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# Multiple Domination in An Intuitionistic Fuzzy Graph

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## Abstract

In this paper we introduce the concept multiple domination in intunionistic fuzzy graphs and obtain some interesting results for this new parameter in intunionistic fuzzy graphs.

**Keywords—** intunionistic Fuzzy graphs, intunionistic Fuzzy domination, mixed intunionistic fuzzy domination number.

## I. Introduction

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram,S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs . C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph . The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R.parvathi and G.Thamizhendhi . In this paper develop the concept of Multiple Domination and obtain some interesting results in operations inuitionistic fuzzy graph.

## II. Preliminaries

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V=\{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to [0,1]$ denote the degree of membership and nonmember ship of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, (i = 1,2,...n)$. $E \subseteq V \times V$ where
$\mu_2 : V \times V \to [0,1],$ *and* $\gamma_2 : V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j)$ and
$0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$

An arc $(v_i, v_j)$ of an IFG G is called an *strong arc* if
$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) = \gamma_1(v_i) \wedge \gamma_1(v_j)$.

Let G = (V,E) be an IFG. Then the *cardinality of G* is defined to be
$|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right| + \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2}]\right|$.
Let G = (V, E) be an IFG. The *vertex cardinality* of G is defined to be
$|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right|$ for all $v_i \in V, (i = 1,2,...n)$. Let G = (V, E) be an IFG.
An *edge cardinality* of G is defined to be $|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2}]\right|$ for all $(v_i, v_j) \in V \times V$, Let G = (V, E) be an IFG. A set $D \subseteq V$ is said to be a *dominating set* of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.
An *intunionistic fuzzy dominating* set D of an IFG, G is called minimal dominating set of G if every node $u \in D$ , $D - \{u\}$ is not a dominating set in G. An *intunionistic fuzzy domination number* $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

A set $S \subseteq V$ in an IFG, G is said to be an *independent* if there is no strong between the vertices $v \in S$. An independent set S of IFG ,G is said to be *maximal independent* set if every node $v \in V - S$ then the set $S \cup \{v\}$ is not an independent set in G. The minimum cardinality among all the maximal independent sets in an IFG, G is called the intunionistic fuzzy independent number.

## III. Intunionistic fuzzy multiple domination

**Definition 3.1**

Let G be an IFG and D be a subset of V. A vertex $v \in V - D$ is said to be an intunionistic fuzzy K-dominated if it is dominated by at least k vertices in D

**Definition 3.2**

In an intunionistic fuzzy graph G every vertex in V-D is fuzzy k-dominated, then D is called a fuzzy K-dominating set.

**Definition 3.3**

The minimum cardinality of an intunionistic fuzzy k-dominating set is called the intunionistic fuzzy k-domination number $\gamma_k(G)$

**Definition 3.4**

An intunionistic fuzzy k-domination number of an intunionistic fuzzy graph G and the intunionistic fuzzy domination number of G are equal when k=1 that is if k=1 then $\gamma_1(G) = \gamma(G)$.

**ALGORITHM**

An algorithm to find the K dominating set of an IFG.

For a given IFG ,G

- Collect the strong arcs (u,v) in G
- Remove all the remaining arcs in G and the resulting IFG is $G_1$
- Choose $v_1 \in V(G_1)$ such that $d(v_1) = \Delta(G_1)$ and find $N_1 = N(v_1)$. The resulting IFG is $G_2$
- Choose $v_2 \in V(G_2)$ in $V(G_1) - \{v_1\}$ such that $d(v_2) = \Delta(G_2)$ [Here $G_2 = G_1 - \{v_1\}$] and find $N_2 = N(v_2)$.
- Continue this process until we get the isolated nodes
- $K = min[|N_1|, |N_2| \cdots]$ the remaining isolated vertices form the K- dominating set for an IFG $G_1$

**Theorem: 3.1**

If $G_1 \times G_2$ is an IFG , N be the number of edges in $G_1 \times G_2$, m and n be the number of vertices in $V_1$ *and* $V_2$ respectively. p , q be the number of edges in $G_1$ and $G_2$ then N=mq +np

