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Fuzzy Possibilistic Pessimistic Criterion

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Abstract

In most practical life situations only inadequate or partial information are available to decision maker about the states and consequences. Thus in this chapter we consider decision making under uncertainty when the probability of the states of nature are not known apriori and the outcomes of each alternative are characterized only approximately. There are many models designed for preference modeling in such cases, both in the crisp and fuzzy cases, but they have many drawbacks. The fuzzy approach is very useful to handle such situations. Here in this paper we construct a new fuzzy pessimistic criterion based on the pessimistic attitude of the decision maker, that he expects the worst to occur and the best not to occur. It has been proved that when ranking of alternatives is not possible using the existing criteria, it is possible to clearly rank the alternatives using this fuzzy criterion constructed. An example is given to illustrate the same.

AMS subject classification:
Keywords: Decision Maker (DM), 1-Fuzzy Pessimistic Criterion (1-FOC) Fuzzy Aspiration Degree (α_D), Fuzzy Reservation Degree (β_D), Fuzzy Aspiration Class w.r.t 'β_D' (FRC[β_D]), Fuzzy Non-Aspiration Class w. r. t 'β_D' (FNRC[β_D]), 3-Fuzzy Optimistic Criterion (3-FPC).

1. Introduction

For decision making under uncertainty in the crisp environment, the most commonly used criterion is the expected utility criterion axiomatized by Savage [10], despite early criticisms by Allais [1], Ellsberg [8] and later by Kahneman and Tversky [9]. Here the subjective values attached to each consequence as well as the degree of confidence of the possible outcomes commensurate and are specifically quantified. But in most problems which deal with practical life situations, this is not always possible. There are many models for preference modeling in the presence of poor information like the max–min rule by Wald [13] axiomatized by Arrow and Hurcwicz [2]. If possibility ordering on states are available, then the above criterion can be redefined [3, 4]. Another refinement of the Wald Criterion is the Possibilistic Criterion [5, 6, 7] based on a utility function 'u' on X, the set of consequences of the decision problem and a possibility distribution π on S (the states of nature) representing the relative plausibility of states both mapping on the same totally ordered scale and can be compared.

In this paper, we use the fuzzy approach, which is very useful in real life situation, when the probability of the states of nature is not known apriori and the outcomes of each alternative are characterized only approximately.

2. Important Results

Definition 2.1. [14] Let X be the universal set. A **Fuzzy set** A in X is defined as $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A : X \to [0, 1]$ is called the membership function of A in X and $[0, 1]$ is the membership set or valuation set.

Definition 2.2. [5, 6, 7] The **Possibilistic Pessimistic Criterion** (PPC) is given by

$$d_i \geq d_j \Leftrightarrow V^-(d_i) \geq V^-(d_j),$$
$$\text{where } V^-(d_i) = \bigwedge_{s_j \in S} [(1 - \pi(s_j)) \vee u(d_i(s_j))]$$

Here $u(d_i(s_j))$ is the utility of the consequence x_{ij}, where $d_i(s_j) = x_{ij} \in X$ and $\pi(s_j)$ is the plausibility of the state s_j.

Note. Here the alternatives are ranked on the basis of worst consequences restricted to most plausible states

Drawbacks of PPC: Through the Criterion PPC is the refinement of many criteria in the crisp case, it has many drawbacks, one major drawback being that it fails to satisfy the Principle of Strict Pareto Dominance.

That is, an alternative d_1 can be ranked equal to another alternative d_2 even if d_1 is at least 'as good as' d_2 in all states and better in some states (including most plausible ones), which is absurd.

Thus we see whether we can modify PPC such that this drawback is rectified.

3. Possibilistic Decision Making in a Fuzzy Environment

We now define the possibilistic decision problem in a fuzzy environment as follows

Definition 3.1. The possibilistic decision problem in a fuzzy environment is given by (S, D, I, μ_A), where

S-the non-empty set of states the nature
D-the non-empty set of feasible alternatives

I-information about the states represented by the possibility degree and $\mu_A : X \rightarrow [0, 1]$ is the membership function on the set of consequences X in the fuzzy set 'satisfaction' A.

Note. Here the membership value $\mu_A(x_{ij}) \in [0, 1]$ that is assigned to the consequence $x_{ij} \in X$ in the fuzzy set 'satisfaction', is the preference measure of the consequences that are ordinally ranked, based on the preference of the decision maker.

