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Thinking Analysis in Middle School Mathematics Problem Solving Process- Taking Circle Synthesis as an Example

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Abstract

The round comprehensive problem is a common problem in middle school mathematics. Most students have certain ideas about this problem, but in the process of solving the problem, the idea is not clear, lack of order, and the blindness of solving the problem is larger. Therefore, faced with this problem type, it can only solve part of the problem, and cannot form the ability to solve general problems. Therefore, it is necessary for teachers to analyze the problems and present the simple links in front of students, explain the thinking process of exploring the way to solve problems clearly and systematically, and improve the ability to analyze and solve problems.

Key words: problem-solving thinking process; Process analysis; Synthesis of circles; Middle school mathematics

At present, in the problem solving teaching, teachers commonly used teaching method is to show one or several correct problem solving methods to students; Then, through the variation exercise, tell the students "this is the correct way to solve this type of math problem". In fact, what the students have grasped is only the "routine" to solve the problem, and the idea of how to find the correct solution to the problem is vague and blind. When it comes to comprehensive math problems, I don't know how to start. So, on the one hand, it requires our teachers to attach great importance to the thinking analysis of problem solving process in the teaching of problem solving. The analysis of problem solving process is the manifestation of the thinking process of problem solving and the simplification of problem understanding. Help students to understand the problem deeply, but also develop the ability to analyze and solve problems. Therefore, focusing on the thinking analysis of the problem solving process is an effective way to solve problems [1-2].

This paper analyzes the thinking process of solving a comprehensive question about circle in the high school entrance examination in Chengdu, Sichuan province in 2021. It can help students deepen their understanding of basic knowledge and realize the internal relationship between various parts of mathematical knowledge. Experience how all kinds of basic mathematical methods organically cooperate, so as to sum up and induce methods and ideas with universal significance.

1 PRESENTATION OF THE CASE

1.1 Title: As shown in Figure 1, AB is the diameter of $\odot O$, $\odot O$ is the upper point, connection AC, BC, D is the point on the AB extension line, connection CD, and $\angle BCD = \angle A$.

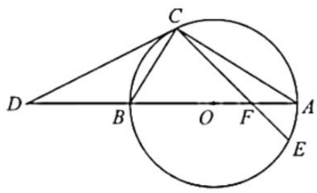

Figure 1

(1) Verify: CD is tangent;

(2) If the radius of is $\sqrt{5}$, the area of is $2\sqrt{5}$,Long for CD

(3) under the condition of (2), E is a point above $\odot O$, Connect CE intersection segment OA at point F, if $\dfrac{EF}{CF} = \dfrac{1}{2}$, take the length of BF.

1.2 Details: (1) Proof: Connect OC (FIG.2) :

$\because AB$ is $\odot O$ diameter, $\therefore \angle ACB = 90°$, $\angle CAB + \angle CBO = 90°$

Again, $\because OB = OC$, $\therefore \angle CBO = \angle BCO$, $\angle CAB + \angle BCO = 90°$

$\because \angle BCD = \angle A$, $\therefore \angle BCD + \angle BCO = 90°$, 则$OC \perp CD$,

$\therefore CD$ is tangent to $\odot O$;

(2) Cross point C acts as $CM \perp AB$ at point M (FIG. 2) :

$\because$the radius of $\odot O$ is $\sqrt{5}$, $\therefore AB = 2\sqrt{5}$

$\because$the area of $\triangle ABC$ is equal to $2\sqrt{5}$, $\therefore CM = 2$,

In $Rt\triangle CMO$, $CO = \sqrt{5}$, $CM = 2$, $\therefore OM = 1$.

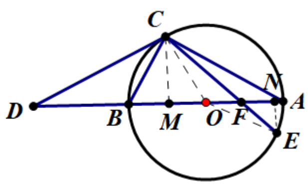

Figure 2

From（1）to $\angle OCD = \angle CMO = 90°$,

$\because \angle COM = \angle COD$, $\therefore \triangle COM \backsim \triangle DOC$, $\therefore \frac{DC}{CM} = \frac{OC}{OM}$, get $CD = 2\sqrt{5}$.