**Proof:**

By the definition of $G_1 \times G_2$, the edges of the form $(x\, x_2)(x\, y_2)$ for all $x \in V_1$ and $(x_2 y_2) \in E_2$ and $(x_1, z)(y_1, z)$ for all $x_1 y_1 \in E_1$ and $z \in V_2$. We take edges of the form $(x\, x_2)(x\, y_2)$ for all $x \in V_1$ and $(x_2 y_2) \in E_2$

$|(x\, x_2)(x\, y_2)| = x|V_1| \times |x\, y_2|$ for all $x_1 \in V_1$ and for all $(x_2 y_2) \in E_2$

$= |V_1| \times |E_1|$

$= mq$

Similarly, we can get for $(x_1,z)(y_1,z)$ for all $x_1y_1 \in E_1$ and $z \in V_2$. $|(x_1,z)(y_1,z)|$ =np. Therefore the number of edges in $G_1 \times G_2$ is $|(x_1x_2)(x_1y_2)| + |(x_1z)(y_1z)| = mq + np$ for all $x_1 \in V_1$ and $(x_2y_2) \in E_2$ for all $x_1y_1 \in E_1$ and $z \in V_2$.N= mq+ np
Hence proved.

**Definition 3.5**
Let $G_1$ =($V_1$,$E_1$) and $G_2$ =($V_2$,$E_2$) be intuitionistic fuzzy graphs on $V_1$,$V_2$ respectively with $V_1 \cap V_2 = \phi$. The join of $G_1$ and $G_2$ is the intuitionistic fuzzy graph G on $V_1 \cup V_2$ defined by $G = (G_1 + G_2) = ((\mu_1 + \mu_1'), (\gamma_1 + \gamma_1'), (\mu_2 + \mu_2'), (\gamma_2 + \gamma_2'))$ where

$$(\mu_1 + \mu_1')(u) = \begin{cases} \mu_1(u) \ if\ u \in V_1 \\ \mu_1'(u) \ if\ u \in V_2 \end{cases}$$

$$(\gamma_1 + \gamma_1')(u) = \begin{cases} \gamma_1(u) \ if\ u \in V_1 \\ \gamma_1'(u) \ if\ u \in V_2 \end{cases}$$

$$(\mu_2 + \mu_2')(uv) = \begin{cases} \mu_2(uv) \ if\ \ uv \in E_1 \\ \mu_2'(uv) \ if\ \ uv \in E_2 \\ \mu_1(u) \wedge \mu_1'(v) \ if\ \ u \in V_1 \ \&v \in V_2 \end{cases} \quad \text{and}$$

$$(\gamma_2 + \gamma_2')(uv) = \begin{cases} \gamma_2(uv) \ if\ \ uv \in E_1 \\ \gamma_2'(uv) \ if\ \ uv \in E_2 \\ \gamma_1(u) \vee \gamma_1'(v) \ if\ \ u \in V_1 \ \&v \in V \end{cases}$$

**Remark:**
Let $G_1 \& G_2$ be an IFG and $|V_1| = k_1 \&$ $|V_2| = k_2$. Then $V_1$ is a $k_2$ dominating set of $G_1 + G_2$ and Then $V_2$ is a $k_1$ dominating set of $G_1 + G_2$ and

**Theorem: 3.2**
If D is a minimal vertex dominating set of an IFG G then there exist a vertex in (V-D) is not dominated by multiple vertices.
**Proof:**
Let D be a minimum dominating set in an IFG $G_1$. Assume that every vertex in (V-D) is dominated by multiple vertices in D.
Let u $\in V - D$ . Let v and w be two vertices in D which dominates u. It follows from our assumption that every vertex in V-D is dominated by at least one vertex in $D - \{v, w\}$
Therefore the set $D' = D - \{v, w\} \cup \{u\}$ is the minimal dominating set of $G_1$. Note that, $|D'| < |D|$
This is contradict to D is minimal dominating set in V-D is not dominated by multiple vertices in D.
Hence proved.

