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The Application of the Thought of Transformation and Conversion in Solving Mathematical Problems in Senior High School—take Multiple Solutions to One Problem as an Example

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Abstract

High school students have a wide range of knowledge, strong logic, extensive connections, and a huge system. To learn high school mathematics well, we need to have a variety of mathematical abilities. Transformation and transformation is the most basic ability that high school students must have, and it is also the core of studying and solving mathematical problems The application of multiple solutions in solving mathematical problems can also exercise the sensitivity of students' thinking, integrate the learned knowledge, and meet the requirements of transformation and transformation. It is an effective means to cultivate students' transformation and transformation ability by using multiple solutions to one problem.

Keywords: high school mathematics; Conversion and transformation; Multiple solutions to one problem

1 Introduction

The idea of transformation is to transform a problem into a problem that is easy to solve through a certain transformation process when studying and solving mathematical problems. Transformation always transforms abstract into concrete, complex into simple, unknown into known, and quickly and reasonably searches for and selects ways and methods to solve problems through transformation.

2 Embodiment of the Thought of Transformation and Conversion in Multiple Solutions to One Problem

To solve mathematical problems by using the thought of transformation and conversion is to use the thought of transformation to reduce one problem to another that is easy to solve under the mathematical knowledge system, and transform unfamiliar problems into familiar ones. In short, it is to make problems familiar, simple and harmonious.

Example 1: If x、y □ R is known, and $\sqrt{x+2}+\sqrt{y-5}=6$,then the value range of $x+2y$ is.

[Analysis] The objective of observation and solution is to find a value range containing two unknowns. The conventional approach is to use the function idea to convert two unknowns into one unknown, simplify the integral formula, and reduce it to the value range of a single function.

Method 1: Order $\sqrt{x+2}=t,\sqrt{y-5}=6-t.$

Then, $x=t^2-2, y=(6-t)^2+5.$ In which $t\in[0,6].$

$\therefore x+2y=t^2-2+2[(6-t)^2+5].$

$\therefore x+2y=3(t-4)^2+32.$ The minimum value 32 is obtained when t=4, and the maximum value 80 is obtained when t=0.

To sum up, the value range of $x+2y$ is [32, 80].

By using this method, we can transform the integer into the value range of the quadratic function of one variable, and transform the unknown into the familiar mathematical knowledge. Teachers should point out the mathematical ideas and methods used in teaching, which is more conducive to the broadening of students' thinking.

Method 2: Order $\sqrt{x+2}=u,\sqrt{y-5}=v$,then $u+v=6.$ In which u、$v\geq 0.$

$\therefore x+2y=u^2-2+2(v^2+5)=u^2+2v^2+8.$

By introducing $v=6-u$ into the above equation, we get a unary quadratic function about u, $x+2y=3u^2-24u+80.$ In which $u\in[0,6].$

In the same way, we can find that the value range of $x+2y$ is [32, 80].

The solution ideas of Method 2 and Method 1 are the same. Use the learned knowledge to get familiar with the problem, and finally convert it into a quadratic function with one variable to solve the range; The difference is that method one is more direct, the transformation is unified, and the thinking is clearer.

[Analysis] According to the structural relationship of the condition $\sqrt{x+2}+\sqrt{y-5}=6$, if the formula is regarded as a whole, the condition can be transformed into the sum of two "numbers", which is 6. In connection with our definition of the middle term of the equal difference sequence, "if the quantity relationship of three numbers 2A=a+b, then three numbers a, A, b are equal difference sequence", then $\sqrt{x+2},3,\sqrt{y-5}$ can be regarded as three items in the equal difference sequence, and the knowledge of the equal difference sequence can be used to solve the problem.

Method 3: Let $\sqrt{x+2},3,\sqrt{y-5}$ be an equal difference sequence. According to the knowledge of equal difference sequence, if the tolerance of equal difference sequence is d, then: $\sqrt{x+2}=3-d,\sqrt{y-5}=3+d$.

The original formula can be changed into: $x+2y=(3-d)^2-2+2[(3+d)^2+5]$.

The simplified formula is: $x+2y=3(d+1)^2+32.$ The minimum value 32 is obtained when d=- 1, and the maximum value 80 is obtained when d=3.

To sum up, the value range of $x+2y$ is [32, 80].

[Analysis] Study the formal structure of the condition $\sqrt{x+2}+\sqrt{y-5}=6$. The sum of the two formulas is a constant. It is associated with the knowledge of trigonometric functions learned, including "$\sin^2\theta+\cos^2\theta=1$", which can be converted into a similar form to achieve the purpose of solving.

