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Common Fixed Point Theorems for Non-continuous, Compatible of Type (E) Self-mappings in Fuzzy Metric Space

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Abstract

The significante motive of this paper is to introduce the new concept of compatible which is a generalized form of compatible of type (E) and weaker then owc or semi compatible, with the help of this new concept we establish some common fixed point theorems in FMS,which is improvement and generalization of known recent results of N. Hoda et.als and many other existing in the literature.
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1. INTRODUCTION AND PRELIMINARIES

Fixed point theory on various fuzzy metric space emerged as a preeminant famous area of research to many mathematician, because of its wide applicability in various field of pure and applied mathematics.In (1965) I.A.Zadeh[9] gave the theory of fuzzy sets,which give the permission for sequentially appraisal of membership in a set.In(1994) George and Veeramani[1] established a revised definition of fuzzy metric space with t-norms, which is modified definition of K.M.[10] fuzzy metric space launched in (1975).Jungck and Rhoades [6,7] presented the concept of compatible and weakly compatible.On the same way M.R.Singh and et-al [18,19] introduced the notion of compatible of type (E) and compaired with compatible maps of various types (A, B, C,& P).In this sequence, many results of fixed point theorem using this concept in defferent metric spaces can be seen in [4, 11, 12, 18, 19, 22, 25].In contemporary

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decades, various authors have introduced numerous concepts related to compatibility and continuity and using these concepts they have proven many significant outcomes of fixed point in different metric space.
Before we move forward, let‘s familiarize ourselves with these concepts.

Definition 1.1: Let (X, d) be a metric space,and A,S:X→X are two self-mappings.Then set of coincidence point is $C(A, S) = \{x \in X : Ax = Sx\}$**,the set of point of coincidence is** $PC(A, S) = \{y \in X : y = Ax = Sx$ **for** $x \in X\}$ **and the set of sequence** $\{x_n\}$ **in X with condition** $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ **for some** $t \in X$ **is**

$$\mathcal{S}(A, S) = \big\{\{x_n\} : \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n\big\}$$

Definition 1.2[15]: E.A.Property: Let (X, d) be a metric space.Then self-maps A, S satisfy E.A. property, if $\mathcal{S}(A, S) \neq \phi$**.**
Note: Class of E.A. property contains class of compatible and noncompatible mappings.
Definition 1.3[6]: Compatible: Let (X, d) be a metric space.Then self-maps A, S are said to be compatible, iff $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0,$ **whenever** $\{x_n\} \in \mathcal{S}(A, S).$
Definition 1.4[21]: Conditionally compatible:Let (X, d) be a metric space.Then self-maps A, S are said to be conditionally compatible, iff whenever $\mathcal{S}(A, S) \neq \phi,$ **then there exist a seq**^{n} $\{x_n\} \in \mathcal{S}(A, S)$ **such that** $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0.$
Definition 1.5[7]: Weakly compatible:Let (X, d) be a metric space.Then self-maps A,S are said to be weakly compatible,if for all $x \in C(A, S) \Rightarrow Ax, Sx \in C(A, S).$
Definition 1.6[8]: Subsequentially continuous: Two self-maps A and S of metric space (X, d) are said to be subsequentially continuous, iff there exist a sequence$\{x_n\} \in X$ **such that** $\{x_n\} \in \mathcal{S}(A, S),$ **then**

$$\lim_{n\to\infty} ASx_n = At \text{ and } \lim_{n\to\infty} SAx_n = St.$$

Notice that If A and S are continous, then they are subsequentially continuous, but the converse is not true.
Based on definition(1.6), S.Beloul released the following new definition in 2015.