**Definition 3.6**

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be intuitionistic fuzzy graphs on $V_1, V_2$ respectively then the Cartesian product of $G_1$ and $G_2$ denoted by $G_1 \times G_2$, is the intuitionistic fuzzy graph G on $V_1 \times V_2$ defined by

$G = (G_1 \times G_2) = ((\mu_1 \times \mu_1^{'}), (\gamma_1 \times \gamma_1^{'}), (\mu_2 \times \mu_2^{'}), (\gamma_2 \times \gamma_2^{'}))$ where

$$(\mu_1 \times \mu_1^{'})(u_1, u_2) = \mu_1(u_1) \wedge \mu_1^{'}(u_2)$$

$$(\gamma_1 \times \gamma_1^{'})(u_1, u_2) = \gamma_1(u_1) \vee \gamma_1^{'}(u_2)$$

$$(\mu_2 \times \mu_2^{'})(u_1 u_2)(v_1 v_2) = \begin{cases} \mu_1(u_1) \wedge \mu_2^{'}(u_2 v_2) \; if \; u_1 = v_1 \\ \mu_1^{'}(u_2) \wedge \mu_1(u_1 v_1) \; if \; u_2 = v_2 \end{cases}$$

and

$$(\gamma_2 \times \gamma_2^{'})(u_1 u_2)(v_1 v_2) = \begin{cases} \gamma_1(u_1) \vee \gamma_2^{'}(u_2 v_2) \; if \; u_1 = v_1 \\ \gamma_1^{'}(u_2) \vee \gamma_1(u_1 v_1) \; if \; u_2 = v_2 \end{cases}$$

**Theorem: 3.3**

If $G_1$ or $G_2$ be an IFG, $D_1$be the $K_1$ dominating set of $G_1$ then $D_1 \times V_1$ is the minimum $K_1$ dominating set of $G_1 \times G_2$.

**Proof:** Let $D_1$be the $K_1$ dominating set of an IFG $G_1$. There exist every $x \in (V_1 - D_1)$ is dominated by $K_1$ vertices in $D_1$. There is $K_1$ strong arc between x and vertices in $D_1$ Now we prove $(V_1 - D_1) \times V_2$ is dominated by $K_1$ vertices in $D_1$.Let $x \in (V_1 - D_1)$, x is dominated by $K_1$in $D_1$.Let $x \times V_2 \in (V_1 - D_1) \times V_2$ , Now we prove $x, v_2$ is dominated by $K_1$ vertices in $D_1 \times V_2$.

There exist $y \in D_1$ such that, $\mu_{B(xy)} = \mu_{A(x)} \wedge \mu_{A(y)}$.Let $(x, v_2) \in (V_1 - D_1) \times V_2$ and $(y, v_2) \in (D_1 \times V_2)$ .

Therefore,$(\mu_B \times \mu_{B'})((x, v_2)(y, v_2)) = \mu_{B(xy)} \wedge \mu_{A'}(v_2)$

$$= \mu_{A(x)} \wedge \mu_{A(y)} \wedge \mu_{A'}(v_2)$$

$$= (\mu_{A(x)} \wedge \mu_{A'}(v_2)) \wedge \mu_{A(y)} \wedge \mu_{A'}(v_2)$$

$$= (\mu_A \times \mu_{A'})(x, v_2) \wedge (\mu_A \times \mu_{A'})(y, v_2)$$

Hence, $(x, v_2)$ is dominated by $(y, v_2)$.

In $D_1$, there are $K_1$ vertices dominates x .Therefore, $(x, V_2) \in [(V_1 - D_1) \times V_2]$ is dominated by $K_1$ vertices in $(D_1 \times V_2)$ by the definition of $G_1 \times G_2$. Clearly, every vertex in $(V_1 - D_1) \times V_2$ is dominated by $K_1$ vertices in $D_1 \times V_2$ .

Hence $D_1 \times V_2$ is a $K_1$ dominating set of $G_1 \times G_2$.

Now we prove $D_1 \times V_2$ is minimum. Assume $(D_1 \times V_2) - (x_1, v_2)$ is a minimum $K_1$ dominating set of $G_1 \times G_2$. $x_1$is dominated by $K_1$vertices in $D_1$. Since $D_1$ is a $K_1$ dominating set of $G_1$, $then$ $(D_1 - x_1)$ is also $K_1$ dominating set. This is contradict to $D_1$is minimum $K_1$ dominating set of $G_1$ . Therefore our assumption is wrong.

Hence, $D_1 \times V_2$ is a $K_1$ dominating set of$G_1 \times G_2$.

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