Fuzzy Baire Spaces and Functions

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Abstract

In this paper some results concerning functions that preserve fuzzy Baire spaces in the context of images and preimages are obtained. Several examples are given to illustrate the concepts introduced in this paper.

KEY WORDS: Fuzzy nowhere dense set, Fuzzy first category set, Fuzzy Baire Space, Fuzzy continuous, Fuzzy semi-continuous, Somewhat Fuzzy continuous, Fuzzy open and Somewhat Fuzzy open functions.

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INTRODUCTION

The fuzzy concept has invaded almost all branches of Mathematics ever since the Introduction of fuzzy set by L. A. ZADEH [12]. The theory of fuzzy topological spaces was introduced and developed by C. L. CHANG [2]. Since then much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of Fuzzy Topology has been developed. In recent years, fuzzy topology has been found to be very useful in solving many practical problems. In [4][El. Naschie showed that the notion of fuzzy topology might be relevant to Quantum Particle Physics and Quantum Gravity in connection with String Theory and $\infty$Theory. Tang [5] used a slightly changed version of Chang’s fuzzy topological spaces to model spatial objects for GIS data bases and Structured Query Language (SQL) for GIS. In classical topology functions that preserve Baire spaces in the context of images and pre-images are studied in [3], [9], [10] and [11]. In this paper some results concerning functions that preserve fuzzy Baire spaces in the context of images and pre-images are obtained. Several examples are given to illustrate the concepts introduced in this paper.
PRELIMINARIES

By a fuzzy topological space we shall mean a non-empty set \( X \) together with a fuzzy topology \( T \) (in the sense of Chang) and denote it by \((X, T)\).

Let \( \lambda \) and \( \mu \) be any two fuzzy sets in \((X, T)\). Then we define \( \lambda \lor \mu : X \to [0, 1] \) and \( \lambda \land \mu : X \to [0, 1] \) as follows:

\[
(\lambda \lor \mu)(x) = \text{Max} \{ \lambda(x), \mu(x) \} \quad \text{and} \quad (\lambda \land \mu)(x) = \text{Min} \{ \lambda(x), \mu(x) \}.
\]

Let \((X, T)\) be any fuzzy topological space and \( \lambda \) be any fuzzy set in \((X, T)\). We define \( \text{Cl} (\lambda) = \bigwedge \{ \mu / \lambda \leq \mu, \; 1-\mu \in T \} \) and \( \text{int} (\lambda) = \bigvee \{ \mu / \mu \leq \lambda, \; \mu \in T \} \). For any fuzzy set \( \lambda \) in a fuzzy topological space \((X, T)\), it is easy to see that \( 1 - \text{cl} (\lambda) = \text{int} (1 - \lambda) \) and \( 1 - \text{int} (\lambda) = \text{cl} (1 - \lambda) \).

Let \((X, T)\) and \((Y, S)\) be any two fuzzy topological spaces. Let \( f \) be a function from the fuzzy topological space \((X, T)\) to the fuzzy topological space \((Y, S)\). Let \( \lambda \) be a fuzzy set in \((Y, S)\). The inverse image of \( \lambda \) under \( f \) written as \( f^{-1}(\lambda) \) is the fuzzy set in \((X, T)\) defined by \( f^{-1}(\lambda)(x) = \lambda(f(x)) \) for all \( x \in X \). Also the image of \( \lambda \) in \((X, T)\) under \( f \) written as \( f(\lambda) \) is the fuzzy set in \((Y, S)\) defined by

\[
f(\lambda)(y) = \begin{cases} 
\sup \{ \lambda(x) \; | \; x \in f^{-1}(y) \} & \text{if } f^{-1}(y) \text{ is non-empty} \\
0 & \text{otherwise}
\end{cases}
\]

for each \( y \in Y \).

**Lemma [2]:** Let \( f : (X, T) \to (Y, S) \) be a mapping. For fuzzy sets \( \lambda \) and \( \mu \) of \((X, T)\) and \((Y, S)\) respectively, the following statements hold.