Definition 1.7[22]: Weakly subsequentially continuous: Two self-maps A and S of metric space (X, d) are said to be wsc, if there exist a sequence$\{x_n\} \in X$ **such that** $\{x_n\} \in \mathcal{S}(A, S)$, **and**

$$\lim_{n\to\infty} ASx_n = At \text{ or } \lim_{n\to\infty} SAx_n = St.$$

Definition 1.7.1[22]: A- subsequentially continuous: The pair of self-maps $\{A, S\}$is said to be A- subsequentially continuous, if there exist a sequence$\{x_n\} \in X$ **such that** $\{x_n\} \in \mathcal{S}(A, S)$, **and** $\lim_{n\to\infty} ASx_n = At.$

Definition 1.7.2[22]: S- subsequentially continuous: The pair of self-maps $\{A, S\}$is said to be S- subsequentially continuous, if there exist a sequence$\{x_n\} \in X$ **such that** $\{x_n\} \in \mathcal{S}(A, S)$, **and** $\lim_{n\to\infty} SAx_n = St.$

Lemma 1.8 Two self-maps A and S are weakly-subsequentially if and ony if they are A - subsequentially continuous or S- subsequentially continuous.

Proof: It is immediatly proved from the above definition.and they never subsequentially continuos.

Definition 1.9[14]:A binary operation * : [0,1] $\times$ [0,1]$\rightarrow$[0,1] is continuous t-norm if for all p,q,r,s $\in$ [0,1] the following conditions are satisfied.

(1) : (p*q) = (q*p) commutative.

(2) : (p*q)*r = p*(q*r) associative.

(3) : p*1 = p identity element.

(4) : * is continuous.

(5) : p*q $\leq$ r*s whenever p $\leq$ r and q $\leq$ s for all p,q,r,s $\in$ [0,1].

Example 1.10[26]:There are some examples of t-norms.

1 (The product t-norm):A map $T_p : [0, 1]^2 \to [0, 1]$ is defined by $T_P(p, q) = pq$

2 (The mini t-norm):A map$T_m : [0, 1]^2 \to [0, 1]$is defined by $T_M(p, q) = min\{p, q\}$

Lemma 1.11[2] The only t-norm * satisfying $r * r \geq r$ **for all** $r \in [0, 1]$ **is the minimum t-norm.**

Definition 1.12[1]:A3-tuple(X, M, *)is called a GV fuzzy metric space,where X is nonempty set,*is continuous t-norm, and if M is a fuzzy set on X$\times$X$\times$(0, ∞) is satisfy the following conditions.

For all x, y, z $\in$X and s, t $>$ 0

FM-1 M(x, y, t)$>$ 0.

FM-2 M(x, y, t) = 1$\Leftrightarrow$ x = y.

FM-3 M(x, y, t) = M(y, x, t).

FM-4 M(x, y, t)*M(y, z, s)$\leq$M(x, z, t+s).

FM-5 M(x, y, .) :(0, ∞)$\rightarrow$[0,1] is continuous.

Definition 1.13[5]: Semi-compatible: A pair of self-maps {A, S} of fuzzy metric space (X, M,*) is said to be semi-compatible, if $\lim_{n\to\infty} ASx_n = St$, **whenever** $\{x_n\} \in \mathcal{S}(A, S)$,
Definition 1.14[16] see also[2,20,24]: Occasionally weakly compatible: Self-maps A and S of fuzzy metric space (X, M,*) are said to be occasionally weakly compatible, if and only if there exist a point in X with x $\in$ C(A,S),at which A and S commute.
Lemma 1.15[2] see in[17,23] Let(X, M, *)be a fuzzy metric space.If there exists k$\in$(0, 1) such that for all x, y $\in$ X, M(x, y, kt)$\geq$M(x, y, t), $\forall\, t > 0$,**then x = y.**
In(2014) K.B. Manandhar and et-al[11] introduced the notion "Compatible of type (E)" in the framework of fuzzy metric space as follows.

