Pairwise Fuzzy Almost Resolvable Spaces

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Abstract

In this paper, the concepts of pairwise fuzzy almost resolvable spaces and pairwise fuzzy almost irresolvable spaces are studied and the conditions under which fuzzy bitopological spaces become pairwise fuzzy almost resolvable spaces, are also investigated.

KEY WORDS: Pairwise fuzzy dense set, pairwise fuzzy nowhere dense set, pairwise fuzzy residual set, pairwise fuzzy first category space, pairwise fuzzy Baire space, pairwise fuzzy weakly Volterra space, pairwise fuzzy resolvable space.

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1. INTRODUCTION

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L.A.Zadeh [14] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. Among the first fields of Mathematics to be considered in the context of fuzzy sets was general topology. The concepts of fuzzy topology was defined by C.L.Chang [2] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics.

1989, Kandil [5] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. The concept of pairwise fuzzy almost resolvable spaces in fuzzy setting was introduced by G.Thangaraj and S.Sethuraman [8]. In this paper we study several characterizations of pairwise fuzzy almost resolvable spaces and pairwise fuzzy almost irresolvable spaces and the conditions under which fuzzy bitopological spaces become pairwise fuzzy almost resolvable spaces, are investigated. For this study pairwise fuzzy submaximal spaces, pairwise fuzzy first category spaces, pairwise fuzzy Baire spaces, pairwise fuzzy hyperconnected and pairwise fuzzy P-spaces are considered in this article.

2. PRELIMINARIES

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T₁, T₂), where T₁ and T₂ are fuzzy topologies on the non-empty set X. The complement λ’ of a fuzzy set λ is defined by λ’(x) = 1 − λ(x) (x ∈ X).

**Definition 2.1:** Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define λ ∨ μ: X → [0, 1] as follows: (λ ∨ μ)(x) = Max {λ(x), μ(x)}. Also we define λ ∧ μ: X → [0, 1] as follows: (λ ∧ μ)(x) = Min {λ(x), μ(x)}.

For a family {λᵢ/i ∈ I} of fuzzy sets in (X, T), the union ψ = ∨ᵢ(λᵢ) and the intersection δ = ∧ᵢ(λᵢ) are defined respectively as ψ(x) = supᵢ{λᵢ(x), x ∈ X} and δ(x) = infᵢ{λᵢ(x), x ∈ X}.

**Definition 2.2:** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define the interior and the closure of λ respectively as follows:

(i) Int(λ) = {μ / μ ≤ λ, μ ∈ T},
(ii) Cl(λ) = {μ / λ ≤ μ, 1−μ ∈ T}.

**Lemma 2.1 [1]:** For a fuzzy set λ of a fuzzy topological space X,

(i) 1 − Int(λ) = Cl(1−λ),
(ii) 1 − Cl(λ) = Int(1−λ).

**DEFINITION 2.3 [11]:** A fuzzy set λ in a fuzzy bitopological space (X, T₁, T₂) is called a pairwise fuzzy open set if λ ∈ Tᵢ (i = 1, 2). The complement of pairwise fuzzy open set in (X, T₁, T₂) is called a pairwise fuzzy closed set.

**Definition 2.4 [7]:** A fuzzy set λ in a fuzzy bitopological space (X, T₁, T₂) is called a pairwise fuzzy dense set if cl₁T₁cl₂ (λ) = cl₂cl₁T₂ (λ) = 1.

**Definition 2.5 [8]:** A fuzzy set λ in a fuzzy bitopological space (X, T₁, T₂) is called a pairwise fuzzy nowhere dense set if int₁T₁cl₂ (λ) = int₂T₂cl₁T₂ (λ) = 0.
DEFINITION 2.6 [8]: A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy Baire if \(\text{int}_{T_1}(V_{i=1}^\infty (\lambda_i)) = 0, \ (i = 1, 2)\) where \((\lambda_i)'s\) are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\).