- \( ff^{-1}(\mu) \leq \mu \)
- \( f^{-1}f(\lambda) \geq \lambda \)
- \( f(1 - \lambda) \geq 1 - f(\lambda) \)
- \( f^{-1}(1 - \mu) = 1 - f^{-1}(\mu) \)
- If \( f \) is injective, then \( f^{-1} f(\lambda) = \lambda \)
- If \( f \) is surjective, then \( f f^{-1}(\mu) = \mu \)
- If \( f \) is bijective, then \( f(1 - \lambda) = 1 - f(\lambda) \)

A fuzzy set \( \lambda \) in a fuzzy topological space \((X, T)\) is called a fuzzy dense set if there exists no fuzzy closed set \( \mu \) in \((X, T)\) such that \( \lambda \leq \mu < 1 \).

A fuzzy set \( \lambda \) in a fuzzy topological space \((X, T)\) is called a fuzzy nowhere dense set if there exists no non-zero fuzzy open set \( \mu \) in \((X, T)\) such that \( \mu < \text{cl} (\lambda) \). That is, \( \text{int} \text{cl} (\lambda) = 0 \).

A fuzzy set \( \lambda \) in a fuzzy topological space \((X, T)\) is called a fuzzy semi-open if \( \lambda \leq \text{cl} \text{int} (\lambda) \). The complement of \( \lambda \) in \((X, T)\) is called fuzzy semi-closed.

A fuzzy set \( \lambda \) in a fuzzy topological space \((X, T)\) is called fuzzy first category if \( \lambda = \bigvee_{i=1}^{\infty} (\lambda_i) \) where \( \lambda_i \)'s are fuzzy nowhere dense sets in \((X, T)\). Any other fuzzy
set in \((X, T)\) is said to be of secondcategory[6]. Let \(\lambda\) be a fuzzy first category set in \((X, T)\).

Then \(1 - \lambda\) is called a fuzzy residual set in \((X, T)\) [8].

**FUZZY Baire Spaces and Functions**

**Definition 3. 1:** Let \((X, T)\) be a fuzzy topological space. Then \((X, T)\) is called a fuzzy Baire Space if \(\text{int} (\bigcap_{i=1}^{n} \lambda_i) = 0\) where \(\lambda_i's\) are fuzzy nowhere dense sets in \((X, T)\) [8].

**Definition 3. 2:** A function \(f: (X, T) \to (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is said to be fuzzy open if the image of every fuzzy open set in \((X, T)\) is fuzzy open in \((Y, S)\) [1].

**Definition 3. 3:** A function \(f: (X, T) \to (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is called fuzzy continuous if \(f^{-1}(\lambda)\) is fuzzy open in \((X, T)\) for each fuzzy open set \(\lambda\) in \((Y, S)\) [2].

**Definition 3. 4:** A function \(f: (X, T) \to (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is called fuzzy semi-continuous if \(f^{-1}(\lambda)\) is fuzzy semi-open in \((X, T)\) for each fuzzy open set \(\lambda\) in \((Y, S)\) [1].

**Definition 3. 5:** A function \(f: (X, T) \to (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is called somewhat fuzzy continuous if \(\lambda \in S\) and \(f^{-1}(\lambda) \neq 0\) implies that there exist a fuzzy open set \(\mu\) in \((X, T)\) such that \(\mu \neq 0\) and \(\mu \leq f^{-1}(\lambda)\) [6].

**Definition 3. 6:** A function \(f: (X, T) \to (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is called somewhat fuzzy open if \(\lambda \in T\) and \(\lambda \neq 0\) implies that there exists a fuzzy open set \(\mu\) in \((Y, S)\) such that \(\mu \neq 0\) and \(\mu \leq f(\lambda)\) [6].

**Theorem 3. 1**[8]: Let \((X, T)\) be a fuzzy topological space. Then the following are equivalent:

1. \((X, T)\) is a fuzzy Baire space.
2. \(\text{Int}(\lambda) = 0\) for every fuzzy first category set \(\lambda\) in \((X, T)\).
3. \(\text{cl}(\mu) = 1\) for every fuzzy residual set \(\mu\) in \((X, T)\).