Definition 3.2. The **Principle of Strict Pareto Dominance** in the fuzzy environment states that, if $\mu_A(d_1(s)) \geq \mu_A(d_2(s))$, for all $s \in S$ and if for $s^* \in S$, $\pi(s^*) > 0, \mu_A(d_1(s^*)) > \mu_A(d_2(s^*))$, then $d_1 > d_2$.

We now see whether we can modify this criterion PPC such that the drawback is rectified and clear ranking is always possible. For this, we construct 1-Fuzzy Pessimistic Criterion (1-FPC) as follows.

4. 1-Fuzzy Pessimistic Criterion (1-FPC)

This criterion is based on a membership function μ_A on X, where $\mu_A(x_{ij})$ represents the fuzzy set membership of a consequence $x_{ij} \in X$ in the fuzzy set 'satisfaction' and the possibility distribution π on S representing the plausibility of states, both mapping on the totally ordered scale and can be compared.

Definition 4.1. Let (S, D, I, μ_A) be a decision problem. Then the **1-Fuzzy Pessimistic Criterion (1-FPC)** is given by,

$$d_i \geq d_j \Leftrightarrow S_1^-(d_i) \geq S_1^-(d_j), \text{ for all } d_i, d_j \in D$$

where

$$S_1^-(d_i) = \wedge_{s_j \in S}[J_{1,i}^-(s_j, x_{ij})],$$

$$J_{1,i}^-(s_j, x_{ij}) = \vee[(1 - \pi(s_j)), \mu_A(x_{ij})] \in [0, 1].$$

Remark 4.2. 1 FPC is the fuzzy version of 1-PPC. But the drawbacks of PPC are seen in 1-FPC also. Hence the need of construction of a new Fuzzy Pessimistic Criterion with the drawbacks rectified.

5. Fuzzy Aspiration and Reservation Degrees

A pessimist always expects the worst to occur and the best not to occur. Thus based on the attitude of the decision maker a new fuzzy pessimistic criterion is constructed. For this we introduce some definitions as follows.

Definition 5.1. Let (S, D, I, μ_A) be a decision problem, and let a consequence $x_{ij} \in X$ be totally satisfying to the DM. Then the membership of x_{ij} in the fuzzy set satisfaction is 1, i.e., $\mu_A(x_{ij}) = 1$. Such consequences $x_{ij} \in X$ are called **efficient consequences**.

Definition 5.2. Let (S, D, I, μ_A) be a decision problem, and let a consequence $x_{ij} \in X$ be highly satisfying (includes totally satisfying also) to the DM. Then the membership of x_{ij} in the fuzzy set satisfaction is very close or equal to 1. Such consequences $x_{ij} \in X$ are called **highly satisfying consequences**.

Next we define the Fuzzy Aspiration Degree as follows.

Definition 5.3. Let (S, D, I, μ_A) be a decision problem. Define

$$\alpha_D = \min\{\mu_A(x_{ij}) : x_{ij} \in X \text{ is highly satisfying}\}.$$

Then, 'α_D' is called the **Fuzzy Aspiration Degree** of the decision problem for the DM.

Definition 5.4. Let (S, D, I, μ_A) be a decision problem with fuzzy aspiration degree 'α_D'. Then a consequence $x_{ij} \in X$ is a **best consequence**, if $\mu_A(x_{ij}) \geq \alpha_D$.

Example 5.5. If $\alpha_D = .9$, then the best consequences of the decision problem are all consequences whose membership values are ≥ 0.9.

Properties of α_D:

(1) The fuzzy aspiration degree varies according to the decision maker and the decision problem, since highly satisfying is a fuzzy concept.

(2) The fuzzy aspiration degree specifies the best consequences of the decision problem.

(3) When $\alpha_D = 1$, the best consequence becomes the efficient consequence.

(4) Since α_D is the minimum degree of satisfaction for the best consequences, without loss of generality we can take $\alpha_D > .5$. Thus, we can assume $\alpha_D \in (.5, 1]$ (even though in most cases its value is higher than 0.5 and close to 1).

A pessimist also expects the worst to occur. Thus we introduce the Fuzzy Reservation Degree which specifies the 'worst consequences' as follows.

Definition 5.6. Let (S, D, I, μ_A) be a decision problem and let $x_{ij} \in X$ be totally dissatisfying to the DM. Then the membership of x_{ij} in the fuzzy set satisfaction is 0, i.e., $\mu_A(x_{ij}) = 0$. Such consequences $x_{ij} \in X$ are called **inefficient consequences**.

Definition 5.7. Let (S, D, I, μ_A) be a decision problem, and let a consequence $x_{ij} \in X$ be highly dissatisfying (includes totally dissatisfying also) to the DM. Then the membership of x_{ij} in the fuzzy set satisfaction is very close or equal to 0. Such consequences $x_{ij} \in X$ are called **highly dissatisfying** consequences.

