average geoid heights with a correlation coefficient of .630, which thus provided a plausible explanation of the pear-shape of the Earth. In this study, the above method of analysis was applied to the 'Land' and 'Water Hemispheres' of the Earth, where the correlation coefficient improved to .669 and .725, respectively. Further, the mean elevations of the various hemispheres were calculated or estimated. The results are the following: (1) For an ideal pear-shaped Earth, the mean elevations of the Northern and Southern Hemispheres are -2.546 m and 2.546 m, respectively; (2) For the actual Earth, the mean elevations are -1.888 m and 2.010 m, respectively; and (3) For the 'Land' and 'Water Hemispheres', the respective mean elevations are -3.370 m and 3.511 m, respectively.

INTRODUCTION

The shape of the Earth, like that of any other sizeable heavenly body, is determined largely by gravitation and centrifugal force of rotation. These two forces, acting together on a uniform Earth, produce an oblate spheroid, symmetrical about its axis of rotation. The surface of this spheroid is referred to as the *reference ellipsoid*. Due to other factors such as in homogeneities, internal convection and surface features, etc., however, the actual surface of the Earth departs slightly but significantly from the reference ellipsoid. The actual surface is called *geoid*, which is an equipotential surface. It coincides with the surface of the oceans, and extends under the continents to the level to which ocean water would settle if connected to the ocean by open channels such as the straights of Gibraltar, Dardanelles and Bosphorus.

Our knowledge about the actual shape of the Earth was greatly refined by the orbital analysis of the artificial Earth satellites. The orbital analysis of Vanguard 1 uncovered the first indication of the departure of the Earth's figure from the reference ellipsoid: the averaged meridional section of the Earth was '*pear-shaped*' with the 'stem' of the pear at the North pole [1].Whilst various attempts at explaining the pear-shape of the Earth have proved to be unsatisfactory, a curious correlation between land-water distribution and the pear-shape of the Earth has recently been uncovered [2]. It was shown that a quantity representing the percentage of water w bears a significant correlation with the average geoid elevation h at any particular latitude. And since the former is an independent variable, it was suggested that it may well have been responsible for the latter, thus creating the pear-shape of the Earth [2]. This suggestion is consistent with the notion of suppression of geoid elevation by mass loading (cf. [3]). The present paper is a continuation of our earlier study on land-water distribution and geoid elevations [2]. But first, we digress to the relatively obscure notions of 'Land' and 'Water Hemispheres'.

The Earth is a watery planet, the only one of its kind in the solar system. A full 71% of the Earth's surface is covered with water, with land comprising the remaining 29%. Also, the distribution of land (and for that matter, water) is lop-sided, with the Northern Hemisphere possessing two-thirds of the land. However, the Earth can be divided into any number of hemispheres bounded by great circles. A hemisphere containing the maximum land area is called the *Land Hemisphere*. Its complementary hemisphere, which then necessarily contains the maximum water area, is called the *Water Hemisphere*. The centre of the Land Hemisphere is determined to be at (47^{o}N, 2^{o}W) near the city of Nantes, France [4, 5]. The centre of the Water Hemisphere is at the anti-podal point (47^{o}S, 178^{o}W) near Bounty Island, New Zealand [4, 5]. The Land Hemisphere contains about 85% of the land surface of the Earth, whereas the Water Hemisphere contains the remaining 15% [6]. However, only 11% of the Water Hemisphere is land, comprising mainly Antarctica, Australasia and southern part of South America. A full 89% of the Water Hemisphere is covered with water. Even on the Land Hemisphere, water dominates land by 53% to 47%. A map of the Land Hemisphere, in *Lambert Projection*, is shown in Fig. 1 (from Ref. [6]). The land-water distribution of the various hemispheres are given in Table I.

