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Mt. Everest vs. Mt. Chimborazo: Which is Higher?

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Abstract

There is an interesting question regarding the highest point on the Earth. It is now a consensus that Mt. Everest is the highest point from the sea level, but Mt. Chimborazo is the farthest point from the center of the Earth. In the former scheme, Mt. Everest is 2.58 km higher than Mt. Chimborazo, whereas in the latter scheme, Mt. Chimborazo is 2.10 km farther away from the center of the Earth than Mt. Everest is. In this paper, we formulate a third scheme to break the tie. It is shown that while a single attracting center of the Earth cannot be found, an attracting disk around the center of the Earth and lying on the equatorial plane can be defined such that the plumb lines at every point on the Earth's surface will pass through that disk. The radius of this disk is determined to be 42.70 km. To find the attracting center between any two locations on the Earth's surface, they are placed on the same meridional plane, whence the intersection between the plumb lines at the two locations is deemed to be attracting center associated with the two locations. The attracting center associated with Mts. Everest and Chimborazo is determined in this way, and the distances of their summits from this center are calculated. Mt. Everest is found to be 2.29 km farther away from this attracting center than Mt. Chimborazo in this scheme, thus making it the farthest point on Earth from the 'attracting center'.

1. INTRODUCTION

Mount Everest was first recognized as the World's highest peak in 1851 [1]. After enjoying this lofty status for over a century, it is now under assault from other contenders [2, 3]. First, ***Mauna Kea***, a volcanic mountain in Hawaii, as measured from its base in the Pacific Ocean, is 10.210 km high, as compared with Mt. Everest's height of 8.848 km from the sea level, which makes the former at least 1.36 km higher than the latter when reckoned from the base level [3]. Second, due to the oblateness of

the Earth, ***Mount Chimborazo***, as well as a host of other peaks on the Andes mountains, are farther away from the center of the Earth than Mt. Everest is [2, 3]. This is widely discussed and debated in the World Wide Web [1 – 3]. It is now generally agreed that: (1) Mt. Everest's summit (at 8.848 km) is higher than that of Chimborazo (6,267 km) from the sea level by 2.58 km; but (2) Mt. Chimborazo (at 6,384.4 km) is farther away from the center of the Earth than Mt Everest (8,382.3 km) is, by 2.1 km. The consensus appears to be a draw of sorts between Everest and Chimborazo as claimant to the title of the highest point on the Earth. In this paper, we analyze the two schemes of measurements (from sea level and the center of the Earth) and formulate a third to break the tie between Everest and Chimborazo.

2. PHYSICAL AND MATHEMATICAL CONSIDERATIONS

The shape of the rotating Earth is determined by (1) ***gravitational force***; and (2) ***centrifugal force of rotation***. Due to the dominant gravitational force alone, the Earth would be ***spherical***. The addition of the centrifugal force modifies this shape into an ***oblate spheroid*** or an ***ellipsoid of revolution about the minor axis*** which is the ***axis of rotation***. The ***sea level*** at any location on this surface defines the ***local horizontal plane***. Perpendicular to this plane is the ***vertical direction*** along which the resultant of the gravitational and centrifugal forces act, which also is, the ***local plumb line direction***. With the exception of the equatorial and polar locations, this direction does not pass through the center of the Earth. Thus there is no unique point which defines the ***center of attraction of the Earth***. However, we shall show that an ***attracting disk around the center of the Earth and lying on the equatorial plane*** can be defined such that the plumb lines at every point of the oblate spheroid will pass through that disk.