Lemma 2.2 [1]: For a family \(\mathcal{A}\) of \(\{\lambda, \alpha\}\) of fuzzy sets of a fuzzy topological space \((X, T)\), \(\text{cl} (\lambda, \alpha) \leq \text{cl} (\lor \lambda, \alpha)\). In case \(\mathcal{A}\) is a finite set, \(\lor \text{cl} (\lambda, \alpha) = \text{cl} (\lor (\lambda, \alpha))\). Also \(\text{vint}(\lambda, \alpha) \leq \text{int} (\lor (\lambda, \alpha))\) in \((X, T)\).

Definition 2.7 [8]: Let \((X, T_1, T_2)\) be a fuzzy bitopological space. A fuzzy set \(\lambda\) in \((X, T_1, T_2)\) is called a pairwise fuzzy first category set if \(\lambda = V_{i=1}^\infty (\lambda_i)\), where \((\lambda_i)'s\) are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\). Any other fuzzy set in \((X, T_1, T_2)\) is said to be a pairwise fuzzy second category set in \((X, T_1, T_2)\).

Definition 2.8 [8]: If \(\lambda\) is a pairwise fuzzy first category set in a fuzzy bitopological space \((X, T_1, T_2)\), then the fuzzy set \((1-\lambda)\) is called a pairwise fuzzy residual set in \((X, T_1, T_2)\).

Definition 2.9 [10]: A fuzzy bitopological space \((X, T_1, T_2)\) is called pairwise fuzzy first category space if the fuzzy set \(1_X\) is a pairwise fuzzy first category set in \((X, T_1, T_2)\). That is, \(1_X = V_{i=1}^\infty (\lambda_i)\), where \((\lambda_i)'s\) are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\). Otherwise, \((X, T_1, T_2)\) will be called a pairwise fuzzy second category space.

Definition 2.10 [10]: A fuzzy set \(\lambda\) in a fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy \(G_\delta\)-set if \(\lambda = \Lambda_{i=1}^\infty (\lambda_i)\), where \((\lambda_i)'s\) are pairwise fuzzy open sets in \((X, T_1, T_2)\).

Definition 2.11 [10]: A fuzzy set \(\lambda\) in a fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy \(F_\delta\)-set if \(\lambda = \Lambda_{i=1}^\infty (\lambda_i)\), where \((\lambda_i)'s\) are pairwise fuzzy closed sets in \((X, T_1, T_2)\).

3. PAIRWISE FUZZY ALMOST RESOLVABLE SPACES
The concept of fuzzy almost resolvable spaces in fuzzy setting was studied in [8]. Motivated by this, the concept of pairwise fuzzy almost resolvable spaces was introduced in [9].

Definition 3.1 [9]: A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy almost resolvable space if \(\lor V_{k=1}^\infty (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)'s\) in \((X, T_1, T_2)\) are such that \(\text{int}_{T_2} \cap \text{int}_{T_2} (\lambda_k) = \text{int}_{T_2} \cap \text{int}_{T_2} (\lambda_k) = 0\). Otherwise \((X, T_1, T_2)\) is called a pairwise fuzzy almost irresolvable space.

Proposition 3.1 If a fuzzy bitopological space \((X, T_1, T_2)\) is a pairwise fuzzy first category space, then \((X, T_1, T_2)\) is a pairwise fuzzy almost resolvable space.
Proof: Let $(X, T_1, T_2)$ be a pairwise fuzzy first category space. Then, $v_{\lambda_k}^\infty (\lambda_k) = 1$, where $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2)$. Since $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2)$, $\text{int} T_1 \text{cl} T_2 (\lambda_k) = \text{int} T_1 \text{cl} T_1 (\lambda_k) = 0$. But, for a fuzzy set $(\lambda_k)$ in $(X, T_1, T_2)$, we have $\text{int} T_1 \text{int} T_2 (\lambda_k) \leq \text{int} T_1 \text{cl} T_2 (\lambda_k)$ and $\text{int} T_1 \text{int} T_1 (\lambda_k) \leq \text{int} T_1 \text{cl} T_1 (\lambda_k)$. Hence we have $\text{int} T_1 \text{int} T_2 (\lambda_k) \leq 0$ and $\text{int} T_2 \text{int} T_1 (\lambda_k) \leq 0$. That is, $\text{int} T_1 \text{int} T_2 (\lambda_k) = 0$ and $\text{int} T_2 \text{int} T_1 (\lambda_k) = 0$. Hence, $v_{\lambda_k}^\infty (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$ in $(X, T_1, T_2)$ are such that $\text{int} T_1 \text{int} T_2 (\lambda) = \text{int} T_2 \text{int} T_1 (\lambda) = 0$, implies that $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space.