Table I. The Earth Hemispheres with Mean Geoid Elevations

Hemisphere	Coordinates of Hemisphere Centre			Land %	Water %	Mean geoid elevation
	Co-latitude	Latitude	Longitude			
Northern Hemisphere	0^o	90^{o}N	-	39	61	-1.888 m
Southern Hemisphere	180^o	90^{o}S	-	19	81	2.010 m
Land Hemisphere	43^o	47^{o}N	2^{o}W	47	53	-3.370 m
Water Hemisphere	137^o	47^{o}S	178^{o}E	11	89	3.511 m
Pear-shaped N.H.	0^o	90^{o}N	-	-	-	-2.546 m
Pear-shaped S.H.	180^o	90^{o}S	-	-	-	2.546 m

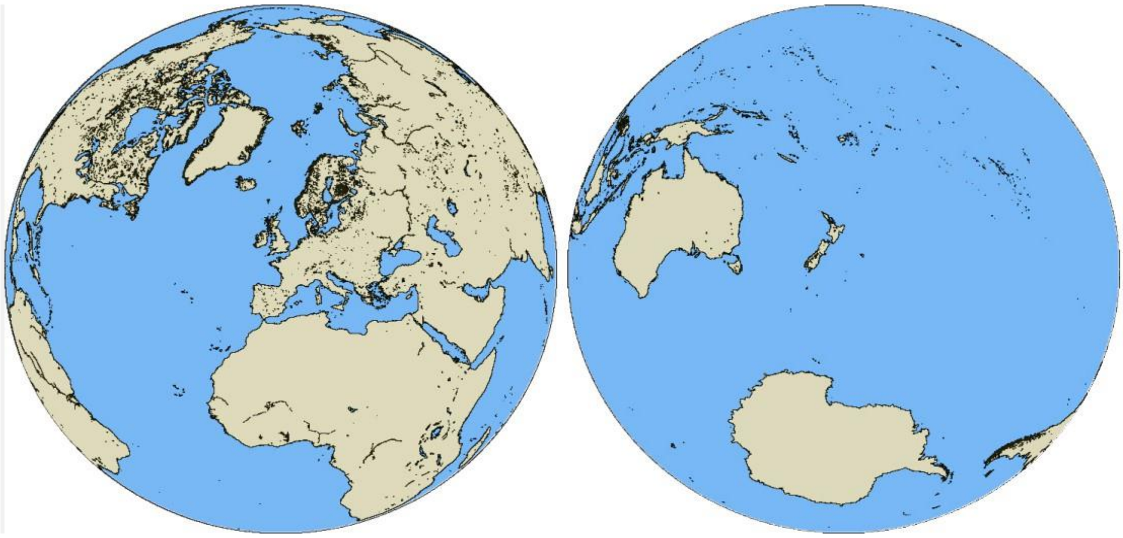

Fig. 1: Land and Water Hemispheres of the Earth.

In this paper, the method of analysis of our earlier study [2] is extended to the Land and Water Hemispheres. Further, the mean elevations of the various hemispheres are calculated or estimated, including those of the Northern and Southern Hemispheres of an ideal pear-shaped Earth.

METHOD OF ANALYSIS

The *correlation coefficient*is a quantity which illustrates a measure of dependence of some sorts between two variables *x* and *y*and is usually denoted by *r*. Given a set of *n* values of *x* and *y*, it is defined by [7]

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)}\sqrt{(\sum y^2 - n\bar{y}^2)}} \quad (1)$$

where $\bar{x}$ and $\bar{y}$ are the average values of *x* and *y* respectively, and the summation Σ runs from 1 to *n*. The correlation coefficient is independent of scale (multiplication by a constant) and origin (translation by a constant) [8]. In our earlier study [2], correlation coefficients were calculated between three relevant quantities, which are the following:

(1) The geoid height *h* at co-latitude θ averaged over longitudes as provided by satellite orbital analysis data;

(2) The third harmonic in Legendre's polynomial expansion of the Earth's gravitational potential (cf. [9]):

$$P_3(cos\theta) = \frac{1}{2}(5cos^3\theta - 3cos\theta) \quad (2)$$

and

(3) The percentage of (*water – land*),defined in three equivalent ways:

$$w = water\% - land\% \quad (3)$$

or

$$w = 2water\% - 100 \quad (4)$$

or

$$w = 100 - 2land\% \quad (5)$$

w is thus a measure of the percentage of water or a negative measure of the percentage of land at a particular co-latitude θ.