Let \(f\) be a function from the fuzzy topological space \((X, T)\) to the fuzzy topological space \((Y, S)\). Under what Conditions on “\(f\)” may we assert that if \((X, T)\) is a fuzzy Baire Space, then \((Y, S)\) is a fuzzy Baire Space? It may be noticed that the fuzzy continuous image of a fuzzy Baire Space may fail to be a fuzzy Baire Space. For, consider the following example.
Example 3.1: Let X = {a, b}. The fuzzy sets λ, μ and ν are defined on X as follows:

- λ: X → [0, 1] is defined as λ(a) = 0.5; λ(b) = 0.7.
- μ: X → [0, 1] is defined as μ(a) = 0.8; μ(b) = 0.4.
- ν: X → [0, 1] is defined as ν(a) = 0.2; ν(b) = 0.6.

Then, T = {0, λ, μ, ν, λ ∨ μ, λ ∧ μ, μ ∨ ν, 1} and S = {0, λ, μ, λ ∨ μ, λ ∧ μ, 1} are fuzzy topologies on X. Now the fuzzynownhere dense sets in (X, T) are 1 − λ and 1 − (λ ∨ μ) and {1 − λ} ∨ (1 − (λ ∨ μ)) = 1 − λ and int (1 − λ) = 0. Hence (X, T) is a fuzzy Baire Space. Define a function f: (X, T) → (X, S) by f(a) = a and f(b) = b. Clearly f is not a fuzzy Baire Space.

Example 3.2: Let X = {a, b, c}. The fuzzy sets λ, μ and ν are defined on X as follows:

- λ: X → [0, 1] is defined as λ(a) = 0.6; λ(b) = 0.4; λ(c) = 0.3.
- μ: X → [0, 1] is defined as μ(a) = 0.5; μ(b) = 0.7; μ(c) = 0.2.
- ν: X → [0, 1] is defined as ν(a) = 0.7; ν(b) = 0.5; ν(c) = 0.6.

Then, T = {0, λ, 1} and S = {0, λ, μ, ν, λ ∨ μ, λ ∧ μ, μ ∨ ν, 1} are fuzzy topologies on X. Now the fuzzynownhere dense sets in (X, T) are 1 − λ, 1 − ν and {1 − λ} ∨ (1 − ν) = 1 − λ and int (1 − λ) = 0. Hence (X, T) is a fuzzy Baire Space. Define a function f: (X, T) → (X, S) by f(a) = a and f(b) = b and f(c) = c. Clearly f is a fuzzy Baire Space.

Proposition 3.1: If a function f: (X, T) → (Y, S) from a fuzzy topological space (X, T) into another fuzzy topological space (Y, S) is fuzzy continuous, 1-1, onto and fuzzy open function, then for any fuzzy set λ in (X, T) int cl [f(λ)] ≤ f int (cl (λ)).

Proof: Let λ be a fuzzy set in (X, T). Then λ ≤ cl (λ) in (X, T) implies that 1 − cl (λ) ≤ (1 − λ). Then f(1 − cl (λ)) ≤ f(1 − λ). Since f is a fuzzy open function and 1 − cl (λ) is a fuzzy open set in (X, T), f(1 − cl (λ)) is a fuzzy open set in (Y, S) such that f(1 − cl (λ)) ≤ f(1 − λ). But int f(1 − λ) is the largest fuzzy open in (Y, S) such that int f(1 − λ) ≤ f(1 − λ). Hence we have that f(1 − cl (λ)) ≤ int f(1 − λ). Since f is 1 − 1 and onto, f(1 − λ) = 1 − f(λ). Hence (1 − f [cl (λ)]) ≤ int [1
— \( f(\lambda) \) which implies that \((1 - f(\lambda)) \leq (1 - \text{cl}(\lambda)) \). Then \( \text{cl}(\lambda) \leq f(\lambda) \), which implies that \( \text{int} \{ f(\lambda) \} \leq \text{int} \{ \text{cl}(\lambda) \} \). Therefore we have \( f^{-1}(\text{int} \{ f(\lambda) \}) = \text{int} \{ f^{-1}(\text{int} \{ f(\lambda) \}) \} \). Now \( \text{int} \{ f(\lambda) \} \) is an fuzzy open set in \((Y, S)\). Since \( f \) is fuzzy continuous \( f^{-1}(\text{int} \{ f(\lambda) \}) \) is a fuzzy open set in \((X, T)\).