Proposition 3.2: If a bitopological space $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space and if $\text{cl} T_1 \text{int} T_2 (\lambda) = 1$ and $\text{cl} T_2 \text{int} T_1 (\lambda) = 1$, for each pairwise fuzzy dense set $\lambda$ in $(X, T_1, T_2)$, then $(X, T_1, T_2)$ is a pairwise fuzzy first category space.

Proof: Let $(X, T_1, T_2)$ be a pairwise fuzzy almost resolvable space. Then, $v_{\lambda_k}^\infty (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$’s in $(X, T_1, T_2)$ are such that $\text{int} T_1 \text{int} T_2 (\lambda_k) = \text{int} T_2 \text{int} T_1 (\lambda_k) = 0$. Now, $1 - \text{int} T_1 \text{int} T_2 (\lambda_k) = 1$ and $1 - \text{int} T_2 \text{int} T_1 (\lambda_k) = 1$. Then, $\text{cl} T_1 \text{cl} T_2 (1 - \lambda_k) = 1$ and $\text{cl} T_2 \text{cl} T_1 (1 - \lambda_k) = 1$ and hence $(1 - \lambda_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Then, by hypothesis, we have $\text{cl} T_1 \text{int} T_2 (1 - \lambda_k) = 1$ and $\text{cl} T_2 \text{int} T_1 (1 - \lambda_k) = 1$. This implies that $\text{int} T_1 \text{cl} T_2 (\lambda_k) = \text{int} T_2 \text{cl} T_1 (\lambda_k) = 0$ and hence $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2)$. Thus, $v_{\lambda_k}^\infty (\lambda_k) = 1$, where $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2)$, implies that $(X, T_1, T_2)$ is a pairwise fuzzy first category space.

Proposition 3.3: If a fuzzy bitopological space $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space if and only if $A_{\lambda_k}^\infty (\mu_k) = 0$, where $(\mu_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$.

Proof: Let $(X, T_1, T_2)$ be a pairwise fuzzy almost resolvable space. Then, $v_{\lambda_k}^\infty (\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$’s in $(X, T_1, T_2)$ are such that $\text{int} T_1 \text{int} T_2 (\lambda_k) = \text{int} T_2 \text{int} T_1 (\lambda_k) = 0$. Now, $1 - \text{int} T_1 \text{int} T_2 (\lambda_k) = 1$ and $1 - \text{int} T_2 \text{int} T_1 (\lambda_k) = 1$. Then, $\text{cl} T_1 \text{cl} T_2 (1 - \lambda_k) = 1$ and $\text{cl} T_2 \text{cl} T_1 (1 - \lambda_k) = 1$ and hence $(1 - \lambda_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Now $v_{\lambda_k}^\infty (\lambda_k) = 1$, implies that $A_{\lambda_k}^\infty (1 - \lambda_k) = 0$, where $(1 - \lambda_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Let $(1 - \lambda_k) = \mu_k$. Then, we have $A_{\mu_k}^\infty (\mu_k) = 0$, where $(\mu_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$.

Conversely, suppose that $A_{\lambda_k}^\infty (\mu_k) = 0$, where $(\mu_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Then, $1 - (A_{\lambda_k}^\infty (\mu_k)] = 1$. This implies that $v_{\lambda_k}^\infty (1 - \mu_k) = 1$. Since $(\mu_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Then, $\text{cl} T_1 \text{cl} T_2 (\mu_k) = \text{cl} T_1 \text{cl} T_1 (\mu_k) = 1$ and hence $1 - [\text{cl} T_1 \text{cl} T_2 (\mu_k)] = 0$ and $1 - [\text{cl} T_1 \text{cl} T_1 (\mu_k)] = 0$. Then, $\text{int} T_1 \text{int} T_2 (1 - \mu_k) = \text{int} T_2 \text{int} T_1 (1 - \mu_k) = 0$. Let $(1 - \mu_k) = \lambda_k$. Hence,
we have $\bigvee_{k=1}^{N}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$ in $(X, T_1, T_2)$ are such that $\text{int}_{T_1}\text{int}_{T_2}(\lambda_k) = \text{int}_{T_2}\text{int}_{T_1}(\lambda_k) = 0$.