The co-latitudinal variations of the three quantities, showing significant similarities in trend are displayed in Fig. 2 (from [2]). It was found that all three variables were positively correlated with one another having correlation coefficients of $r(h, P_3)$ = .963; $r(w, h)$ = .630; and $r(w, P_3)$ = .516. The very high positive value of $r(h, P_3)$ indicates that the geoid is mainly shaped by the third harmonic, i.e., it has a predominantly pear-shape. The significantly large positive value of $r(w, h)$ indicates that there is a significant inter-dependence between the variables w and h. By definition (3), this means that the distribution of water is directly correlated with the geoid height h. Since w is an independent variable, it may well have been responsible for the geoid heights in the first place. In this paper, we extend this analysis to the Land and Water Hemispheres.

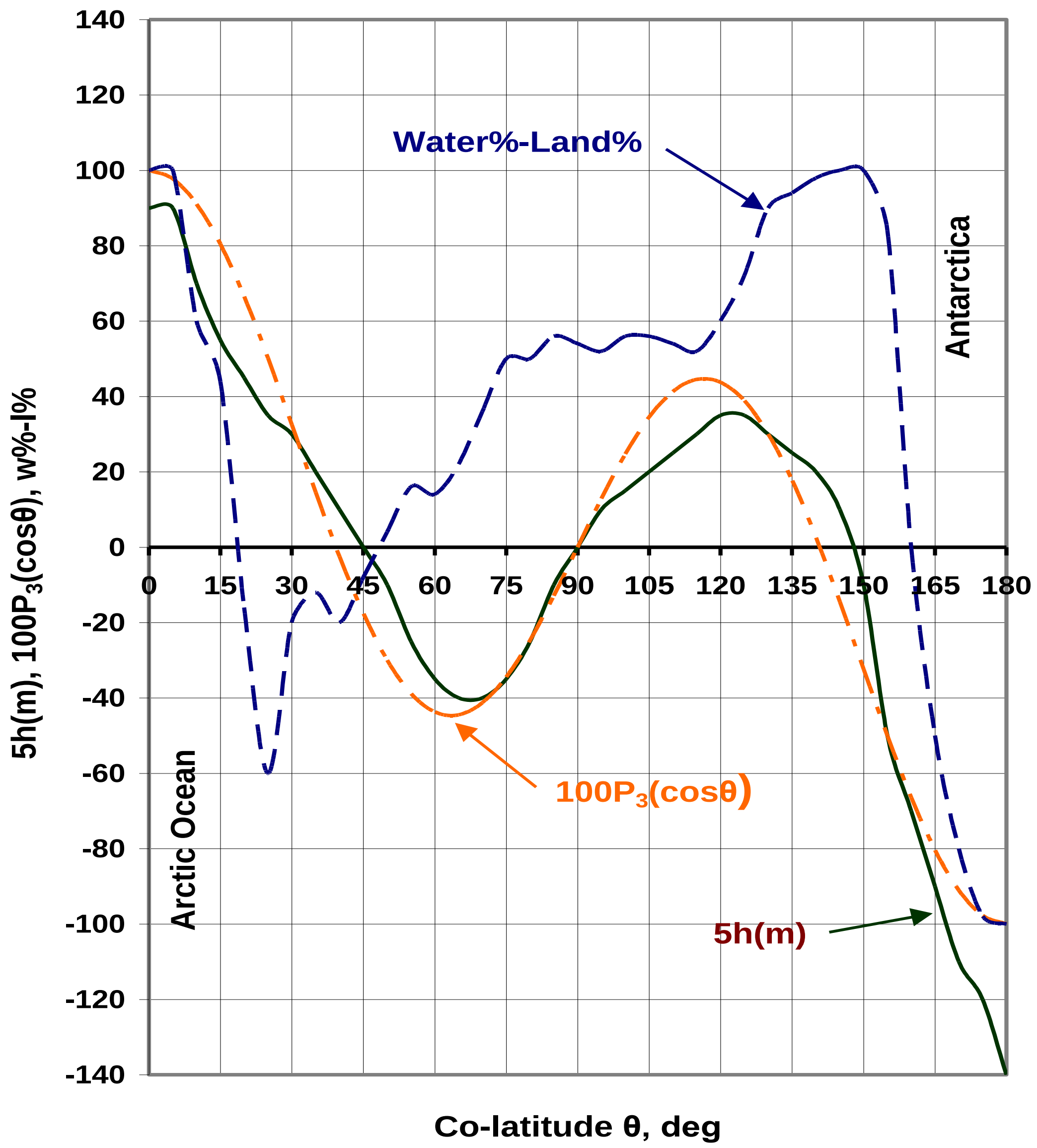

Fig. 2: Co-latitudinal variations of geoid height, third harmonic of Legendre function and water percentage of the Earth.

WATER PERCENTAGE AND GEOID IN LAND AND WATER HEMISPHERES

Since the launch of the first satellites, geoid height estimations have improved continuously. Today, the EGM 2008 model [10] is widely used by researchers. Recently, the GRIM5-S1 model [11] has further improved the geoid height accuracies. Fig. 3 (adapted from [12]) is a world-wide geoid map in *Mollweide Projection* based on the latest model. The boundary between the Land and Water Hemispheres is marked on that figure. Significantly, the most intense geoid high in the western Pacific Ocean falls entirely within the Water Hemisphere. Additionally, the most intense low of geoid in the northern Indian Ocean lies predominantly in the Land Hemisphere.