Definition 1.16[11]: Compatible of type (E) :The self-maps A and S of fuzzy metric space (X, M, *) are said to be compatible of type (E), if and only if $\lim_{n\to\infty} M(AAx_n, ASx_n, t) = 1$, $\lim_{n\to\infty} M(AAx_n, Sx, t) = 1$, $\lim_{n\to\infty} M(ASx_n, Sx, t) = 1$, **and** $\lim_{n\to\infty} M(SSx_n, SAx_n, t) = 1$, $\lim_{n\to\infty} M(SSx_n, Ax, t) = 1$, $\lim_{n\to\infty} M(SAx_n, Ax, t) = 1$, **whenever a sequence**$\{x_n\} \in X$ **such that** $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ **for some** $x \in X$ **and** $t > 0$**.**
Definition 1.17[13]:Let (X, M, *) be a fuzzy metric space

(1) Sequence $\{x_n\}$ in X is called convergent to x if $\lim_{x\to\infty} M(x_n, x, t) = 1$

(2) Sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{x\to\infty} M(x_n, x_{n+m}, t) = 1$ $\forall$ t $>$ 0 and n, m$\in$ N.

2. GENERALIZATION OF COMPATIBLE OF TYPE (E).

According to Al-thagafi[16] weakly compatible self-maps form a proper subclass of the owc self-maps.Since,owc mappings commute at some coincidence point, but these maps do-not guarantee the existence of coincidence point. Therefore, according to lemma[(2.1),24],the concept of owc,would not be a necessary condition for the existence of common fixed point,
To introduce new notion of compatibility, first of all see the following examples.

Example(2.1)[(5),4]:Let X = [0,10] be endowed with a symmetric d. Let S and T be define as follows.

$$Sx = \begin{cases} 3 - x & 0 \leq x \leq 2 \\ \frac{1}{2} & 2 < x \leq 10 \end{cases} \qquad Tx = \begin{cases} \frac{5-x}{2} & 0 \leq x < 2 \\ \frac{1}{2} & 2 \leq x \leq 10 \end{cases} \tag{1}$$

Here $C(S,T) = \{1\} \cup (2,10]$ and $\mathcal{S}(S,T) \neq \phi$.
Think about the constant sequence and other seq^n contained in $\mathcal{S}(S,T)$.We observed that pair (S,T) are niether S-compatible of type (E) nor T-compatible of type (E), and also not non-trivially owc.

Example(2.2):Let $X = [0,\infty)$ be a nonempty set with usual metric d,and A & S are self-mapping defined as follows.

$$Ax = \begin{cases} x & if\ 0 \leq x < 1 \\ \frac{2}{3} & if\ 1 \leq x < \infty \end{cases} \qquad Sx = \begin{cases} 2x & if\ 0 \leq x < 1 \\ \frac{x}{1+x} & if\ 1 \leq x < \infty \end{cases} \tag{2}$$

Here $C(A,S) = \{0,2\}$ and $\mathcal{S}(A,S) \neq \phi$.
Then clearly pair (A,S) is owc, but not compatible of type (E).
Motivated by definition [(1.15)11], and taking into account of lemma[(2.1),24] and the above examples, we introduce a new notion of conditionally compatible of type (E), which is a generalization of compatible of type (E), and also weaker form of owc.

Definition 2.3: Let (X, M, *) be fuzzy metric space.Two self-mapings A and S are called conditionally compatible of type (E),if and only if whenever $\mathcal{S}(A,S) \neq \phi$, then ther exist a sequence$\{x_n\} \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \ \ forsome\ \ x \in X\ \ and\ \ t > 0$$

with,

$$\left.\begin{aligned} \lim_{n\to\infty} M(AAx_n, ASx_n, t) = 1 \\ \lim_{n\to\infty} M(AAx_n, Sx, t) = 1 \\ \lim_{n\to\infty} M(ASx_n, Sx, t) = 1 \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{array}{r}\lim_{n\to\infty} M(SSx_n, SAx_n, t) = 1\\ \lim_{n\to\infty} M(SSx_n, Ax, t) = 1\\ \lim_{n\to\infty} M(SAx_n, Ax, t) = 1\end{array}\right\} \quad (4)$$

In general it can be seen that compatible of type (E) is conditionally compatible of type (E), but its converse is not true. It can be verified this fact with the help of examples (2.2) and [(1),16], now in the same account.