Note

(1) In the crisp case, inefficient consequences are called worst consequences, since for such consequences the DM is totally dissatisfied.

(2) Inefficient consequences are always highly dissatisfying consequences, but the converse need not be true.

Definition 5.8. Let (S, D, I, μ_A) be a decision problem. Define,

$$\beta_D = \max\{\mu_A(x_{ij}) : x_{ij} \in X \text{ is highly dissatisfying}\}.$$

Then β_D is called the **Fuzzy Reservation Degree** of the decision problem for the DM.

Definition 5.9. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree β_D. Then a consequence $x_{ij} \in X$ is a **worst consequence**, if

$$\mu_A(x_{ij}) \leq \beta_D.$$

5.1. Properties of β_D

1. The 'Fuzzy Reservation Degree' varies according to the decision maker and the decision problem, since highly dissatisfying is a fuzzy concept.

2. When $\beta_D = 0$, the worst consequence becomes the inefficient consequence.

3. Clearly $\beta_D < 0.5$ (though in most cases its value is even very less than 0.5 and close to 0).

For the construction of 3-Fuzzy Pessimistic Criterion (3-FPC), which is based on the attitude of the decision maker that the worst will occur and the best will not occur, we now introduce certain definitions as follows.

6. Fuzzy Reservation and Non-Reservation Classes

Definition 6.1. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree 'β_D'. Then the **Fuzzy Reservation Class** w.r.t 'β_D' denoted by $FRC[\beta_D]$, is defined as

$$FRC[\beta_D] = \{x_{ij} \in X : \mu_A(x_{ij}) \leq \beta_D\}$$

Here $FRC[\beta_D]$ is the set of 'worst consequences' of the decision problem.

Definition 6.2. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree 'β_D'. Then the **Fuzzy Non-Reservation Class - 1** w.r.t 'β_D' denoted by $FNRC_1[\beta_D]$, is defined as

$$FNRC_1[\beta_D] = \{x_{ij} \in X : \mu_A(x_{ij}) > \beta_D, \mu_A(x_{ij}) < \pi(s_j), \\ \mu_A(x_{ij}) < \alpha_D\}$$

Definition 6.3. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree 'β_D'. Then the **Non-Fuzzy Reservation Class-2** w.r.t 'β_D' denoted by $FNRC_2[\beta_D]$, is defined as

$$FNRC_2[\beta_D] = \{x_{ij} \in X : \mu_A(x_{ij}) > \beta_D, \mu_A(x_{ij}) \geq \pi(s_j), \mu_A(x_{ij}) < \alpha_D\}.$$

Definition 6.4. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree 'β_D'. Then the **Non-Fuzzy Reservation Class-3** w.r.t 'β_D' denoted by $FNRC_3[\beta_D]$, is defined as

$$FNRC_3[\beta_D] = \{x_{ij} \in X : \mu_A(x_{ij}) \geq \alpha_D\}.$$

We now construct the **3-Fuzzy Pessimistic Criterion** as follows.

Definition 6.5. Let (S, D, I, μ_A) be a decision problem with fuzzy reservation degree 'β_D'. Define

$$J^-_{3,i}(s_j, x_{ij}) = \begin{cases} \frac{\beta_D}{2}(\mu_A(x_{ij}) \vee (1 - \pi(s_j)) + (1 - \frac{\beta_D}{2}) \\ \quad (\mu_A(x_{ij}) \wedge (1 - \pi(s_j)) \text{ if } x_{ij} \in FRC[\beta_D] \\ (1 - \frac{\beta_D}{2})(\mu_A(x_{ij}) \wedge \pi(s_j)) + \frac{\beta_D}{2}(\pi(s_j) \vee \mu_A(x_{ij})) \\ \quad \text{if } x_{ij} \in FNRC_1[\beta_D] \\ (1 - \beta_D)(\mu_A(x_{ij}) \vee \pi(s_j)) + \beta_D(\pi(s_j) \wedge \mu_A(x_{ij})) \\ \quad \text{if } x_{ij} \in FNRC_2[\beta_D] \\ (1 - \frac{\beta_D}{2})(\pi(s_j)) \vee \mu_A(x_{ij})) + \frac{\beta_D}{2}(\pi(s_j) \wedge \mu_A(x_{ij})) \\ \quad \text{if } x_{ij} \in FNRC_3[\beta_D] \end{cases}$$

Here $J^-_{3,i}(s_j, x_{ij}) \in [0, 1]$, is the score of the alternative $d_i \in D$, corresponding to the pair (s_j, x_{ij}). Then the **3-Fuzzy Pessimistic Criterion (3-FPC)** is given by

$$d_i \geq d_j \Leftrightarrow S^-_3(d_i) \geq S^-_3(d_j), \forall d_i, d_j \in D$$

where $S^-_3(d_i) = \frac{1}{n} \sum_{s_j \in S} J^-_{3,i}(s_j, x_{ij})$, is the 'degree of satisfaction' of the alternative 'd_i' for the decision maker.