Therefore $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space.

**Proposition 3.4:** If $\bigvee_{k=1}^{N}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in a fuzzy bitopological space $(X, T_1, T_2)$, then $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space.

**Proof:** Suppose that $\bigvee_{k=1}^{N}(\lambda_k) = 1$, where the fuzzy sets $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in a fuzzy bitopological space $(X, T_1, T_2)$. Then, $\Lambda_{k=1}^{N}(1 - \lambda_k) = 0$. Since $(\lambda_k)$’s are pairwise fuzzy nowhere dense sets in $(X, T_1, T_2)$, $\text{int}_{T_1}\text{cl}_{T_2}(\lambda_k) = \text{int}_{T_2}\text{cl}_{T_1}(\lambda_k) = 0$. Now $\text{int}_{T_1}\text{int}_{T_2}(\lambda_k) \leq \text{int}_{T_1}\text{cl}_{T_2}(\lambda_k)$ and $\text{int}_{T_2}\text{int}_{T_1}(\lambda_k) \leq \text{int}_{T_2}\text{cl}_{T_1}(\lambda_k)$. Therefore $\text{int}_{T_1}\text{int}_{T_2}(\lambda_k) = \text{int}_{T_2}\text{int}_{T_1}(\lambda_k) = 0$. Hence $(1 - \lambda_k)$’s (k = 1 to N) are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Let $\mu_k (k=1$ to $\infty$) be pairwise fuzzy dense sets in $(X, T_1, T_2)$ in which the first N pairwise fuzzy dense sets be $(1 - \lambda_k)$. Now $\Lambda_{k=1}^{N}(\mu_k) \leq \Lambda_{k=1}^{N}(1 - \lambda_k)$. Then, we have $\Lambda_{k=1}^{N}(\mu_k) \leq 0$. That is, $\Lambda_{k=1}^{N}(\mu_k) = 0$, where $(\mu_k)$’s are pairwise fuzzy dense sets in $(X, T_1, T_2)$. Then, by proposition 3.3, $(X, T_1, T_2)$ is a pairwise fuzzy almost resolvable space.

**Theorem 3.1 [9]:** If a fuzzy bitopological space $(X, T_1, T_2)$ is a pairwise fuzzy Baire space, then $(X, T_1, T_2)$ is a pairwise fuzzy almost irresolvable space.

**Theorem 3.2 [10]:** If $(X, T_1, T_2)$ is a pairwise fuzzy Baire space, then,

(i) $\text{int}_{T_1}\text{int}_{T_2}(\lambda) = 0$ for a pairwise fuzzy first category set $\lambda$ in $(X, T_1, T_2)$.

(ii) $\text{cl}_{T_1}\text{cl}_{T_2}(\lambda) = 0$ for a pairwise fuzzy residual set $\lambda$ in $(X, T_1, T_2)$.

**Proposition 3.5:** If $\Lambda_{k=1}^{\infty}(\lambda_k) = 0$, where the fuzzy sets $(\lambda_k)$’s are pairwise fuzzy residual sets in a pairwise fuzzy Baire space, then $(X, T_1, T_2)$ is a pairwise fuzzy almost irresolvable space.