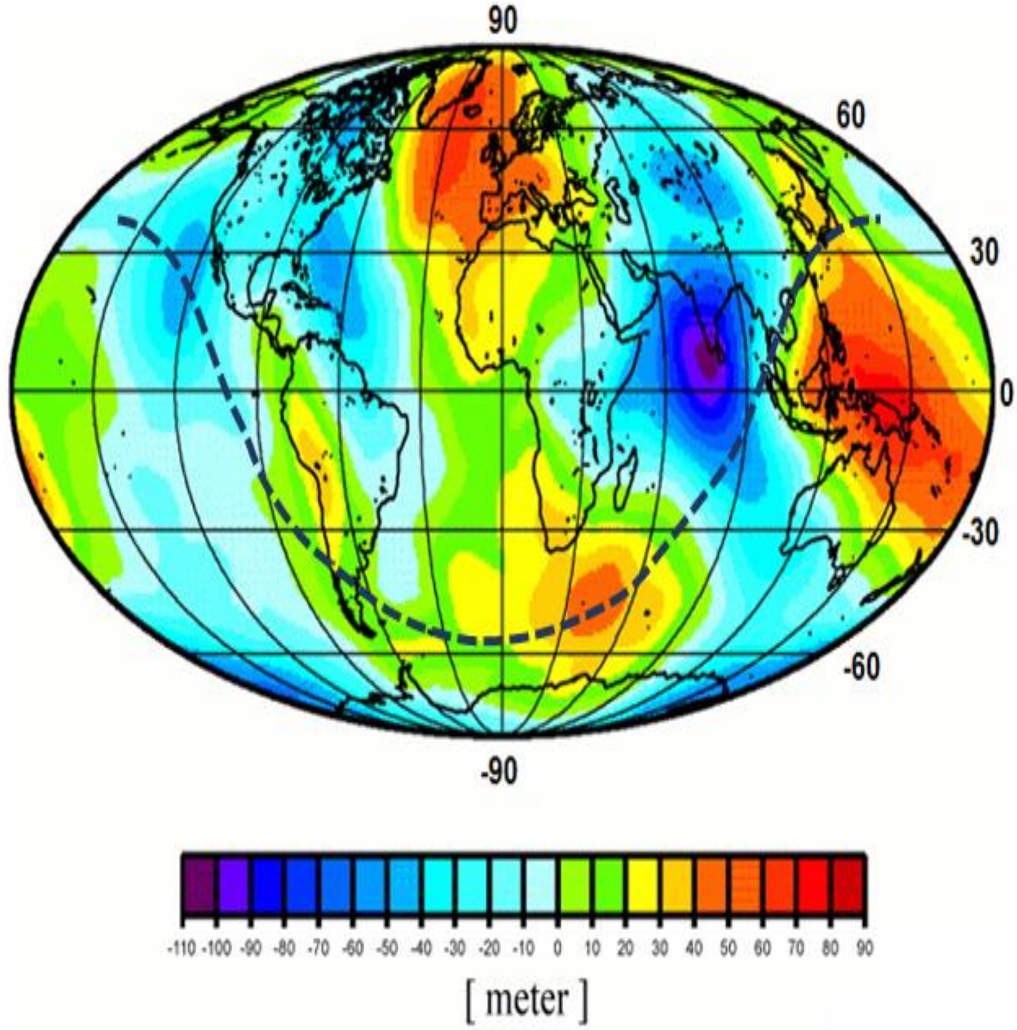

Fig. 3: Geoid map of the world in Mollweide Projection showing the boundary between the Land and Water Hemispheres.

The water percentage *w* and geoid height *h* are calculated as functions of co-latitude θ for both the Land and Water Hemispheres. This is done more conveniently on a world-wide geoid