Lemma 2.4: Let (X, M,*) be fuzzy metric space. Then every pair of owc type mappings are conditionally compatible of type(E).

Proof: Let (X, M,*) be fuzzy metric space.Suppose that any pair (A,S) of self-mapping which is owc, then there exist a point $u \in X$ such that

A(u) = S(u) = w (say), with AS(u) = SA(u).

$\Rightarrow$

$$Aw = Sw$$

Now we take a constant sequence $\{x_n\} = \{u\}$ in X.Then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = w$$

Now

$$\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} ASx_n = Aw = Sw$$

$$\lim_{n\to\infty} SSx_n = \lim_{n\to\infty} SAx_n = Sw = Aw$$

Hence pair (A,S) is conditionally compatible of type (E).

Note:-(1)If Aw=Sw,then(A, S) is conditionally compatible of type (E)$\Leftrightarrow$(A, S) is conditionally compatible.

(2) If Aw$\neq$Sw,then (A, S) is conditionally compatible of type(E) $\not\Leftrightarrow$ (A, S) is conditionally compatible.

Thus above result can be seen in examples (2.2),[(1),16] and [(3.1),5], while the result is generally not true in the opposite direction.See following example.

Example(2.5):Let (X, M,*) be a fuzzy metric space.Where X = [0, 1], and two self-mapping A & S are defined as follows.

$$Ax = \begin{cases} \frac{2}{3} & if\ x \in [0, \frac{1}{2}) - \{\frac{1}{4}\} \\ \frac{1}{3} & if\ x = \frac{1}{4} \\ \frac{1-x}{2} & if\ x \in [\frac{1}{2}, 1] \end{cases} \qquad Sx = \begin{cases} \frac{1}{3} & if\ x \in [0, \frac{1}{2}) - \{\frac{1}{4}\} \\ \frac{2}{3} & if\ x = \frac{1}{4} \\ \frac{x}{2} & if\ x \in [\frac{1}{2}, 1] \end{cases} \tag{5}$$

Here $C(A,S) = \{\frac{1}{2}\},\ \ and\ \ \mathcal{S}(A,S) \neq \phi$, then there exist a sequence $\{x_n\} = \{\frac{1}{2} + \frac{1}{n}\}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \frac{1}{4} = x\ \ (say)$$

Now for $t > 0$

$$\lim_{n\to\infty} M(AAx_n,\ ASx_n, t) = M(\frac{2}{3},\ \frac{2}{3}, t) = 1$$

$$\lim_{n\to\infty} M(AAx_n,\ Sx, t) = M(\frac{2}{3},\ S(\frac{1}{4}), t) = 1$$

$$\lim_{n\to\infty} M(ASx_n,\ Sx, t) = M(\frac{2}{3},\ S(\frac{1}{4}), t) = 1$$

and

$$\lim_{n\to\infty} M(SSx_n,\ SAx_n, t) = M(\frac{1}{3},\ \frac{1}{3}, t) = 1$$

$$\lim_{n\to\infty} M(SSx_n,\ Ax, t) = M(\frac{1}{3},\ A(\frac{1}{4}), t) = 1$$

$$\lim_{n\to\infty} M(SAx_n,\ Ax, t) = M(\frac{1}{3},\ A(\frac{1}{4}), t) = 1$$

So pair (A,S) is conditionally compatible of type(E).In the second part at $x = \frac{1}{2}$

$$A(\frac{1}{2}) = S(\frac{1}{2}) = \frac{1}{4}$$

But

$$AS(\frac{1}{2}) = A[S(\frac{1}{2})] = A[\frac{1}{4}] = \frac{1}{3}$$

$$SA(\frac{1}{2}) = S[A(\frac{1}{2})] = S[\frac{1}{4}] = \frac{2}{3}$$

So

$$AS(\frac{1}{2}) \neq SA(\frac{1}{2})$$

Hence maps A and S are not owc, here A, S are not continuous at $x = \frac{1}{2}, \frac{1}{4}$, and if we take constant seq^n $x_n = \{\frac{1}{2}\}$ then pair (A,S) and (S,A) are not semi-compatible.
So conditionally compatible of type(E) is weaker form of owc or semi-compatible.
We define following definition by partitioning the notion of conditionally compatible of type (E),which will be useful for new results