**Proof:** Let $(X, T_1, T_2)$ be a pairwise fuzzy Baire space in which $\Lambda_{k=1}^{\infty}(\lambda_k) = 0$, where the fuzzy sets $(\lambda_k)$’s are pairwise fuzzy residual sets in $(X, T_1, T_2)$. Now $\Lambda_{k=1}^{\infty}(\lambda_k) = 0$. Therefore $1 - [\Lambda_{k=1}^{\infty}(\lambda_k)] = 1$. Then, we have $\bigvee_{k=1}^{N}(1 - \lambda_k) = 1$. Since $(\lambda_k)$’s are pairwise fuzzy residual sets, $(1 - \lambda_k)$’s are pairwise fuzzy first category sets in $(X, T_1, T_2)$. Since $(X, T_1, T_2)$ is a pairwise fuzzy Baire space, by theorem 3.2, $\text{int}_{T_1}\text{int}_{T_2}(1 - \lambda_k) = 0$ and $\text{int}_{T_2}\text{int}_{T_1}(1 - \lambda_k) = 0$. Let $\mu_k = (1 - \lambda_k)$. Therefore, we have $\bigvee_{k=1}^{N}(\mu_k) = 1$, where $(\mu_k)$’s are such that $\text{int}_{T_1}\text{int}_{T_2}(\mu_k) = 0$ and $\text{int}_{T_2}\text{int}_{T_1}(\mu_k) = 0$. Therefore $(X, T_1, T_2)$ is a pairwise fuzzy almost irresolvable space.
**Definition 3.2** [9]: A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy submaximal space if each pairwise fuzzy dense set \(\lambda\) in \((X, T_1, T_2)\) is a pairwise fuzzy open set in \((X, T_1, T_2)\). That is, if \(\lambda\) is a pairwise fuzzy dense set in a fuzzy bitopological space \((X, T_1, T_2)\), then \(\lambda \in T_i\) \((i = 1, 2)\).

**Proposition 3.6:** If a fuzzy bitopological space \((X, T_1, T_2)\) is a pairwise fuzzy almost resolvable and pairwise fuzzy submaximal space, then \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), where \((\lambda_k)\)’s are pairwise fuzzy closed sets in \((X, T_1, T_2)\).

**Proof:** Let \((X, T_1, T_2)\) be a pairwise fuzzy almost resolvable space. Then, \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)\)’s in \((X, T_1, T_2)\) are such that \(\text{int}_{T_1} \text{int}_{T_2} (\lambda_k) = \text{int}_{T_1} (\lambda_k) = 0\). Now, \(1 - \text{int}_{T_1} \text{int}_{T_2} (\lambda_k) = 1\) and \(1 - \text{int}_{T_2} \text{int}_{T_1} (\lambda_k) = 1\). Then, \(\text{cl}_{T_1} \text{cl}_{T_2} (1 - \lambda_k) = 1\) and \(\text{cl}_{T_1} \text{T}_{T_2} (1 - \lambda_k) = 1\) and hence \((1 - \lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Since \((X, T_1, T_2)\) is a pairwise fuzzy submaximal space, the pairwise fuzzy dense sets \((1 - \lambda_k)'s\) are pairwise fuzzy open sets in \((X, T_1, T_2)\) and hence \((\lambda_k)'s\) are pairwise fuzzy closed sets in \((X, T_1, T_2)\). Thus \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), where \((\lambda_k)'s\) are pairwise fuzzy closed sets in a pairwise fuzzy almost resolvable and pairwise fuzzy submaximal space \((X, T_1, T_2)\).

**Proposition 3.7:** If each fuzzy set \((\lambda_k)\) is a pairwise fuzzy \(F_\sigma\)-set in a pairwise fuzzy almost resolvable space \((X, T_1, T_2)\), then \(\lambda_{k=1}^{\infty} (1 - \lambda_k) = 0\), where \((1 - \lambda_k)'s\) are pairwise fuzzy dense and pairwise fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\).