map in *Lambert's Equidistant Cylindrical Projection* (Fig. 4, adapted from [13]).

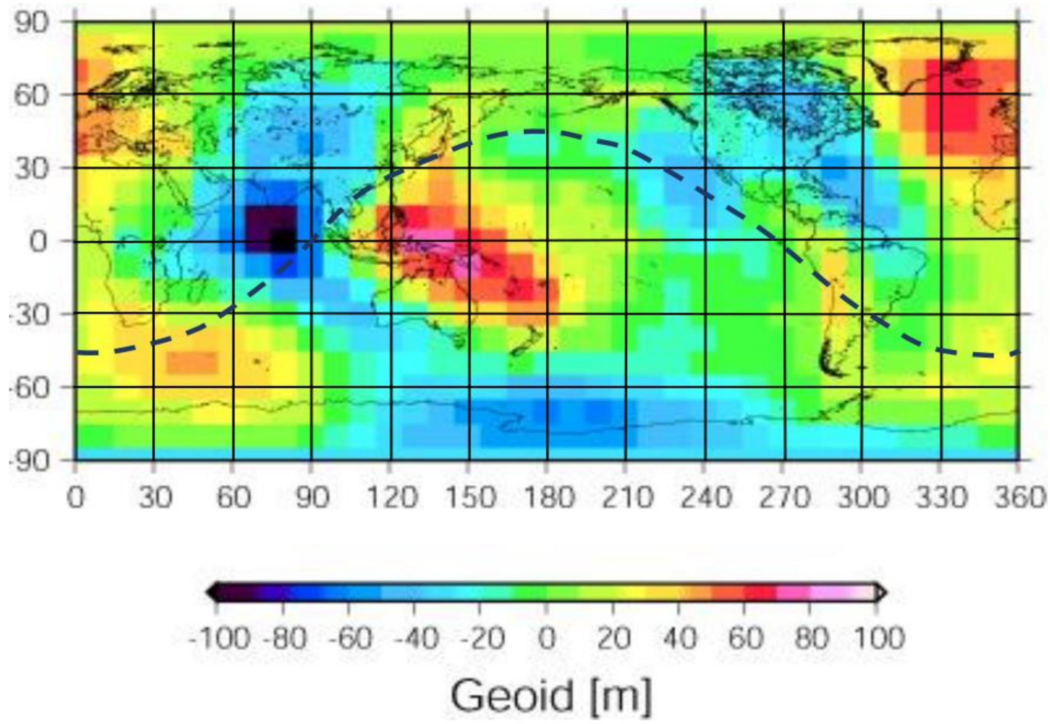

Fig. 4: Geoid map of the world in Lambert's Equidistant Cylindrical Projection with average geoid colour for $10^o \times 10^o$ areal sections. The boundary between the Land and Water Hemispheres is also marked.