In the ***rectangular coordinate system*** (x, y, z), the equation of the oblate spherical Earth with its axis in the y-direction and equator on the z-x plane is (Fig. 1):

$$\frac{z^2 + x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

where a is the ***equatorial radius*** of the Earth (6,378.137 km) and b is the ***polar radius*** (6,356.752 km). Setting $z = 0$, the equation of meridional section is (Fig. 1):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

where a is the ***semi-major axis*** and b is the ***semi-minor axis*** of the elliptical section. The ***flattening of the Earth*** f is defined by [4]:

$$f = \frac{a-b}{a} \tag{3}$$

The ***eccentricity*** of the ellipse e is related to the flattening by the relation [4]:

$$e^2 = 1 - \frac{b^2}{a^2} = f(2-f) \tag{4}$$

According to the ***World Geodetic System WGS84*** [5], we have a = 6,378.137 km; b = 6,356.752 km; and $1/f$ = 298.257; whence e = .081819.

The ***geographic*** or ***geodetic latitude*** φ of a place on the Earth's surface is the angle between the vertical line perpendicular to the surface and the equatorial plane (Fig. 1) [6, 7]. This is the angle used on maps. The ***geocentric latitude*** φ_0, on the other hand, is the angle between the radius vector of that place from the center of the Earth and the equatorial plane (Fig. 1) [4]. By simply ***latitude*** is generally meant the geographic latitude. The latitude is greater than the geocentric latitude everywhere except at the poles. The difference, called the ***reduction for latitude*** $d\varphi = \varphi - \varphi_0$ is calculated in a series expansion to be [4]:

$$d\varphi = \varphi - \varphi_0 = \ell \sin 2\varphi - \frac{\ell^2}{2}\sin 4\varphi + \frac{\ell^3}{3}\sin 6\varphi - ... \tag{5}$$

where

$$\ell = \frac{e^2}{2-e^2} \tag{6}$$

Equations (3) and (4) yield for the Earth: e = .081819191; whence $\boldsymbol{\ell}$ = .003358425. Since $\boldsymbol{\ell}$ is small, its higher powers can be neglected to give [4]:

$$d\varphi = \varphi - \varphi_0 = \ell \sin 2\varphi \tag{7}$$

The reduction in latitude $d\varphi$ is maximum when φ is ± 45°.

We can calculate the distance from the Earth's center of the intersection point of the plumb line with the equatorial plane dx. We have (vide Fig. 2):

$$dx = \frac{r\,d\varphi}{\sin\varphi} = 2r\ell\cos\varphi \tag{8}$$

where r is the radius of the Earth at the latitude φ. The value for dx is maximum for equatorial locations (φ = 0°), which is equal to $2r\boldsymbol{\ell}$ = 42.6979 km. This means that the plumb lines at all locations on the Earth's surface will pass through a circular disk of radius of radius 42.6979 km in the equatorial place with its center coinciding with that

of the Earth (Fig. 3). In lieu of a singular point, this disk can be viewed as a ***circle of attraction of the Earth***. All plumb lines on the Earth's surface must pass through this circle, including those on the equatorial locations. The ***center of attraction*** between two locations on the Earth's surface will be at a point on or very close to this disk where the plumb lines from the two locations will intersect.

3. FARTHEST POINT FROM THE CENTER OF ATTRACTION

We can now go down to the business of determining the distances of Mts. Everest (latitude φ = 27º58'17") and Chimborazo (φ = – 1º28'9") from their mutual center of attraction. As stated earlier, we first place them in the same meridional plane (Fig. 4). Since Chimorazo is very close to the equator, its plumb line lies practically in the equatorial plane. In Fig. 4, O is the center of the Earth; PC is the plumb line from Mt. Everest; and PD is the perpendicular from Mt. Everest to the equatorial plane. Thus C is the attracting center of the Earth related to Mts. Everest and Chimborazo. We can now calculate the distances of Mts. Everest and Chimborazo and from this attracting center C.

First, the geographic latitude of Mt. Everest φ = 27.988º. The geocentric latitude is calculated from Eq. (7) to be φ_0 = 27.829º. Next, the relevant linear distances can be evaluated with reference to Fig. 4: ***geocentric abscissa*** x = OD; ***geocentric ordinate*** y = PD; ***geodetic abscissa*** x' = CD; and ***abscissa difference*** $dx = x - x'$ = OC. From Eq (2), we get upon simplification:

$$x = \frac{ab}{\sqrt{b^2 + a^2 \tan^2 \varphi_0}} \tag{9}$$

Substituting for the values of a, b and φ_0: x = 5,636 km; whence $y = x\tan \varphi_0$ = 2,975.333 km.