**Proof:** Let \((X, T_1, T_2)\) be a pairwise fuzzy almost resolvable space. Then, \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)\)’s in \((X, T_1, T_2)\) are such that \(\text{int}_{T_1} \text{int}_{T_2} (\lambda_k) = \text{int}_{T_1} (\lambda_k) = 0\). Now, \(1 - \text{int}_{T_1} \text{int}_{T_2} (\lambda_k) = 1\) and \(1 - \text{int}_{T_2} \text{int}_{T_1} (\lambda_k) = 1\). Then, \(\text{cl}_{T_1} \text{cl}_{T_2} (1 - \lambda_k) = 1\) and \(\text{cl}_{T_2} \text{cl}_{T_1} (1 - \lambda_k) = 1\) and hence \((1 - \lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Now \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), implies that \(\lambda_{k=1}^{\infty} (1 - \lambda_k) = 0\), where \((1 - \lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Since the fuzzy sets \((\lambda_k)'s\) in \((X, T_1, T_2)\), are pairwise fuzzy \(F_\sigma\)-sets, \((1 - \lambda_k)'s\) are pairwise fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\). Hence, we have \(\lambda_{k=1}^{\infty} (1 - \lambda_k) = 0\), where \((1 - \lambda_k)'s\) are fuzzy dense and fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\).

**Theorem 3.3** [8]: If \(\lambda\) is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space \((X, T_1, T_2)\), then \(1 - \lambda\) is a pairwise fuzzy dense set in \((X, T_1, T_2)\).

**Definition 3.3** [11]: A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy weakly Volterraff \((\lambda_{k=1}^{\infty} (\lambda_k) \neq 0\), where \((\lambda_k)'s\) are pairwise fuzzy dense and pairwise fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\).

**Proposition 3.8:** If \(\mathcal{V}_{k=1}^{\infty} (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy \(F_\sigma\)-sets and pairwise fuzzy nowhere dense sets in a fuzzy bitopological space \((X, T_1, T_2)\),
then \((X, T_1, T_2)\) is not pairwise fuzzy weakly Volterra space, but is pairwise fuzzy almost resolvable space.

**Proof:** Let \(V^N_{i=1} (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)'s\) are fuzzy pairwise \(F_\sigma\)-sets and pairwise fuzzy nowhere dense sets in a fuzzy bitopological space \((X, T_1, T_2)\). Then, \(\text{int}_{T_1} [V^N_{i=1} (\lambda_k)] = \text{int}_{T_1} [1]\). Now \(1 - \text{int}_{T_1} [V^N_{i=1} (\lambda_k)] = 0\), implies that \(\cap_{T_1} (A^N_{i=1} (1 - (\lambda_k)) = 0\). Since \((\lambda_i)'s\) are pairwise fuzzy \(F_\sigma\)-sets in \((X, T)\), \((1 - \lambda_k)'s\) are pairwise fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\). Also since \((\lambda_k)'s\) are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\), by theorem 3.3, \((1 - \lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Now \(\cap_{T_1} (A^N_{i=1} (1 - (\lambda_k)) = 0\), implies that \(\cap_{T_1} (A^N_{i=1} (1 - \lambda_k)) = 0\), where \((\lambda_k)'s\) are pairwise fuzzy dense and pairwise fuzzy \(G_\delta\)-sets in \((X, T_1, T_2)\). Then, \((X, T_1, T_2)\) is not pairwise fuzzy weakly Volterra space. But, \(V^N_{i=1} (\lambda_k) = 1\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy nowhere dense sets in a fuzzy bitopological space \((X, T_1, T_2)\), implies by proposition 3.4, that \((X, T_1, T_2)\) is pairwise fuzzy almost resolvable space.

**Definition 3.4:** A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy hyperconnected space if each non-null pairwise fuzzy openset \(\lambda\) in \((X, T_1, T_2)\), is a pairwise fuzzy dense set in \((X, T_1, T_2)\). That is, \(\lambda\) is a pairwise fuzzy open set in a fuzzy bitopological space \((X, T_1, T_2)\), then \(\text{cl}_{T_1} \text{cl}_{T_2} (\lambda) = \text{cl}_{T_2} \text{cl}_{T_1} (\lambda) = 1\).

**Proposition 3.9:** If \(\wedge_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy open sets in pairwise fuzzy hyperconnected space \((X, T_1, T_2)\), then \((X, T_1, T_2)\) is a pairwise fuzzy almost resolvable space.

**Proof:** Suppose that \(\wedge_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy opensets inapairwise fuzzy hyperconnected space \((X, T_1, T_2)\). Since \((\lambda_k)'s\) are pairwise fuzzy open sets in a pairwise fuzzy hyperconnected space, \((\lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Hence we have \(\wedge_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Then, by proposition 3.3, \((X, T_1, T_2)\) is a pairwise fuzzy almost resolvable space.