7. Example

Now we give an example to show that 1-FPC fails to satisfy the Principle of Strict Pareto Dominance.

Example 7.1. Let $D = \{d_1, d_2, d_3, d_4\}$ be the set of alternatives and $S = \{s_1, s_2, s_3, s_4\}$ be the states of nature with possibility degree of states given by $\pi(s_1) = 0.3, \pi(s_2) =$

Table 1:

$D \backslash S$	s_1	s_2	s_3	s_4
d_1	.4	.6	.7	.5
d_2	.4	.6	.7	.4
d_3	.6	.4	.2	.5
d_4	.8	.5	.6	.7

$0.5, \pi(s_3) = 1$ and $\pi(s_4) = 0.6$. Let the 'satisfaction table' of the decision problem be given in Table 1.

Then, by 1-FOC, $d_1 = d_2$ even though 'd_1' is as satisfying as 'd_2' in states s_1, s_2 and s_3 and better in s_4.

Thus 1-FPC and also PPC fails to satisfy the Principle of Strict Pareto Dominance. But by 3-FPC for $\beta_D = .2$ (say) and $\lambda_D = .8$ (say), we get $S_3^-(d_1) = .5675, S_3^-(d_2) = .5275, S_3^-(d_3) = .435, S_3^-(d_4) = .62$.

$$\therefore d_4 > d_1 > d_2 > d_3.$$

Hence 3-FPC satisfies the Principle of Strict Pareto Dominance and therefore clear ranking of alternative is always possible.

Next we prove the theorem to show that 3-FPC in general satisfies the Principle of Strict Pareto Dominance.

Theorem 7.2. Let (S, D, I, μ_A) be a decision problem. For $d_1, d_2 \in D$, suppose $\mu_A(d_1(s_j)) \geq \mu_A(d_2(s_j))$, for all $s_j \in S$ and for $s_r \in S, \pi(s_r) > 0$, let

$$\mu_A(d_1(s_r)) > \mu_A(d_2(s_r)), \text{ where } d_i(s_j) = x_{ij} \in X,$$

for all $d_i \in D, \forall s_j \in S$. Then, if by 1-FPC, $d_1 = d_2$, by 3-FPC, $d_1 > d_2$.

Proof. For $d_1, d_2 \in D$, let $\mu_A(d_1(s_j)) = \mu_A(d_2(s_j)), d_1 > d_2, \forall s_j \in S - \{s_r\}$ and let for $s_r \in S$,

$$\pi(s_r) > 0, \mu_A(d_1(s_r)) > \mu_A(d_2(s_r)),$$

where $d_i(s_j) = x_{ij} \in X$, for all $d_i \in D \forall s_j \in S$. Let α_D, β_D be the aspiration degree and reservation degree respectively of the decision maker for the given decision problem. Also, let $FRC[\beta_D]$, denote the fuzzy reservation class and $FNRC_1[\beta_D]$, $FNRC_2[\beta_D], FNRC_3[\beta_D]$ denote the fuzzy non-reservation class w. r. t 'β_D'.

Then

$$\mu_A(x_{1r}) > \mu_A(x_{2r}) \text{ (given).}$$

For $x_{1r}, x_{2r} \in X$, since by 1-FPC, $d_1 = d_2$ two cases arises

Case I: If for $s_k \in S - \{s_r\}, \pi(s_k) > 0$

$$\vee_{s_j \in S}[J_{1,1}^-(s_j, x_{ij})] = J_{1,1}^-(s_k, x_{1k})(\text{ for a fixed } k)$$

