

Reflectance Properties of Thue-Morse 1-D Dielectric Multilayers

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Abstract

Omnidirectional reflectivity is theoretically investigated in four Thue-Morse (T-M) structures S_4 , S_5 , S_6 and S_7 . It is observed that the width of omnidirectional reflection region increases with the order of T-M multilayer i.e. from S_4 to S_7 generation and multiple photonic band gaps exist for both TE and TM modes due to self-similarity of T-M multilayers. It can also be found that multiple omnidirectional bandgaps (OBGs) exist in the higher generation non-periodic T-M dielectric structures due to the self-similarity in the internal structure. These structures can be used as dense optical filters, sensors and multifrequency laser cavities.

INTRODUCTION

Photonic crystals [1-2] are usually characterized by a well-defined lattice periodicity. Since all the applications of these crystals are based on the photonic band gaps, it is very important to find out the means by which the gap parameters like gap-width, position and even its resonant modes can be controlled conveniently. There is increasing importance on the careful control of the transmission characteristics in order to design the photonic crystals used in dense wavelength division multiplexing systems and photonic integrated circuits which derive from the device band properties. There is thus a high premium on the ability to engineer the band properties of a photonic crystal in a systematic manner [3-4].

When the periodicity of PBG structure is broken, it is called aperiodic or quasiperiodic structure. The concept of a quasicrystal is a natural extension of that of a crystal, in which translational periodicity is relaxed, preserving quasiperiodic order.

Now, the studies on PBG have been extended to include quasiperiodic photonic structures. Compared to the periodic structures, more structural parameters can be tuned in the quasiperiodic designs, thus opening a way to a wide range of technological applications in several different fields [5]. This particular property of these materials is used in opto-electronic devices [6-9]. One-dimensional photonic quasicrystals including Fibonacci [10] and Thue-Morse (TM) [11] sequences have been constructed experimentally.

Photonic quasiperiodic structures are deterministically generated dielectric structures with nonperiodic refractive index modulation. Quasi-periodic photonic crystals are quite different from periodic photonic crystals in many aspects [12-14]. The photonic bandgaps of quasiperiodic photonic crystals are omnidirectional gaps i.e. these gaps are independent of the incident direction, whereas in periodic photonic crystal, the gaps depend on the incident direction.

Now-a-days, there is increasing interest in the physics and applications of one-dimensional spatially periodic, quasiperiodic and random photonic bandgap (PBG) structures [15-18]. Thue-Morse structure [19], Fibonacci sequence [20-22], Cantor layer etc. are some examples of the one dimensional quasiperiodic structures.

The Thue-Morse quasicrystal has been the subject of an extensive theoretical and experimental investigation in the last three decades. Liu [19] investigated Thue-Morse sequences for the normal propagation of light waves. Localization properties of light in this quasi-periodic system were also demonstrated. Qiu *et al* [20] provided a new approach to achieve the omnidirectional reflection in multiple frequency ranges by optical observation in Thue-Morse $\text{TiO}_2\text{-SiO}_2$ multilayers with visible and near-infrared light. The optical resonant transmission of T-M dielectric multilayers is also reported by Qiu *et al* [20]. Dielectric aperiodic Thue-Morse structures up to 128 layers are fabricated by using porous silicon technology and photonic band gap properties are investigated by L. Moretti *et al* [21]. The physical mechanism and properties of the two classes of photonic gaps in the aperiodic Thue-Morse (T-M) lattice: the traditional gaps and the fractal gaps are investigated by H. Lei *et al* [22].

H. A. Banaei *et al.* [23] theoretically examined the design and characterization of optical filters based on photonic crystal Thue-Morse structures. It is shown that by introducing defect layer in the original structure of the proposed filter, main characteristics of it are changed. The main advantage of this defect in Thue-Morse structure is its capability for DWDM communication applications. Jihene Zaghoudi *et al.* have attempted to determine the optical properties of quasi-periodic symmetric one-dimensional photonic systems. In addition, it studies hybrid hetero-structure systems constructed by using periodic and quasi-periodic multilayer systems. The results show also that the width of the PBG depends on the parameters and nature of the built system [24].