**Definition 3.5** [11]: A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy P-space if every non-zero pairwise fuzzy \(G_\delta\)-set in \((X, T_1, T_2)\), is a pairwise fuzzy open set in \((X, T_1, T_2)\). That is, if \(\lambda = \wedge_{k=1}^\infty (\lambda_k)\), where \((\lambda_k)'s\) are pairwise fuzzy open sets in \((X, T_1, T_2)\), then \(\lambda\) is a pairwise fuzzy open set in \((X, T_1, T_2)\).

**Proposition 3.10:** If \(\wedge_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy \(G_\delta\)-sets in a pairwise fuzzy hyperconnected and pairwise fuzzy P-space \((X, T_1, T_2)\), then \((X, T_1, T_2)\) is a pairwise fuzzy almost resolvable space.

**Proof:** Suppose that \(\wedge_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \((\lambda_k)'s\) are pairwise fuzzy \(G_\delta\)-sets in a pairwise fuzzy hyperconnected and pairwise fuzzy P-space \((X, T_1, T_2)\). Since \((\lambda_k)'s\) are pairwise fuzzy \(G_\delta\)-sets in a pairwise fuzzy P-space, \((\lambda_k)'s\) are
pairwise fuzzy open sets in \((X, T_1, T_2)\). Again, since \(\lambda_k\)'s are pairwise fuzzy open sets in a pairwise fuzzy hyperconnected space \((X, T_1, T_2)\), \(\lambda_k\)'s are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Hence we have \(\Lambda_{k=1}^\infty (\lambda_k) = 0\), where the fuzzy sets \(\lambda_k\)'s are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Then, by proposition 3.3, \((X, T_1, T_2)\) is pairwise fuzzy almost resolvable space.

**Definition 3.6 [7]:** A fuzzy bitopological space \((X, T_1, T_2)\) is called a pairwise fuzzy resolvable space if there exists a pairwise fuzzy dense set \(\lambda\) in \((X, T_1, T_2)\) such that \(1 - \lambda\) is also a pairwise fuzzy dense set in \((X, T_1, T_2)\). That is, \(\text{cl}(T_1 \cap T_2) (1 - \lambda) = 1 = \text{cl}(T_2 \cap T_1) (1 - \lambda)\), for a pairwise fuzzy dense set \(\lambda\) in \((X, T_1, T_2)\).

**Theorem 3.4 [12]:** A fuzzy bitopological space \((X, T_1, T_2)\) is a pairwise fuzzy resolvable space if \(\forall_{k=1}^N (\lambda_k) = 1\), where the fuzzy sets \(\lambda_k\)'s are such that \(\text{int}(T_1 \cap T_2) (\lambda_k) = 0 = \text{int}(T_2 \cap T_1) (\lambda_k)\). Let \(\mu_k\)'s \(k = 1\) to \(\infty\) be fuzzy sets in \((X, T_1, T_2)\) for which \(\text{int}(T_1 \cap T_2) (\mu_k) = \text{int}(T_2 \cap T_1) (\mu_k) = 0\) and the first \(N\) fuzzy sets be \(\lambda_k\). Then, we have \(\forall_{k=1}^N (\lambda_k) \leq \forall_{k=1}^N (\mu_k)\) and hence \(1 \leq \forall_{k=1}^N (\mu_k)\). That is, \(\forall_{k=1}^N (\mu_k) = 1\), where the fuzzy sets \(\mu_k\)'s are such that \(\text{int}(T_1 \cap T_2) (\mu_k) = \text{int}(T_2 \cap T_1) (\mu_k) = 0\). Therefore \((X, T_1, T_2)\) is pairwise fuzzy almost resolvable space.

4. PAIRWISE FUZZY ALMOST RESOLVABLE SPACES

**Proposition 4.1:** If a bitopological space \((X, T_1, T_2)\) is a pairwise fuzzy almost irresolvable space and if \(\text{cl}(T_1 \cap T_2) (\lambda) = 1\) and \(\text{cl}(T_2 \cap T_1) (\lambda) = 1\), for each pairwise fuzzy dense set \(\lambda\) in \((X, T_1, T_2)\), then \((X, T_1, T_2)\) is a pairwise fuzzy second category space.