(3) Passing point E acts as $EN \perp AB$ at point N , connecting OE (FIG. 2) :

$\because CM \perp AB$, $EN \perp AB$, $\therefore \triangle FCM \backsim \triangle FEN$,

$\because \frac{EN}{CM} = \frac{FE}{CF} = \frac{NF}{MF} = \frac{1}{2}$,

From（2）to $CM = 2$, $OM = 1$, $\therefore EN = OM = 1$,

$\because OC = OE$, $\therefore Rt\triangle COM \cong Rt\triangle OEN$, $\therefore ON = CM = 2$, $MN = 3$,

$\because \frac{NF}{MF} = \frac{1}{2}$, $\therefore FM = 2$, $\therefore OM = 1$, $OF = 1$,

$\because BF = OB + OF$, $\therefore BF = 1 + \sqrt{5}$.

This topic is the circle of the comprehensive topic, examined the circumference Angle theorem, tangent line judgment, similar triangle judgment and nature, answer this topic requires us to master the content of each part, we should pay attention to the knowledge throughout.

After analyzing the steps of solving each problem, it seems that the whole problem solving process is relatively smooth. However, many students may have problems when solving the (2) question and will not solve it. The most common and deep reason is that we should go back to the starting point of solving the problem, that is to say, whether we have made clear the conditions and problems of the problem?

2. EXPLORING IDEAS

A mathematical problem, first of all, we should make clear the meaning of the question, from the number and formula given in the structure of the word symbol language, the geometric features of the graph, to find what the known conditions are? What's the problem we're trying to solve?

The question is composed of a circle and a number of triangles, so to solve the problem smoothly, we must first be familiar with the geometric characteristics of the circle and triangle. Secondly, look for the relationship between conditions and conditions and between conditions and problems. Finally, key points and breakthrough points are determined [3].

2.1What are the conditions of the problem?

The following 6 points can be obtained by sorting out the conditions of the topic:

1) AB is the diameter of $\odot O$;

2) ΔABC is a right triangle.

3) The position relationship between wired segments AB, AC, BC, CD and the circle;

4) The quantity relationship between the two angles " $\angle BCD = \angle A$ ";

5) The radius of $\odot O$ is $\sqrt{5}$ and the area of ΔABC is $2\sqrt{5}$;

6) The quantitative relationship between line segments EF and CF is $\frac{EF}{CF} = \frac{1}{2}$.

The above information can be easily captured from the graphic and symbolic language in the title. However, how to use the above information and how to effectively combine multiple information in solving problems is a practical problem for students who do not know where to start [4].

2.2What is the conclusion? What do we need from the conclusion?

1)Question 1 is to prove that line CD is tangent to $\odot O$, that is to say, the position relation between line CD and $\odot O$ is tangent, and its essence is to find the radius of line CD perpendicular to the tangent point.

2)Question 2 is to find the length of line segment CD. According to the knowledge reserves of middle school students at the present stage, the length of line segment can only be solved by finding right triangles through the Pythagorean theorem or using the similarity ratio of side lengths of similar triangles. To solve this problem, we need to use the measurement of known line segment, Angle and area, and further find the length of CD through transformation.

3)Question 3 is to find the BF length of line segment, which is carried out under the condition of problem 2, and gives the quantitative relationship of line segment EF and CF as $\frac{EF}{CF} = \frac{1}{2}$. In the synthesis of circle and triangle, the proportion of line segments is often solved by using similar triangles.

The problem set up three small questions, in the middle school mathematics to reduce the difficulty of understanding the problem. Students should consciously use the result as a condition for the next question. The mathematical thought of how to transform the conclusion to the condition is the difficulty in solving the problem.

2.3Communicate the connection between conclusions and conditions

The condition of the problem is the means to inspire the solution of the problem, and the conclusion of the problem is the direction to guide the solution of the problem. Grasping the connection between the condition and the conclusion can help solve the problem [5].