Method 4: Order $\sqrt{x+2}=6\cos^2\theta,\sqrt{y-5}=6\sin^2\theta.$

Then the original formula can be changed to:

$x+2y=36\cos^4\theta-2+2(36\sin^4+5).$

that is $x+2y=36\cos^4\theta+72\sin^4\theta+8.$

Then, use $\sin^2\theta+\cos^2\theta=1$ to replace, and replace the above equation with a functional equation containing only one unknown quantity.

as follows: $x+2y=36(1-\sin^2\theta)^2+72\sin^4\theta+8.$

that is $x+2y=36(3\sin^4\theta-2\sin^2\theta)+44.$ □

Order $\sin^2\theta=t,t\in[0,1].$ Brought into the above formula:

$x+2y=108(t-\frac{1}{3})^2+32.$

This formula obtains the minimum value of 32 at $t=\frac{1}{3}$ and the maximum value of 80 at t=1.

To sum up, the value range of $x+2y$ is [32, 80].

[Analysis] The observation condition is that the sum of the two radical formulas $\sqrt{x+2}+\sqrt{y-5}=6$ is a fixed value and contains two unknowns. This formula is too complex. How can we simplify to get a functional formula containing an unknown number? This uses the elimination method to construct a dual equation and add two equations to eliminate an unknown quantity.

Method 5: $\sqrt{x+2}+\sqrt{y-5}=6,$

Order $\sqrt{x+2}-\sqrt{y-5}=t.$ Add the two equations to get:

$2\sqrt{x+2}=6+t.$ $t\in[-6,6].$

Then both radicals can be expressed by functions containing t, which achieves the goal of elimination, as follows:

$\sqrt{x+2}=\dfrac{6+t}{2},\sqrt{y-5}=\dfrac{6-t}{2}.$ Then $x+2y=\dfrac{(6+t)^2}{4}-2+2[\dfrac{(6-t)^2}{4}+5].$

The simplified formula is: $x+2y=\dfrac{3}{4}(t-2)^2+32.$ The minimum value 32 is obtained when t=2, and the maximum value 80 is obtained when t=-6.

To sum up, the value range of $x+2y$ is [32, 80].

3 The Role of the Thought of Transformation and Conversion in Solving Mathematical Problems

3.1 The thought of transformation and conversion can provide students with thinking strategies to solve problems

Follow the principles of familiarity, simplification and harmony when applying the thought of transformation and conversion, transform the unknown problems into known solvable problems, and contact with the old knowledge to find solutions. The thought of transformation provides students with the thinking strategy of transforming difficult into easy and transforming new into old in solving problems. The general steps are to contact the conditions given by the topic, find the relationship between the conditions and the old knowledge, analyze the goal of transformation, and use the corresponding knowledge to explore the direction of transformation, so as to achieve the goal of transformation.

Example 2: Find the value range of function $y=\sqrt{x-5}+\sqrt{24-3x}$.

Observe the structure of this formula: the addition of two formulas equals to a "value", similar to the structure "$a+b=2c$". In this way, we can solve the problem by using the familiar knowledge of the middle term of the equal difference sequence.

Solution: treat $\sqrt{x-5},\dfrac{y}{2},\sqrt{24-3x}$ as an equal difference sequence.

Order $\sqrt{x-5}=\frac{y}{2}-d$, $\sqrt{24-3x}=\frac{y}{2}+d$, $|d|\le\frac{y}{2}$.

To simplify the elimination of two equations: $9=3(\frac{y}{2}-d)^2+(\frac{y}{2}+d)^2$.

Then, $9=4(d-\frac{1}{4}y)^2+\frac{3}{4}y^2(*)$.

According to the value range of d, the following can be obtained: on the right side of the formula(*), the minimum value $\frac{3}{4}y^2$ is obtained at $d=\frac{y}{4}$ and the maximum value $3y^2$ is obtained at $d=-\frac{y}{2}$.

$\therefore\frac{3}{4}y^2\le 9\le 3y^2$

$\therefore\sqrt{3}\le y\le 2\sqrt{3}.$

This problem can also be solved by the construction of dual formula or substitution method, which is consistent with the solution of Example 1.

3.2 Transformation and conversion ideas provide a bridge between different knowledge

In high school mathematics, the thought of transformation and conversion can strengthen the connection between knowledge, blend knowledge and effectively promote knowledge transfer. In Example 1, the problem of finding the value range in a function field is connected with the knowledge of equal difference sequence and trigonometric function through multiple solutions. By studying the structure of the formula, we can associate it with similar knowledge structures and build a bridge between knowledge, which effectively helps the transfer of knowledge and is more conducive to the construction of students' knowledge system.

4 Summary

The thought of transformation and conversion plays a very important role in the learning of high school mathematics, and is the basic method to solve problems. It is more beneficial to cultivate students' problem-solving strategies, clarifying the links between knowledge and forming a knowledge system to reflect the thought of transformation with multiple solutions to one problem.

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