In this paper, the omnidirectional reflection properties of Thue-Morse Si/MgF₂ multilayers are theoretically investigated.

Theoretical Formulation

Thue-Morse (TM) sequence is one of the well known examples in one-dimensional aperiodic structure. The T-M 1-D structure is constituted by the sequence of two layers A and B with refractive indices n_A and n_B , and thicknesses d_A and d_B respectively. These sequences are produced by repeating application of the substitution rules $A \rightarrow AB$ and $B \rightarrow BA$. For example, the first few generations S_n of Thue-Morse sequence are as follows,

$$\begin{aligned}
 S_0 &= [A] \\
 S_1 &= [AB] \\
 S_2 &= [ABBA] \\
 S_3 &= [ABBABAAB] \\
 S_4 &= [ABBABAABBAABABBA] \\
 &\dots\dots\dots
 \end{aligned}$$

The transfer matrices for the single layer A and B are given by

$$M_A = \begin{bmatrix} \cos \beta_A & -\frac{i}{q_A} \sin \beta_A \\ -iq_A \sin \beta_A & \cos \beta_A \end{bmatrix} \quad \text{and,} \quad M_B = \begin{bmatrix} \cos \beta_B & -\frac{i}{q_B} \sin \beta_B \\ -iq_B \sin \beta_B & \cos \beta_B \end{bmatrix},$$

where $\beta_A = \frac{2\pi}{\lambda} n_A d_A \cos \theta_A$ and $\beta_B = \frac{2\pi}{\lambda} n_B d_B \cos \theta_B$ are the layer phase thicknesses. θ_A and θ_B are the angle of refractions in layers A and B respectively which are determined by the Snell's law and λ is the wavelength of incident wave. Parameters q_A and q_B are given by, $q_A = n_A \cos \theta_A$ and $q_B = n_B \cos \theta_B$ for TE polarization and $q_A = \frac{\cos \theta_A}{n_A}$ and $q_B = \frac{\cos \theta_B}{n_B}$ for TM polarization. Thus, the transfer matrices M_j of the Thue-Morse sequences are $M_2 = M_A M_B M_B M_A$,

$$M_3 = M_A M_B M_B M_A M_B M_A M_A M_B$$

and

$$M_4 = M_A M_B M_B M_A M_B M_A M_A M_B M_B M_A M_A M_B M_A M_B M_B M_A$$

for S_2 , S_3 and S_4 respectively.

Here, an N -period finite structure whose basic cell is the Thue-Morse structure S_j is considered. The overall transfer matrix M of the system is obtained to be [25-26]

$$M = (M_j)^N = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

The reflection coefficient is given by

$$r = \frac{(M_{11} + q_t M_{12})q_i - (M_{21} + q_t M_{22})}{(M_{11} + q_t M_{12})q_i + (M_{21} + q_t M_{22})},$$

where $q_{i,t} = n_{i,t} \cos \theta_{i,t}$ for TE wave and $q_{i,t} = \frac{\cos \theta_{i,t}}{n_{i,t}}$ for TM wave, where i and t

represent incident medium and substrate respectively. The reflectivity is given as