Definition 2.6: Let (X, M, *) be fuzzy metric space.Two self-mapings A and S are called conditionally A- compatible of type (E),if and only if whenever $\mathcal{S}(A, S) \neq \phi$, then ther exist a sequence$\{x_n\} \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \text{ for some } x \in X \text{ and } t > 0$$

with condition (3).

Definition 2.7: Let (X, M, *) be fuzzy metric space.Two self-mapings A and S are called conditionally S- compatible of type (E),if and only if whenever $\mathcal{S}(A, S) \neq \phi$, then ther exist a sequence$\{x_n\} \in X$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \text{ for some } x \in X \text{ and } t > 0$$

with condition (4),
We observe that if self-maps A and S are conditionally compatible of type(E), then they are conditionally A-compatible and S-compatible of type (E).but its converse is not true.

3. MAIN-BODY

Definition 3.1[13,14]: Let φ be the class of all mappings $\phi : [0, 1] \to [0, 1]$ satisfying the condition

(1) : ϕ is continuous and non-decreasing in [0,1].

(2) : $\phi(x) > x \ \forall x \in (0, 1)$.

(3) : $\phi(0) = 0 \ and \ \phi(1) = 1$.

Lemma 3.2:If $\phi \in \varphi$, then

(1) : $\lim_{n\to\infty} \phi^n(t) = 1$

(2) : If for some $t \in (0, 1]$, $t > \phi(t)$, then it is true only for t = 1.

Proof:(1)Let $\phi \in \varphi$, and suppose that there exist a $t_0 \in (0, 1)$ such that

$$\lim_{n\to\infty} \phi^n(t_0) = \nu < 1$$

Since $\phi$is continuous and monotonic, then

$$\nu = \lim_{n\to\infty} \phi^{n+1}(t_0)$$

$$= \lim_{n\to\infty} \phi[\phi^n(t_0)]$$

$$= \phi[\lim_{n\to\infty} \phi^n(t_0)]$$

$$= \phi(\nu) > \nu$$

A controdiction.Hence $\lim_{n\to\infty} \phi^n(t) = 1.$
(2) Suppose for some $t \in (0,1], \quad t > \phi(t).$
But

$$\phi(t) > t \quad \forall\ t \in [0,1]$$

Thus

$$\phi(t) = t \text{ by definition of } \phi$$

$$t = 1 \text{ or } t = 0$$

Hence this true only for t = 1.
In (2017) Pardeep kumar and et-al[20] generalized the result of A.jain, et-al[2]for four self-mapping using semi-compatible and owc as follows.

Theorem 3.3:Let A, B, S, and T be self-mappings of complete fuzzy metric space (X, M, *) satisfying following conditions.

(3.3.1) : $A(X) \subset T(X), \ B(X) \subset S(X).$

(3.3.2) : The pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

(3.3.3) : For all $x, y \in X$and $t > 0$

$$M(Ax, By, t) \geq r\Big\{min \begin{Bmatrix} M(By,Ty,t), M(Sx,Ty,t), M(Ax,Sx,t) \\ M(By,Sx,t), M(Ax,Ty,t) \end{Bmatrix}\Big\}$$

Where $r : [0,1] \to [0,1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1$
Then A, B, S, and T have a unique common fixed point in X.

Remark-1(a):In (2014) A.Jain and et-al[2] have generalized B.singh's result in the sense that the requirement of continuity is totally removed by using semicompatible and owc.

Remark-1(b):In this sequence in (2017) Pardeep kumar and et-al[20] generalized the result of A.jain, et-al[2] by using continuous functions, while continuity is not included in the statement of theorem

In this paper we improve and generalize above result and all relative results which are existing in the liturature, with the help of new concept of generalization of compatible of type(E), as follow.