Hence \( f^{-1}(\text{int} \{ f(\lambda) \}) = \text{int} \{ f^{-1}(\text{int} \{ f(\lambda) \}) \} \leq \text{int} \{ f^{-1}(\text{cl}(\lambda)) \} \leq \text{int} \{ \lambda \} \) \( \text{since} f \) is 1-1. That is, \( f^{-1}(\text{int} \{ f(\lambda) \}) \leq \text{int} \{ \lambda \} \). 

From (1) and (2), we have \( f^{-1}(\text{int} \{ f(\lambda) \}) \leq \text{int} \{ \lambda \} \) which implies that \( f^{-1}(\text{int} \{ f(\lambda) \}) \leq \text{int} \{ \lambda \} \). Since \( f \) is onto, we have \( \text{int} \{ f(\lambda) \} \leq f(\lambda) \).

**Proposition 3.2:** If \( f(S) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is fuzzy continuous and fuzzy open function, then for any fuzzy set \( \lambda \) in \((X, T)\), \( \text{int} \{ f(\lambda) \} \leq \text{int} \{ f(\lambda) \} \).

**Proof:** Let \( \lambda \) be any fuzzy set in \((X, T)\). Then \( \text{int} \{ f(\lambda) \} \) is a fuzzy open set in \((X, T)\).

Since \( f \) is an fuzzy open function, \( f(\text{int} \{ \lambda \}) \) is a fuzzy open set in \((Y, S)\).

Now \( f(\text{int} \{ \lambda \}) \leq f(\text{int} \{ f(\lambda) \}) \). Since \( f \) is fuzzy continuous, \( f(\text{int} \{ \lambda \}) \leq \text{int} \{ f(\lambda) \} \).

Hence we have \( f(\text{int} \{ \lambda \}) \leq \text{int} \{ f(\lambda) \} \). But \( \text{int} \{ f(\lambda) \} \leq \text{int} \{ f(\lambda) \} \).

Therefore, we have \( f(\text{int} \{ \lambda \}) \leq \text{int} \{ f(\lambda) \} \).

**Proposition 3.3:** If \( f(S) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is fuzzy continuous, fuzzy open, 1-1 and onto function, then for any fuzzy set \( \lambda \) in \((X, T)\), \( f(\text{int} \{ \lambda \}) \) = \( \text{int} \{ f(\lambda) \} \).

**Proof:** Let \( \lambda \) be any fuzzy set in \((X, T)\). Since \( f \) is fuzzy continuous, \( 1-1 \), onto and fuzzy open function, by proposition 3.1, we have \( \text{int} \{ f(\lambda) \} \leq f(\text{int} \{ f(\lambda) \}) \).

Since \( f \) is a fuzzy continuous and a fuzzy open function, by proposition 3.2, we have \( f(\text{int} \{ \lambda \}) \leq \text{int} \{ f(\lambda) \} \).

From (1) and (2), we have \( f(\text{int} \{ \lambda \}) \).

**Proposition 3.4:** Let \( f(S) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) be fuzzy continuous, fuzzy open, \( 1 \)-1 and onto function. Then, for any fuzzy set \( \lambda \) in \((X, T)\), \( \lambda \) is a fuzzy nowhere dense set in \((X, T)\) if and only if \( f(\lambda) \) is a fuzzy nowhere dense set in \((Y, S)\).

**Proof:** Let \( \lambda \) be a fuzzy nowhere dense set in \((X, T)\). Then, \( \text{cl}(\lambda) = 0 \). Since \( f \) is fuzzy continuous, fuzzy open, \( 1 \)-1 and onto function, by proposition 3.3, we have \( f(\text{int} \{ \lambda \}) \) \( = \text{int} \{ f(\lambda) \} \). Then, \( \text{int} \{ f(\lambda) \} = f(0) = 0 \). Hence \( f(\lambda) \) is a fuzzy nowhere dense set in \((Y, S)\).

Conversely, let \( f(\lambda) \) be a fuzzy nowhere dense set in \((Y, S)\).

Then, \( \text{int} \{ f(\lambda) \} = 0 \). Hence \( \text{int} \{ \text{cl}(\lambda) \} = \text{int} \{ f(\lambda) \} \) implies that \( f(\text{int} \{ \lambda \}) \) \( = 0 \).