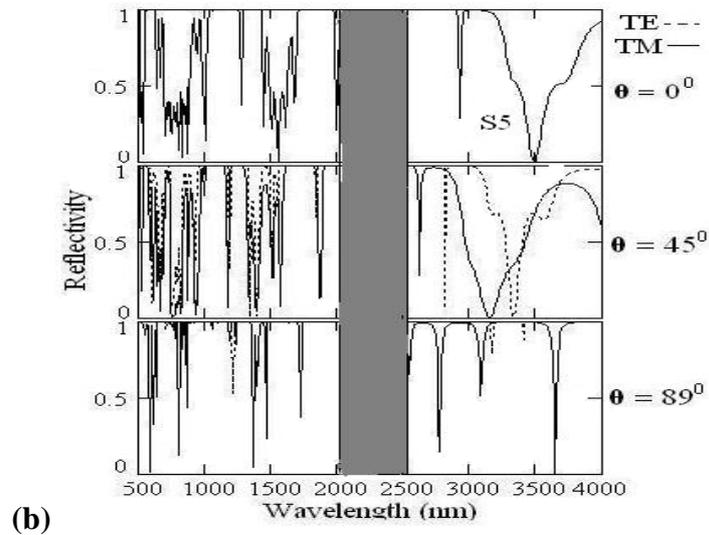
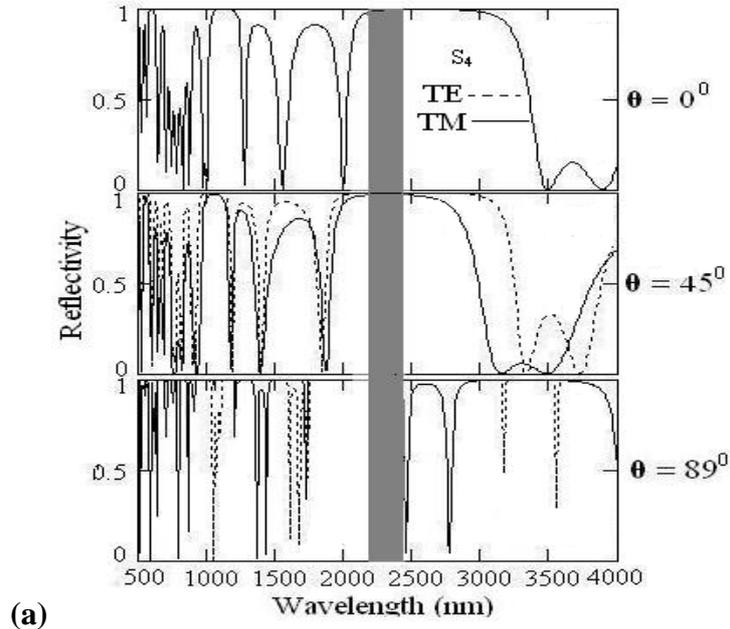
$$R = |r|^2.$$

RESULTS AND DISCUSSION

The Thue-Morse dielectric multilayer consists of two building blocks A and B as Silicon (Si) and magnesium fluoride (MgF_2) respectively. Around the optical telecommunication wavelength of 1550 nm, their respective refractive indices are $n_A = 3.53$ and $n_B = 1.38$. In a multilayer, A and B are arranged according to the Thue-Morse sequence. The electromagnetic wave is incident from air to the top surface of the multilayer. The thickness of two dielectric materials were chosen to satisfy the same phase shift condition $n_A d_A = n_B d_B$. The central wavelength was set to 1550 nm which gives the thickness of the two materials as $d_A = (1550 \text{ nm})/4.n_A = 109.77 \text{ nm}$ and $d_B = (1550 \text{ nm})/4.n_B = 280.80 \text{ nm}$.

The reflectance spectra of four T-M samples S_4 , S_5 , S_6 and S_7 having different number of layers are calculated using transfer matrix method in figures 1(a)-(d). The

reflectivity of different generations are plotted at three different incident angles 0° , 45° and 89° as shown in the figures in the wavelength region 500 nm to 4000 nm. The reflectivity values are calculated for both TE and TM modes at the incident angles 0° and 89° in order to show the omnidirectional reflection region. The grey area region having the 100% reflection between angles 0° and 89° of the TM mode gives omnidirectional reflection region.