Theorem 3.4:LetA,B,S,and T be self-mappings of fuzzy metric space(X, M, *) satisfying following conditions.

(3.4.1) : The pair (A, S) is A-subsequentially continuous, and conditionally A-compatible of type(E).

(3.4.2) : The pair (B, T) is B-subsequentially continuous, and conditionally B-compatible of type(E).

(3.4.3) : For all $x, y \in X$and $t > 0$

$$M(Ax, By, t) \geq \phi\Big\{min \left\{ \begin{array}{c} M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t) \\ M(By, Sx, t), M(Ax, Ty, t) \\ \dfrac{M(Sx, Ty, t) * M(Ax, By, t)}{M(Sx, By, t)} \end{array} \right\}\Big\}$$

Then A, B, S, and T have a unique common fixed point in X.

Proof:Let (X, M, *) be a fuzzy metric space, Suppose that pairs (A, S) and (B, T) are A-subsequentially and B-subsequentially continuous respectively.Then there exist sequences $\{x_n\}$ **and** $\{y_n\}$ **in X, such that** $\{x_n\} \in \mathcal{S}(A, S) \cap \mathcal{S}(B, T)$.

Then

$$\lim_{n\to\infty} ASx_n = Az \text{ and } \lim_{n\to\infty} BTy_n = Bz_1 \text{ for some } z, z_1 \in X$$

Since, (A,S) and (B,T) are conditionally A-compatible and B-compatible of type (E) respectively, and$\mathcal{S}(A, S) \neq \phi$ & $\mathcal{S}(B, T) \neq \phi$, then there exist sequences $\{x_n\} \in \mathcal{S}(A, S)$ & $\{y_n\} \in \mathcal{S}(B, T)$, with for all $t > 0$

$$\lim_{n\to\infty} M(AAx_n,\ ASx_n, t) = 1$$

$$\lim_{n\to\infty} M(AAx_n,\ Sz, t) = 1$$

$$\lim_{n\to\infty} M(ASx_n,\ Sz, t) = 1$$

and

$$\lim_{n\to\infty} M(BBy_n,\ BTy_n, t) = 1$$
$$\lim_{n\to\infty} M(BBy_n,\ Tz_1, t) = 1$$
$$\lim_{n\to\infty} M(BTy_n,\ Tz_1, t) = 1$$

for some $z, z_1 \in X$.
Thus $Az = Sz$ and $Bz_1 = Tz_1$
Case-I: If $\{x_n\} \neq \{y_n\}$, then we can show $z = z_1$ by contraction condition.
Case-II: If $\{x_n\} = \{y_n\}$, then clearly $z = z_1$
So Az = Sz and Bz = Tz.
$\Rightarrow$

$$z \in C(A,S) \cap C(B,T).$$

Now put x = z and y = z in (3.4.3)

$$M(Az, Bz, t) \geq \phi\Big\{min \left\{ \begin{array}{c} M(Bz, Tz, t), M(Sz, Tz, t), M(Az, Sz, t) \\ M(Bz, Sz, t), M(Az, Tz, t) \\ \dfrac{M(Sz, Tz, t) * M(Az, Bz, t)}{M(Sz, Bz, t)} \end{array} \right\} \Big\}$$

$$\geq \phi\Big\{min \left\{ \begin{array}{c} 1, M(Az, Bz, t), 1 \\ M(Bz, Az, t), M(Az, Bz, t) \\ \dfrac{M(Az, Bz, t) * M(Az, Bz, t)}{M(Az, Bz, t)} \end{array} \right\} \Big\}$$

$$> \phi\Big\{min\Big\{1, M(Az, Bz, t), 1, 1\Big\}$$

$$> \phi\Big\{M(Az, Bz, t)\Big\}$$

$\Rightarrow$ M(Az, Bz, t) = 1 $\Rightarrow$ Az = Bz

Therefore Az = Sz = Bz = Tz.
Again put x = z & y = x_n in (3.4.3)

$$M(Az, Bx_n, t) \geq \phi\Big\{min \left\{ \begin{array}{c} M(Bx_n, Tx_n, t), M(Sz, Tx_n, t), M(Az, Sz, t) \\ M(Bx_n, Sz, t), M(Az, Tx_n, t) \\ \dfrac{M(Sz, Tx_n, t) * M(Az, Bx_n, t)}{M(Sz, Bx_n, t)} \end{array} \right\} \Big\}$$