Therefore \( f^{-1}(\text{int} \{ f(\lambda) \}) = f^{-1}(0) = 0 \). Since \( f \) is \( 1 \)-1, \( \text{int} \{ f(\lambda) \} = 0 \). Hence \( \lambda \) is a fuzzy nowhere dense set in \((X, T)\).
Proposition 3.5: Let the function \( f: (X, T) \to (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) be fuzzy continuous, fuzzy open, 1-1 and onto function. If \((X, T)\) is a fuzzy Baire Space, then \((Y, S)\) is a fuzzy Baire Space.

Proof: Let \( \lambda \) be a fuzzy first category set in \((Y, S)\). Then \( \lambda = \bigvee_{i=1}^{\infty} (\lambda_i) \) where \( \lambda_i \)'s are fuzzy nowhere dense sets in \((Y, S)\). Since \( f \) is onto, \( f f^{-1}((\lambda_i)) = (\lambda_i) \) and hence \( \lambda = \bigvee_{i=1}^{\infty} f f^{-1}((\lambda_i)) \) and therefore, \( f[f^{-1}((\lambda_i))] \) are fuzzynowhere dense sets in \((Y, S)\). Since \( f \) is fuzzy continuous, fuzzy open, 1-1 and onto function, by proposition 3.4, we have \( f^{-1}((\lambda_i)) \) is a fuzzy nowhere dense set in \((X, T)\). Let \( \mu = \bigvee_{i=1}^{\infty} f^{-1}((\lambda_i)) \). Then \( \mu \) is a fuzzy first category set in \((X, T)\). Since \((X, T)\) is a fuzzy Baire Space, \( \mu(\lambda) = 0 \). Then we have \( \mu(f^{-1}(\lambda_i)) = 0 \) which implies that \( \mu(f^{-1}((\bigvee_{i=1}^{\infty} (\lambda_i)))) = 0 \). Then, we have \( f(\mu(f^{-1}((\bigvee_{i=1}^{\infty} (\lambda_i)))) = f(0) = 0 \). Since \( f \) is a continuous function, we have \( f^{-1}(\mu(f^{-1}((\bigvee_{i=1}^{\infty} (\lambda_i)))) = \mu(f^{-1}(\bigvee_{i=1}^{\infty} (\lambda_i))) = \mu(\bigvee_{i=1}^{\infty} (\lambda_i)) = 0 \).

Since \( \mu \) is a fuzzy first category set, \( \mu(\lambda) = 0 \). Hence by theorem 3, 1., \((Y, S)\) is a fuzzy Baire Space.

Proposition 3.6: If the function \( f: (X, T) \to (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is fuzzy semi-continuous and somewhat fuzzy open function and if \( \lambda \) is a fuzzy nowhere dense set in \((Y, S)\), then \( f^{-1}(\lambda) \) is a fuzzy nowhere dense set in \((X, T)\).

Proof: Let \( \lambda \) be a fuzzy nowhere dense set in \((Y, S)\). Then, \( 1 - \text{cl} (\lambda) \) is a fuzzy dense set in \((Y, S)\) [8]. Since \( f \) is somewhat fuzzy open, \( f^{-1}(1 - \text{cl} (\lambda)) \) is a fuzzy dense set in \((X, T)\). That is, \( \text{cl} (\bigvee_{i=1}^{\infty} (1 - \text{cl} (\lambda) )) = 1 \) \( \ldots \ldots \ldots (1) \). Now \( 1 - \text{cl} (\lambda) \) is a fuzzy open set in \((Y, S)\). Since \( f \) is fuzzy semi-continuous fuzzy \( f^{-1}(1 - \text{cl} (\lambda)) \) is a fuzzy semi-open set in \((X, T)\). Then \( f^{-1}(1 - \text{cl} (\lambda)) \leq \text{cl} \text{int} f^{-1}(1 - \text{cl} (\lambda)) \) implies that \( f^{-1}(1 - \text{cl} (\lambda)) \leq \text{cl} \text{int} f^{-1}(1 - \text{cl} (\lambda)) \). Hence \( f^{-1}(1 - \text{cl} (\lambda)) \leq \text{cl} \text{int} f^{-1}(1 - \text{cl} (\lambda)) \) \( \bigvee (\lambda) \). Then, from (1), \( 1 \leq \text{cl} \text{int} f^{-1}(1 - \text{cl} (\lambda)) \). That is, \( \text{cl} \text{int} f^{-1}(1 - \text{cl} (\lambda)) \) \( = 1 \) which implies that \( 1 - \text{int} \text{cl} f^{-1}(1 - \text{cl} (\lambda)) \) \( = 1 \) and hence \( \text{int} \text{cl} f^{-1}(1 - \text{cl} (\lambda)) \) \( = 0 \). Therefore we have \( \text{int} \text{cl} f^{-1}(\lambda) = 0 \). Hence \( f^{-1}(\lambda) \) is a fuzzy nowhere dense set in \((X, T)\).