Then, since $J^-_{1,1}(s_j, x_{ij}) = J^-_{1,2}(s_j, x_{ij}) \forall j \neq r$ and

$$J^-_{1,1}(s_r, x_{1r}) \geq J^-_{1,2}(s_r, x_{2r})$$

$$\vee_{s_j \in S}[J^-_{1,2}(s_j, x_{ij})] = J^-_{1,2}(s_k, x_{1k})$$

If in this case,

$$J^-_{1,1}(s_r, x_{1r}) > J^-_{1,2}(s_r, x_{2r}),$$

then it follows directly by applying 3-FPC, that

$$d_1 > d_2$$

If, $J^-_{1,1}(s_r, x_{1r}) = J^-_{1,2}(s_r, x_{2r})$, then proceeding similar to the proof given in Case II, by 3-FPC

$$d_1 > d_2$$

Case II: If,
$\vee_{s_j \in S}[J^-_{1,1}(s_j, x_{ij})] = J^-_{1,1}(s_r, x_{1r})$ and $\vee_{s_j \in S}[J^-_{1,2}(s_j, x_{2j})] = J^-_{1,2}(s_r, x_{2r})$, then since by 1-FPC, $d_1 = d_2$ (given) $\Rightarrow S^-_1(d_1) = S^-_1(d_2)$
$\Rightarrow J^-_{1,1}(s_r, x_{1r}) = J^-_{1,2}(s_r, x_{2r})$
$\Rightarrow \mu_A(x_{1r}) \wedge \pi(s_r) = \mu_A(x_{2r}) \wedge \pi(s_r)$
$\therefore$ since, $\mu_A(x_{1r}) > \mu_A(x_{2r})$, it follows that

$$\mu_A(x_{1r}) > \pi(s_r) \text{ and } \mu_A(x_{2r}) \geq \pi(s_r)$$

Thus, x_{1r} and x_{2r} belong to either FRC[β_D], $FNRC_2[\beta_D]$ or $FNRC_3[\beta_D]$. Hence it follows that if

1. $x_{1r}, x_{2r} \in FRC[\beta_D]$
2. $x_{1r}, x_{2r} \in FNRC_2[\beta_D]$
3. $x_{1r}, x_{2r} \in FNRC_3[\beta_D]$
4. $x_{1r} \in FRC[\beta_D]$ and $x_{2r} \in FNRC_2[\beta_D]$
5. $x_{1r} \in FRC[\beta_D]$ and $x_{2r} \in FNRC_3[\beta_D]$
6. $x_{1r} \in FNRC_2[\beta_D]$ and $x_{2r} \in FNRC_3[\beta_D]$.

In all cases we get

$$J^-_{3,1}(s_r, x_{1r}) > J^-_{3,2}(s_r, x_{2r}).$$

Then by 3-FOC

$$
\begin{aligned}
S_3^-(d_1) &= \frac{1}{n}\sum_{s_j \in S} J_{3,1}^-(s_j, x_{ij}) \\
&= \frac{1}{n}\left[\sum_{s_j \in S-\{s_r\}} J_{3,1}^-(s_j, x_{1j}) + J_{3,1}^-(s_r, x_{1r})\right] \\
&= \frac{1}{n}\left[\sum_{s_j \in S-\{s_r\}} J_{3,2}^-(s_j, x_{2j}) + J_{3,1}^-(s_r, x_{1r})\right] \\
&> \frac{1}{n}\left[\sum_{s_j \in S-\{s_r\}} J_{3,2}^-(s_j, x_{2j}) + J_{3,2}^-(s_r, x_{2r})\right] \\
&= \frac{1}{n}\left[\sum_{s_j \in S} J_{3,2}^-(s_j, x_{2j})\right] \\
&= S_3^-(d_2) \\
&\therefore S_3^-(d_1) > S_3^-(d_2), \text{ and hence} \\
&\quad d_1 > d_2.
\end{aligned}
$$

In cases when $\beta_D = 0$ and $d_1 = d_2$, we can take $\beta_D = 0.0001$ (say) which approximates to 0. Then proceeding as in the proof above, we get $d_1 > d_2$. ■

Remark 7.3. Thus 3-FPC satisfies the Principle of Strict Pareto Dominance > Hence clear ranking of alternatives is always possible in all cases using 3-FPC, when the decision maker is a pessimist.

Note. The above theorem can also be stated as follows. Let d_1 be as satisfying as d_2 in all states and better in some states and suppose by 1-FPC, $d_1 = d_2$, then by 3-FPC, d_1 will be strictly better than d_2 i.e., $d_1 > d_2$.

Conclusion

In this paper, a new Fuzzy Pessimistic Criterian 3-FPC is constructed for ranking of alternatives in a fuzzy environment based on the attitude of the decision maker that the worst will occur and the best will not occur. The drawbacks in the existing criteria are rectified and clear ranking of alternatives is always possible using this criterion. Many other models for ranking of alternatives in a fuzzy environment based on the attitude of the decision maker have been constructed in our papers [11, 12].

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