On taking $\lim_{n\to\infty}$

$$M(Az,z,t) \geq \phi\Big\{min\left\{\begin{array}{c} M(z,z,t), M(Az,z,t), M(Az,Az,t) \\ M(z,Az,t), M(Az,z,t) \\ \dfrac{M(Az,z,t) * M(Az,z,t)}{M(Az,z,t)} \end{array}\right\}\Big\}$$

$$> \phi\Big\{min\Big\{1, M(Az,z,t), 1, 1\Big\}$$

$$> \phi\Big\{M(Az,z,t)\Big\}$$

$\Rightarrow$ M(Az, z, t) = 1 $\Rightarrow$ Az = z

$\Rightarrow$ Az = Sz = Bz = Tz = z.

Therefore z is common fixed point of A, S, B and T.Like this we can prove that common fixed point is unique.Hence z is common fixed point of A, S, B and T.

Remark-2:Our result(3.4) improves and generalizes the result(3.3) as follows.
(2.1) Containment of the range of self-mappings and completeness of fuzzy metric space have been totally removed.
(2.2) The result has been proved without using the continuity of mappings.
(2.3) Condition of owc and semi-compatible,are replaced with the conditionally compatible of type (E), and subsequentially contiinuous, which are weaker form of owc and reciprocal continuous respectively.
Theorem 3.5:Let A, B, S and T be self-mappings of fuzzy metric space(X, M,*) satisfying following conditions.

(3.5.1) : The pair (A, S) is S-subsequentially continuous, and conditionally S- compatible of type(E).

(3.5.2) : The pair (B, T) is T-subsequentially continuous, and conditionally T- compatible of type(E).

(3.5.3) : For all $x, y \in X$and $t > 0$

$$M(Ax,By,t) \geq \phi\Big\{min\left\{\begin{array}{c} M(By,Ty,t), M(Sx,Ty,t), M(Ax,Sx,t) \\ M(By,Sx,t), M(Ax,Ty,t) \\ \dfrac{M(Sx,Ty,t) * M(Ax,By,t)}{M(Sx,By,t)} \end{array}\right\}\Big\}$$

Then A, B, S, and T have a unique common fixed point in X.

Proof:Let (X, M, *) be a fuzzy metric space, Suppose that pairs (A, S) and (B, T) are S-subsequentially and T-subsequentially continuous respectively.Then there exist sequences a $\{x_n\}$ in X,such that $\{x_n\} \in \mathcal{S}(A, S) \cap \in \mathcal{S}(B, T)$.

Then

$$\lim_{n\to\infty} SAx_n = Sz \text{ and } \lim_{n\to\infty} TBx_n = Tz \text{ for some } z, \in X$$

Again, (A,S) and (B,T) are conditionally S-compatible and T-compatible of type (E) respectively, and$\mathcal{S}(A, S) \neq \phi$ & $\mathcal{S}(B, T) \neq \phi$, then there exist sequences $\{x_n\} \in \mathcal{S}(A, S) \cap \mathcal{S}(B, T)$, with for all $t > 0$.

$$\lim_{n\to\infty} M(SSx_n,\ SAx_n, t) = 1 \qquad \lim_{n\to\infty} M(TTx_n,\ TBx_n, t) = 1$$

$$\lim_{n\to\infty} M(SSx_n,\ Az, t) = 1 \quad \text{and} \quad \lim_{n\to\infty} M(TTx_n,\ Bz, t) = 1$$

$$\lim_{n\to\infty} M(SAx_n,\ Az, t) = 1 \qquad \lim_{n\to\infty} M(TBx_n,\ Bz, t) = 1$$

for some z $\in X$.