If h_C is the altitude of the Chimborazo from the sea level (6.267 km), then its distance from the attracting center d_C is (vide Fig. 4):

$$d_C = a - dx + h_C \tag{10}$$

Likewise, if h_E is the altitude of the Everest from the sea level (8.848 km), then its distance from the attracting center d_E is (vide Fig. 4):

$$d_E = \sqrt{x'^2 + y^2} + h_E \tag{11}$$

Upon executing the values of the quantities in Eqs. (10) and (11), we get: d_C = 6,346.664 km; d_E = 6,348.958 km; and $d_E - d_C$ = 2.291 km. Hence ***Mt. Everest is 2.29***

km farther away from the attracting center than Mt. Chimborazo is according to this scheme.

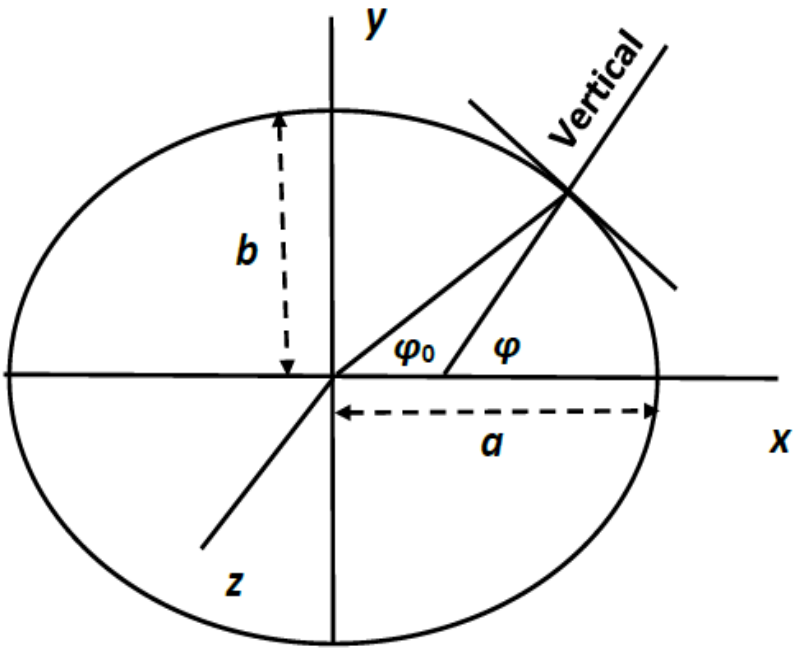

Fig. 1

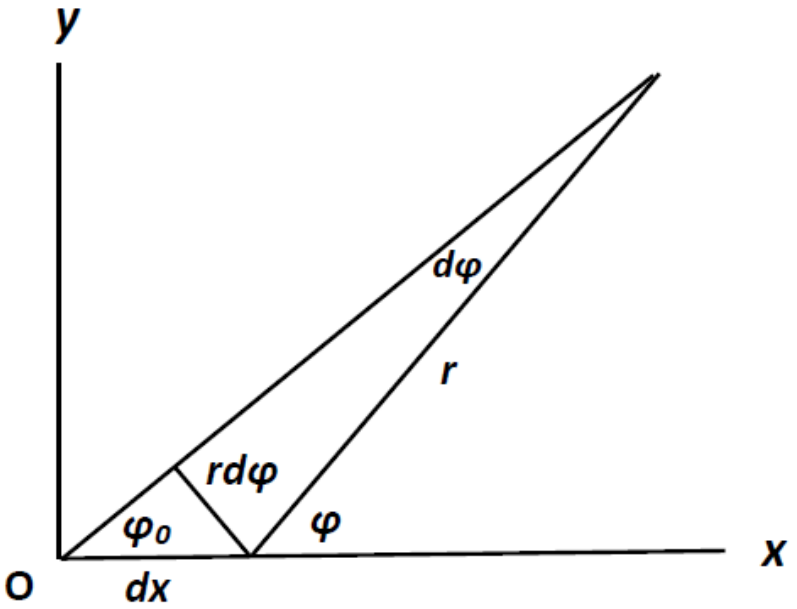

Fig. 2

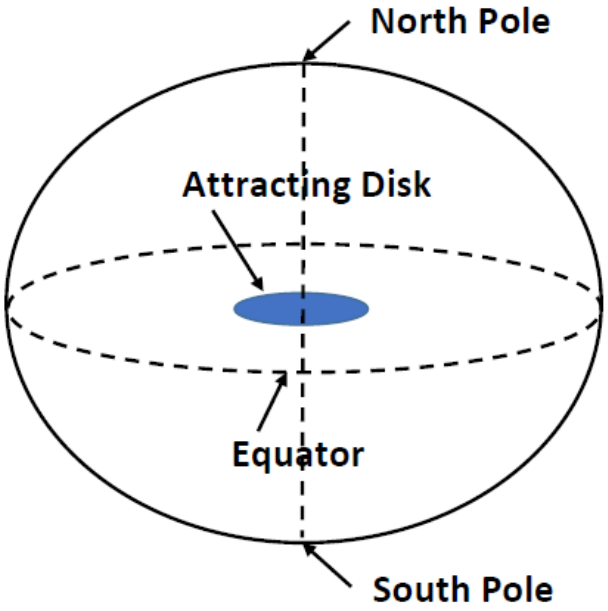

Fig. 3

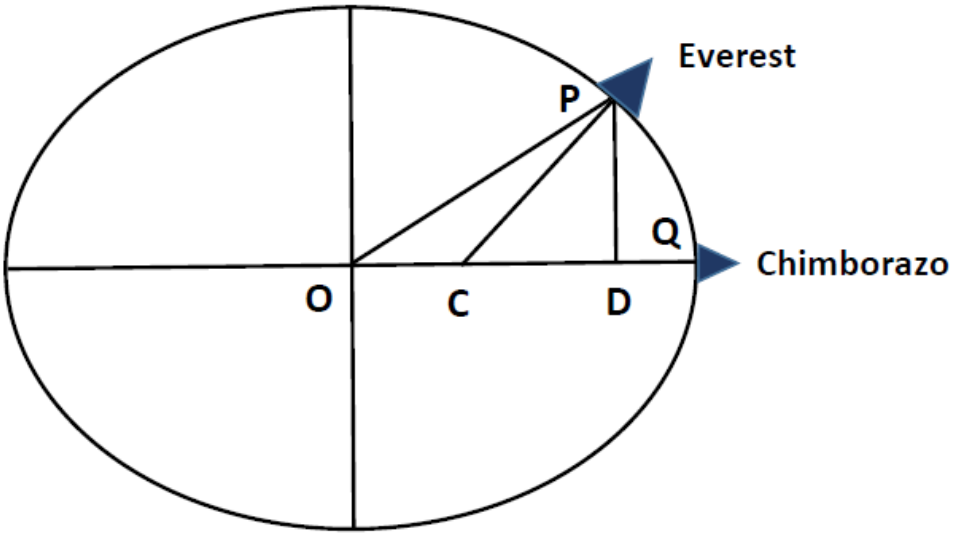

Fig. 4

4. DISCUSSION

Discussion about the highest point on the Earth must include a reference level or reference point from which the height or the distance is measured. The most natural reference level is, of course the sea level, from which the summit of Mt. Everest is the highest point. It has been lately advanced that Mt. Chimborazo is the highest point

when measured from the center of the Earth. In this study it has been argued that since the resultant attractive force at Mt. Everest is not directed at this center, perhaps an alternative center could be found at which the attractive forces at both Everest and Chimborazo are directed. In this last scheme, Mt. Everest edges out Mt. Chimborazo to reclaim the title of the highest point on the Earth.

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