In that projection, the latitudes and longitudes are equally spaced. Areal sections bounded by latitudes and longitudes at 10^o intervals are considered. The average colour of each section gives the average geoid height of that section (Fig. 4). A section on the boundary between the hemispheres is assigned to the proper hemisphere by virtue of the larger fraction. The geoid heights of the sections are then averaged over the longitudes within the hemisphere to yield h for that hemisphere. The water percentage w is also calculated graphically for each co-latitude within the hemisphere. Fig. 5 shows the plot of w and h as functions of θ for both the Land and Water Hemispheres. As in Fig. 2 for the Northern and Southern Hemispheres, the general trends between the two curves betray unmistakable similarities between the two variables w and h for both hemispheres. The correlation coefficients between the two variables for the Land and Water Hemispheres are now .669 and .725, respectively. Both represent significant increases over the values of .630 for the Earth as a whole [2]. This study reinforces the notion of dependence of the geoid heights on the water percentage at a particular co-latitude.

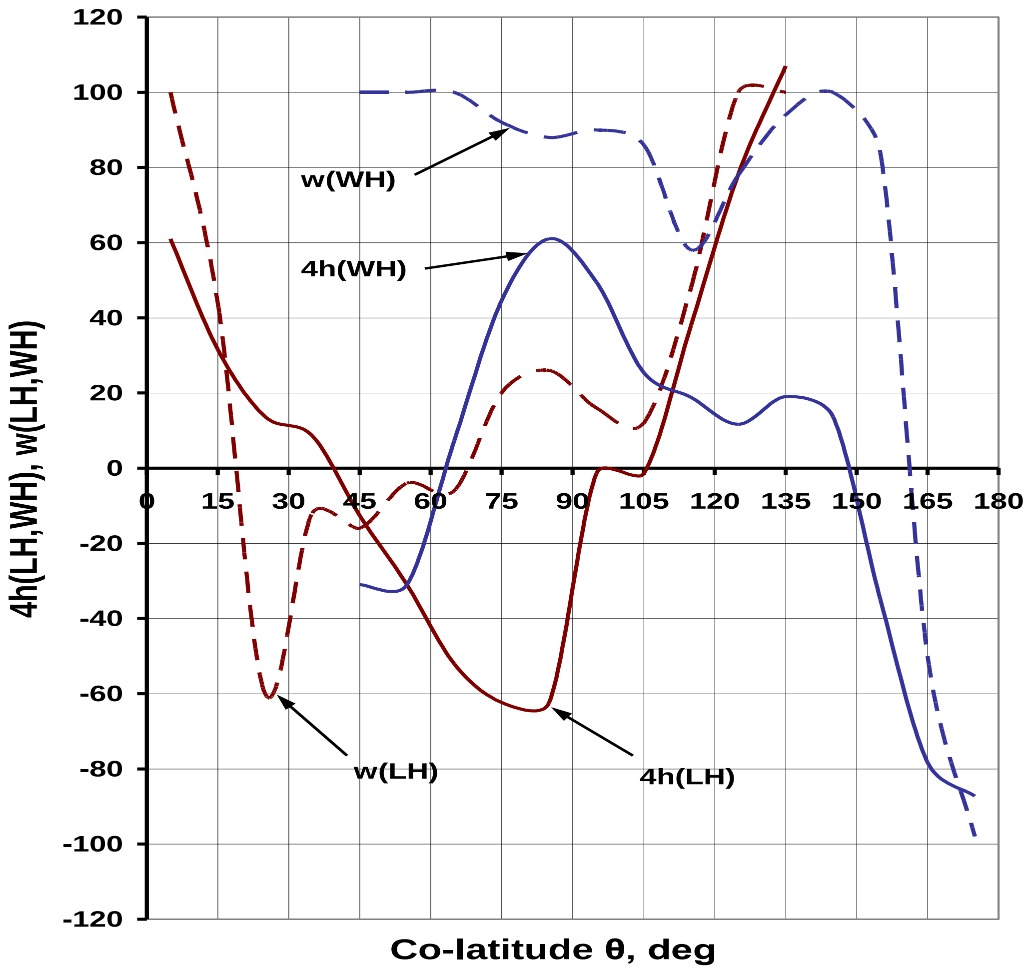

Fig. 5: Co-latitudinal variations of water percentage and geoid elevations of the Land and Water Hemispheres.