Thus $Az = Sz$ and $Bz = Tz$

After this the rest theorem can prove according to the main result.

Corollary 3.6:Let (X, M,*) be a fuzzy metric space, andA, B, S, and T are four self-mappings of X, If they are satisfying following conditions.

(3.6.1) : The pairs (A, S) and (B, T) are weakly-subsequentially continuous.

(3.6.2) : The pairs (A, S) and (B, T) are conditionally compatible of type(E).

(3.6.3) : For all $x, y \in X$and $t > 0$

$$M(Ax, By, t) \geq \phi\Big\{min \begin{Bmatrix} M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t) \\ M(By, Sx, t), M(Ax, Ty, t) \\ \dfrac{M(Sx, Ty, t) * M(Ax, By, t)}{M(Sx, By, t)} \end{Bmatrix}\Big\}$$

Then A, B, S, and T have a unique common fixed point in X.

Proof:Let (X, M, *) be a fuzzy metric space, Suppose that pairs (A, S) and (B, T) are wsc and conditionally compatible of type (E).

(1) Since pairs (A, S) and (B, T) are wsc, then by lemma (1.8), they are A-subsequentially continuous or S-subsequentially contnuous.

(2) Every pair of conditionally compatible of type (E) is conditionally A or S compatible of type (E).Thus further result follows from theorem (3.4) and (3.5).

Corollary 3.7:Let (X, M,*) be a fuzzy metric space, andA, B, S, and T are four self-mappings of X, If they are satisfying following conditions.

(3.7.1) : The pairs (A, S) and (B, T) are wsc and owc.

(3.7.2) : For all $x, y \in X$and $t > 0$

$$M(Ax, By, t) \geq \phi\Big\{min \begin{Bmatrix} M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t) \\ M(By, Sx, t), M(Ax, Ty, t) \\ \dfrac{M(Sx, Ty, t) * M(Ax, By, t)}{M(Sx, By, t)} \end{Bmatrix}\Big\}$$

Then A, B, S, and T have a unique common fixed point in X.

Proof:Let (X, M, *) be a fuzzy metric space, Suppose that pairs (A, S) and (B, T) are wsc and owc.Then by lemma (2.4) they are conditionally compatible of type (E).After this, the remaining part of corollary follows corollary (3.6).

Theorem 3.8:LetA,B,S,and T be self-mappings of fuzzy metric space(X, M, *) satisfying following conditions.

(3.8.1) : The pair (A, S) is A-subsequentially continuous, and conditionally A-compatible of type(E).

(3.8.2) : The pair (B, T) is B-subsequentially continuous, and conditionally B-compatible of type(E).

(3.8.3) : there exist $k \in (0, 1)$ such that for all $x, y \in X$and $t > 0$

$$M(Ax, By, kt) \geq \phi\Big\{min \begin{Bmatrix} M(By, Ty, t), M(Sx, Ty, t), M(Ax, Sx, t) \\ M(By, Sx, t), M(Ax, Ty, t) \\ \dfrac{M(Sx, Ty, t) * M(Ax, By, t)}{M(Sx, By, t)} \end{Bmatrix}\Big\}$$

Then A, B, S, and T have a unique common fixed point in X.

Proof:Let (X, M, *) be a fuzzy metric space, By using lemma (1.15)[2], we proceed further calculation of theorem as follows theorem (3.4).

Note-:We can prove all above results with contraction condition (3.8.3),by using lemma 1.15[2],in the same way.

Remark-3:According to remark(2),our all results are better than other related results present in the literature.

4. CONCLUSION

We introduced new concept of conditionally compatible of type (E), which is weaker form of owc.With the help of this concept we improved and generalized the result of [20] and many known results,existing in the liturature, and also proved some common fixed point theorem for non-continuous mapping.This concept is a necessary condition for common fixed point.

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