Proposition 3.7: If the function \( f: (X, T) \to (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is fuzzy semi-continuous and somewhat fuzzy open function and if \( \lambda \) is a fuzzy first category set in \((Y, S)\), then \( f^{-1}(\lambda) \) is a fuzzy first category set in \((X, T)\).

Proof: Let \( \lambda \) be a fuzzy first category set in \((Y, S)\). Then \( \lambda = \bigvee_{i=1}^{\infty} (\lambda_i) \) where \( \lambda_i \)'s are fuzzy nowhere dense sets in \((Y, S)\). Since \( f \) is a fuzzy semi-continuous and
somewhat fuzzy open function, by proposition 3. 6, \( f^{-1} (\lambda) \) is a fuzzynowhere dense set in \((X, T)\) for each fuzzynowhere dense set \(\lambda_i\) in \((Y, S)\). Then we have \( f^{-1} (\lambda) = f^{-1} \left( \bigvee_{i=1}^{\infty} (\lambda_i) \right) = \left( \bigvee_{i=1}^{\infty} f^{-1} (\lambda_i) \right) \). Since \( f^{-1} (\lambda_i) \)’s are fuzzy nowhere dense sets in \((X, T)\), \( f^{-1} (\lambda) \) is a fuzzy first category set in \((X, T)\).

**Proposition 3. 8:** If the function \( f: (X, T) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is fuzzy semi-continuous and somewhat fuzzy open and somewhat fuzzy continuous function and if \((X, T)\) is a fuzzy Baire Space \((Y, S)\), then \((Y, S)\) is a fuzzy Baire Space.

**Proof:** Let \(\lambda\) be a fuzzy first category set in \((Y, S)\). Then, by proposition 3. 7 \( f^{-1} (\lambda) \) is a fuzzy first category set in \((X, T)\). Since \((X, T)\) is a fuzzy Baire Space \(\text{int} (f^{-1} (\lambda)) = 0\). We now claim that \(\text{int} (\lambda) = 0\). Suppose \(\text{int} (\lambda) \neq 0\). Then there exists a non-zero fuzzy open set \(\mu\) in \((Y, S)\) such that \(\mu \leq \lambda\). Then \( f^{-1} (\mu) \leq f^{-1} (\lambda) \).

Since \(\mu\) is a non-zero fuzzy open set \(\mu\) in \((Y, S)\) and \( f^{-1} (\mu) \neq 0 \) and since \(f\) is a somewhat fuzzy continuous function, there exists a non-zero fuzzy open set \(v\) in \((X, T)\) such that \(v \leq f^{-1} (\mu) \). Hence \(v \leq f^{-1} (\mu) \leq f^{-1} (\lambda) \). That is, \(\text{int} (f^{-1} (\lambda)) \neq 0\), which is a contradiction to \(\text{int} (f^{-1} (\lambda)) = 0\). Hence we must have \(\text{int} (\lambda) = 0\) and therefore \((Y, S)\) is a fuzzy Baire Space.

**Theorem 3. 2 [7]:** If the function \( f: (X, T) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is somewhat fuzzy continuous and 1-1 and onto and if \(\text{cl} (\text{int} (\lambda)) = 1\) for any non-zero fuzzy set \(\lambda\) in \((X, T)\) then \(\text{cl} [\text{int} (f(\lambda))] = 1\) in \((Y, S)\).