AVERAGE GEOID ELEVATIONS OF HEMISPHERES

As a continuation of this study, we now calculate or estimate the average geoid elevations of the hemispheres of the Earth. We do this for three models of the Earth's geoid: (1) An idealized pear-shaped model of the Earth for both the Northern and Southern Hemispheres; (2) The actual geoid model of the Earth for the Northern and Southern Hemispheres; and (3) The actual geoid model of the Earth for the Land and Water Hemispheres. Since the area of a section of co-latitudinal width $d\theta$ and longitudinal width $d\phi$ is $a^2 sin\theta d\theta d\phi$ with a being the reference radius of the Earth, the geoid height of each section is multiplied by the factor of $sin\theta$ in order to yield its weighted average. The weighted average geoid heights for the northern and southern hemispheres are then as follows:

$$< h >_{NH} = \frac{\int_0^{\pi/2} hsin\theta d\theta}{\int_0^{\pi/2} sin\theta d\theta} = \int_0^{\pi/2} hsin\theta d\theta \quad (6)$$

and

$$< h >_{SH} = \frac{\int_{\pi/2}^{\pi} hsin\theta d\theta}{\int_{\pi/2}^{\pi} sin\theta d\theta} = \int_{\pi/2}^{\pi} hsin\theta d\theta \qquad (7)$$

since the denominators of both expressions are unity.

As our first example, we consider an idealized pear-shaped Earth, where the geoid height is expressed as

$$h = AP_3(cos\theta) = \frac{A}{2}(5cos^3\theta - 3cos\theta) \qquad (8)$$

The least-squares fit amplitude *A* is obtained by multiplying the observed geoid height *h* in Eq. (8) by $cos\theta$ and summing over the data points, giving:

$$A = \frac{2\sum hcos\theta}{\sum(5cos^4\theta - 3cos^2\theta)} = 20.365\, m \qquad (9)$$

This value is close to the value of the amplitude (20) used in the earlier study [2].

Eq. (8), when substituted in Eqs. (6) and (7), yields the weighted average geoid heights for the Northern and Southern Hemispheres of a pear-shaped Earth:

$$< h >_{PNH} = \frac{A}{2}\int_0^{\pi/2}(5cos^3\theta - 3cos\theta)sin\theta d\theta = -\frac{A}{8} = -2.546\, m \qquad (10)$$

and

$$< h >_{PSH} = \frac{A}{2}\int_{\pi/2}^{\pi}(5cos^3\theta - 3cos\theta)sin\theta d\theta = \frac{A}{8} = 2.546\, m \qquad (11)$$

The average geoid heights of the Northern and Southern Hemispheres of a pear-shaped Earth are equal in magnitude but opposite in sign. This is to be expected, since $P_3(cos\theta)$ is an odd function about the equator. Further, because of the signs of the average geoids, it can be stated that *the Southern Hemisphere has a larger surface area than the Northern Hemisphere for an idealized pear-shaped Earth*.

These results can be better grasped as follows. $P_3(cos\theta)$ is a cubic polynomial whose three roots are obtained by setting it to zero, giving:

$$cos\theta(5cos^3\theta - 3cos\theta) = 0 \qquad (12)$$

The roots of Eq. (12) are: (1) $\theta_1 = cos^{-1}\sqrt{3/5} \approx 39.23^o$; (2)$\theta_2 = cos^{-1}0 = 90^o$; and (3) $\theta_3 = \pi - cos^{-1}\sqrt{3/5} \approx 140.77^o$. The Northern Hemisphere is divided into two parts: (1) The polar section north of $\theta = \theta_1$, where geoid elevations are positive; and (2) The tropical section south of $\theta = \theta_1$, where geoid elevations are negative. The surface area of the polar section is calculated as follows:

$$A_{polar} = a^2\int_0^{\theta_1} sin\theta d\theta \int_0^{2\pi} d\phi = 2\pi a^2(cos\theta_1 - 1) \approx .225(2\pi a^2) \qquad (13)$$