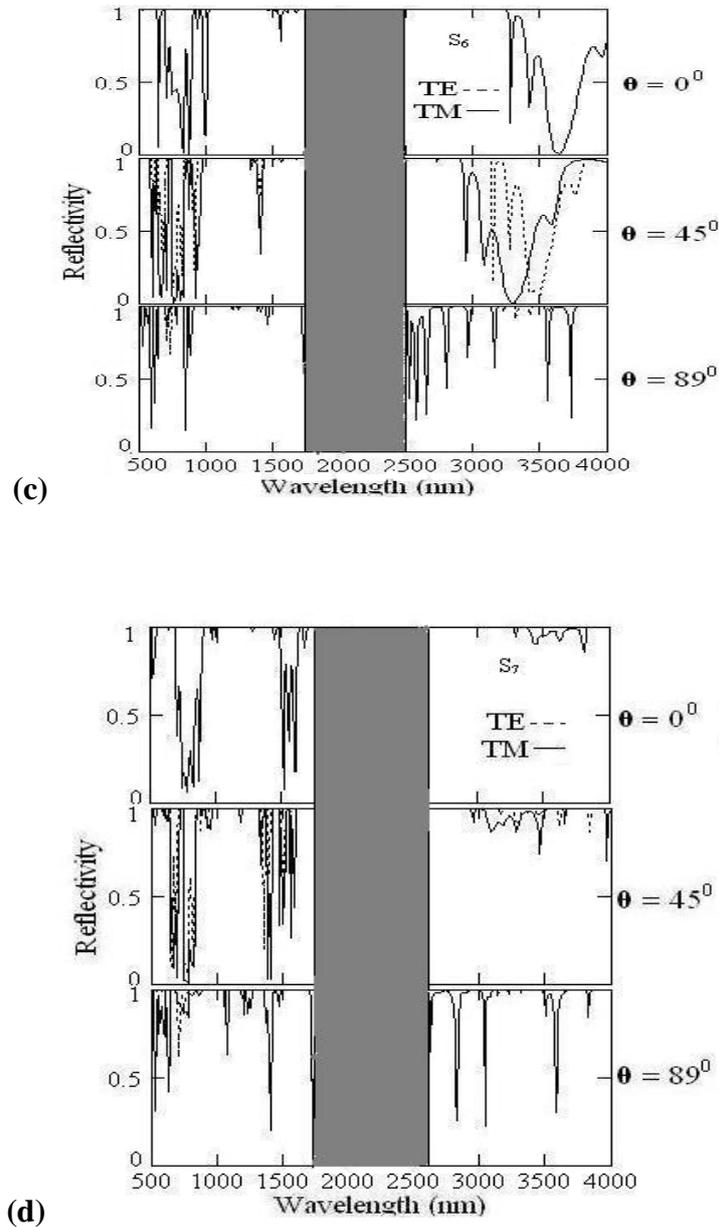


Figure 1: Calculated reflectivity spectra of the Thue-Morse structure with different generations (a) S_4 T-M structure (b) S_5 T-M structure (c) S_6 T-M structure and (d) S_7 T-M structure.

Various resonant transmission peaks are obtained at different wavelengths depending on the modulation of optical thickness in the case of S_4 T-M dielectric multilayer. It can be found that even-order Thue-Morse multilayer has the characteristics of mirror symmetry. Omnidirectional reflection region is found around the wavelength 2345 nm which is represented by the grey area existing between the angles 0° and 89° having reflectivity values around one for TM wave.

The structures S_5 , S_6 and S_7 are also theoretically investigated for omnidirectional reflectivity. For S_5 structure, omnidirectional reflection region of width 498 nm (approx.) is obtained. Omnidirectional reflection region of width around 595 nm is obtained for the S_6 structure. For the S_7 structure, omnidirectional reflection region having width 745 nm (approx.) is obtained. Various transmission peaks are also obtained for these multilayers. From figs. 1(a)-(d), it is seen that the width of omnidirectional reflection region increases with order of T-M multilayer i.e. from S_4 to S_7 generation. It can be found that multiple omnidirectional photonic band gaps (PBGs) exist in the higher generation non-periodic T-M dielectric structures as can be seen from figures of S_6 and S_7 generation due to the self-similarity in the internal structure.

CONCLUSION

Four S_4 - S_7 T-M dielectric multilayers are theoretically investigated for the omnidirectional PBGs. Multiple photonic band gaps exist for both TE and TM modes due to self-similarity of T-M multilayers. It is seen that the number and width of omnidirectional PBGs depend on the order of generation of T-M structure and increases with the order. It also depends on the refractive index contrast and thickness of the dielectric materials. The presence of sharp band-edge states can provide an alternate route to photonic crystals for the fabrication of dense optical filters, sensors and multifrequency laser cavities. as multichannel optical devices and all-dielectric waveguides.

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