**Proposition 3. 9:** If the function \( f: (X, T) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is somewhat fuzzy continuous, 1-1 and onto function and if \(\lambda\) is a fuzzy nowhere dense set in \((X, T)\) for any fuzzy set \(\lambda\) in \((X, T)\), then \(f(\lambda)\) is a fuzzy nowhere dense set in \((Y, S)\).

**Proof:** Let \(\lambda\) be a fuzzy nowhere dense set in \((X, T)\). Then, \(\text{cl} (\lambda) = 0\). Then \(1 = \text{int} \text{cl} (\lambda) = 1\) which implies that \(\text{cl} \text{int} (1 - (\lambda)) = 1\). Since \(f\) is a somewhat fuzzy continuous function, by theorem 3. 2, \(\text{cl} \text{int} f [ (1 - (\lambda)) ] = 1\). Since \(f\) is 1-1 and onto, \(f(1 - \lambda) = 1 - f(\lambda)\). Then \(\text{cl} \text{int} [ (1 - f(\lambda)) ] = 1\) and hence \(1 = \text{int} \text{cl} f(\lambda) = 1\). That is, \(\text{cl} (f(\lambda)) = 0\). Hence \(f(\lambda)\) is a fuzzy nowhere dense set in \((Y, S)\).

**Proposition 3. 10:** If the function \( f: (X, T) \rightarrow (Y, S) \) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is somewhat fuzzy continuous, 1-1 and onto function and if \(\lambda\) is a fuzzy first category set in \((X, T)\) then \(f(\lambda)\) is a fuzzy first category set in \((Y, S)\).

**Proof:** Let \(\lambda\) be a fuzzy first category set in \((X, T)\). Then \(\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)\) where \(\lambda_i\)’s are fuzzy nowhere dense sets in \((X, T)\). Then \(f(\lambda) = f(\bigvee_{i=1}^{\infty} (\lambda_i))\). Since \(f\) is 1-1 and onto, \(f(\lambda) = \bigvee_{i=1}^{\infty} f(\lambda_i)\). Since \(f\) is a somewhat fuzzy continuous and \(\lambda\) is a
fuzzy nowhere dense set in \((X, T)\), by proposition 3. 9, \(f (\mathcal{L}_4)\) is a fuzzynowhere dense set in \((Y, S)\). Therefore \(f (\mathcal{L})\) is a fuzzy first category setin \((Y, S)\).

**Proposition 3. 11:** If the function \(f: (X, T) \rightarrow (Y, S)\) from a fuzzy topological space \((X, T)\) into another fuzzy topological space \((Y, S)\) is a somewhat fuzzycontinuous, somewhat fuzzy open and 1-1 and onto function and if \((Y, S)\) is a fuzzy Baire Space \((Y, S)\), then \((X, T)\) is a fuzzy Baire Space.

**Proof:** Let \(\lambda\) be a fuzzy first category set in \((X, T)\). Since \(f\) is a fuzzy somewhat continuous, 1-1 and onto function, by proposition 3. 10, \(f (\mathcal{L})\) is a fuzzy first category set in \((Y, S)\). Since \((Y, S)\) is a fuzzy Baire space, we have \(\text{int} [f (\mathcal{L})] = 0\). We now claim that \(\text{int} (\mathcal{L}) = 0\). Suppose \(\text{int} (\mathcal{L}) \neq 0\). Then there exists a non-zero fuzzy open set \(\mu\) in \((X, T)\) such that \(\mu \leq \mathcal{L}\). Then \(f (\mu) \leq f (\mathcal{L})\). Since \(\mu\) is a non-zero fuzzy open set in \((X, T)\) and \(f (\mu) \neq 0\) and \(f\) is a somewhat fuzzy open function, there exists a non-zero fuzzy open set \(\nu\) in \((Y, S)\) such that \(\nu \leq f (\mu)\). Hence \(\nu \leq f (\mu) \leq f (\mathcal{L})\). That is, \(\text{int} [f (\mathcal{L})] \neq 0\), which is a contradiction to \(\text{int} [f (\mathcal{L})] = 0\). Hence \(\text{int} (\mathcal{L}) = 0\) for a fuzzy first category set in \((X, T)\) and therefore, by theorem 3. 1, \((X, T)\) is a fuzzy Baire space.

**REFERENCES**