**Proof:** Let \((X, T_1, T_2)\) be a pairwise fuzzy almost irresolvable space. Then, \(\forall_{k=1}^N (\lambda_k) \neq 1\), where the fuzzy sets \(\lambda_k\)'s in \((X, T_1, T_2)\) are such that \(\text{int}(T_1 \cap T_2) (\lambda_k) = \text{int}(T_2 \cap T_1) (\lambda_k) = 0\). Now, \(1 - \text{int}(T_1 \cap T_2) (\lambda_k) = 1\) and \(1 - \text{int}(T_2 \cap T_1) (\lambda_k) = 1\). Then, \(\text{cl}(T_1 \cap T_2) (1 - \lambda_k) = 1\) and \(\text{cl}(T_2 \cap T_1) (1 - \lambda_k) = 1\) and hence \((1 - \lambda_k)\)'s are pairwise fuzzy dense sets in \((X, T_1, T_2)\). Then, by hypothesis, we have \(\text{cl}(T_1 \cap T_2) (1 - \lambda_k) = 1\) and \(\text{cl}(T_2 \cap T_1) (1 - \lambda_k) = 1\). This implies that \(\text{int}(T_1 \cap T_2) (\lambda_k) = \text{int}(T_2 \cap T_1) (\lambda_k) = 0\) and hence \(\lambda_k\)'s are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\). Thus, \(\forall_{k=1}^\infty (\lambda_k) \neq 1\), where \(\lambda_k\)'s are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\), implies that \((X, T_1, T_2)\) is a pairwise fuzzy second category space.
**Proposition 4.2:** If \( \text{cl}_{T_1} \text{int}_{T_2} (\lambda_k) = 1 \) and \( \text{cl}_{T_2} \text{int}_{T_1} (\lambda_k) = 1 \), for each pairwise fuzzy dense set \((\lambda_k)\) in a fuzzy bitopological space \((X, T_1, T_2)\) and if \( \text{cl}_{T_1} \left( \bigwedge_{k=1}^{n} (\lambda_k) \right) = 1 \), \(i = 1, 2\), then \((X, T_1, T_2)\) is a pairwise fuzzy almost irresolvable space.

**Proof:** Suppose that \( \text{cl}_{T_1} \left( \bigwedge_{k=1}^{n} (\lambda_k) \right) = 1 \), \(i = 1, 2\) where the fuzzy sets \((\lambda_k)\)'s are pairwise fuzzy dense sets in a fuzzy bitopological space \((X, T_1, T_2)\). Now \( \text{cl}_{T_1} \left( \bigwedge_{k=1}^{n} (\lambda_k) \right) = 1 \), implies that \( \text{int}_{T_1} \left( \bigvee_{k=1}^{n} (1 - \lambda_k) \right) = 0 \). By hypothesis, \( \text{cl}_{T_1} \text{int}_{T_2} (\lambda_k) = 1 \) and \( \text{cl}_{T_2} \text{int}_{T_1} (\lambda_k) = 1 \), for the pairwise fuzzy dense sets \((\lambda_k)\) in \((X, T_1, T_2)\). Then \( \text{int}_{T_1} \text{cl}_{T_2} (1 - \lambda_k) = 0 \) if \( \text{int}_{T_2} \text{cl}_{T_1} (1 - \lambda_k) \) and hence \( (1 - \lambda_k) \)'s are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\). Thus, from (A), we have that \( \text{int}_{T_1} \left( \bigvee_{k=1}^{n} (1 - \lambda_k) \right) = 0 \), where \((1 - \lambda_k) \)'s are pairwise fuzzy nowhere dense sets in \((X, T_1, T_2)\). Hence \((X, T_1, T_2)\) is a pairwise fuzzy Baire space and therefore, by theorem 3.1, \((X, T_1, T_2)\) is a pairwise fuzzy almost irresolvable space.

**REFERENCES**