Thus the polar section has only 22.5% of the area of the Northern Hemisphere. The remaining 77.5% of the Northern Hemisphere belongs to the tropical section. This accounts for the overall negative elevation of the entire Northern Hemisphere. A similar calculation explains the overall positive elevation of the Southern Hemisphere.

The actual geoid of the Earth, averaged over longitudes, is nearly pear-shaped with a correlation coefficient $r(h, P_3) = .963$. Thus, it is expected that the average geoid elevations of the actual Northern and Southern Hemispheres should not be much different from those of the ideal pear-shape Earth. The average geoid elevations were estimated numerically from the observed average geoid heights using Eqs. (6) and (7). The results are: (1) $< h >_{NH} = -1.888\ m$; and (2) $< h >_{SH} = 2.010\ m$. These figures depart considerably from those of the idealized pear-shaped Earth of $< h >_{PNH} = -2.546\ m$and $< h >_{PSH} = 2.546\ m$, respectively. Thus $< h >_{NH}$ is numerically 26% smaller than $< h >_{PNH}$ whereas $< h >_{SH}$ is 21% smaller than $< h >_{PSH}$. In view of the fact that the Earth's figure is nearly pear-shaped, the result most likely point to the role of the water percentage *w* in the average geoid elevations of the hemispheres $< h >$.

Finally, the average elevations of the Land and water Hemispheres were also estimated numerically. The results are: (1) $< h >_{LH} = -3.370\ m$; and (2) $< h >_{WH} = 3.511\ m$. These results compare with $< h >_{NH} = -1.888\ m$; and $< h >_{SH} = 2.010\ m$. Thus, $< h >_{LH}$ is numerically 78% greater than $< h >_{NH}$ whereas $< h >_{WH}$ is 75% greater than $< h >_{SH}$. This will be more surprising if we consider how much common surface area the Northern and Land Hemispheres (and the Southern and Water Hemispheres) share, which is calculated as follows. The area not common to the Land and Northern Hemispheres is that of a *lune* between two great circles separated by the angle between the poles of the two hemispheres, which is $\alpha = 90^o - 47^o = 43^o$. The area of this lune is $S = (43/180)(2\pi a^2) \approx .2389(2\pi a^2)$, or 23.89% of the area of a hemisphere. The area common to the Northern and Land Hemisphere is then $1 - S \approx .7611(2\pi a^2)$, or 76.11% of the area of a hemisphere. The same analysis holds between the Water and Southern Hemispheres. The mystery is resolved if one scrutinizes world geoid map of Fig. 3. The Land Hemisphere excludes much of the great Western Pacific geoid high (which is a part of the Northern Hemisphere) and includes much of the great Northern Indian Ocean low (which is not a part of the Northern Hemisphere). Both of these factors act to lower the average geoid elevation of the Land Hemisphere. The same arguments explain the enhancement of the average geoid elevations of the Water Hemisphere.

DISCUSSION

The pear-shape of the Earth remained one of the great unsolved problems of the Geophysical era. In a recent study, we obtained a possible causality between the land-water distribution and the pear-shape of the Earth, where it was shown that the percentage of water was positively correlated to the longitudinally-averaged geoid height consistent with the principle of mass loading. The present study showed that when the Land and Water Hemispheres were considered in lieu of the Northern and

Southern Hemispheres, the correlation coefficient between the percentage of water and average geoid elevation increased significantly. This happened, of course, when the percentage of water increased and the percentage of land decreased, which then bolsters the notion of causality between the land-water distribution and the shape of the Earth. The mean geoid elevations of the various hemispheres further enhances the notion of that causality.

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