1）Question 1 requires that line CD is tangent to $\odot O$; We just have to prove that line CD is perpendicular to the radius past C. Therefore, it is easy for students to think of connecting OC, so now turn the problem into proving $OC \perp CD$. Further, the problem can be transformed into proof of $\angle OCD = 90°$.

Using known conditions $\angle BCD = \angle A$, $\angle ACB=90°$, $\angle CBO=\angle BCO$ and then through the transformation of Angle $\angle OCD = \angle BCD + \angle BCO=90°$. immediately proved.

2）Problem 2 gives the radius of $\odot O$ as $\sqrt{5}$, the area of ΔABC as $2\sqrt{5}$, and requires the length of CD. First, only the radius and diameter length of $\odot O$ can be known through the known conditions. And then you have to think about what does the area of ΔABC do? The formula for the area of the triangle is base×height, which can wait for the quantitative relationship. However, the problem cannot be solved. Therefore, we try to make point C as $CM \perp AB$ at point M, and get $CM = 2$, $OM = 1$. Further analysis shows that the length of OC, and OM can be obtained in $Rt\triangle OCM$, but the value of DM cannot be calculated, so the Pythagorean theorem cannot be used to solve the problem. If we can find two congruent triangles and convert line segment CD into known line segment, we will find that this idea still doesn't work; Therefore, once again observing the graphic features and known quantities, it is not difficult to think of using similar triangle method to solve the problem, and then prove $\triangle COM \backsim \triangle DOC$, , and then get $CD = 2\sqrt{5}$. We can solve it.

3）On the basis of (2), question 3 gives the position relation of point E and point F and the quantity relation of line segment EF and CF. Given the length and ratio relation of line segment, the length of BF is obtained. I'm not going to do a detailed analysis of the process here. The crossing point E acts as $EN \perp AB$ at point N and connects OE to prove $\triangle FCM \backsim \triangle FEN$. $EN = 1$ can be obtained by similarity. $Rt\triangle COM \cong Rt\triangle OEN$ can be obtained by congruence, and then $BF = 1 + \sqrt{5}$ can be obtained by given conditions.

After the above exploration, we can find that if we get a mathematical problem, we can make an analogy with the problem solving experience accumulated in the past, the familiar topic content, the familiar problem solving ideas and the new problem. From the second question of this comprehensive question, we can feel that students have only these kinds of methods to solve this kind of problem. To answer this question successfully, we need to connect the condition with the conclusion, and constantly build Bridges between the two. If the transfer is successfully realized, students can form the analytical thinking to solve the problem, so that the new problem can be clearly answered. However, students often can not complete the whole thinking process of solving problems smoothly in the actual process.

3 REFLECTION QUESTIONS

Through the demonstration of the above three problem analysis processes, students clearly understand the important role of "geometric conditions" in solving analytic geometry problems and how to use these "geometric conditions". In addition, students can clearly get how to analyze a problem and how to analyze the conditions and conclusions in an orderly way [4]. In this way, students can make their unclear problem-solving ideas clear and orderly. After exploring the thinking process of mathematical problem-solving, each step is exposed, and students' geometric demonstration ability will also be improved in a subtle way.

To capture useful information from understanding the meaning of the question is mainly to obtain "symbolic information" from the description of the question and "image information" from the picture of the question. To extract the relevant information from the memory storage, mainly from the concept, theorem, formula, basic model and so on to extract the basis of problem solving or problem solving. Combine the two groups of information effectively to form a harmonious logical structure [4]. Therefore, for this type of geometric synthesis problem, the following steps can be used to solve the problem.

The first step: make clear the conditions and problems of the topic, and then convert the symbols in the topic, and convert the text description into symbols. In the process of transformation, a preliminary understanding of conditions and conclusions is generated. In order to clarify the general scope of the problem or the overall direction, this is to think for directional regulation.

The second step: from the mathematical method level of the solution, this is about the circle and triangle combined with the comprehensive problem, first of all to the circle and triangle related properties and theorems to master. Then the conclusion has the means to solve the problem, common, the use of the circumference Angle theorem, tangent judgment, similar triangle judgment and properties. Finally, choose a solution to the problem.

Step 3: from the solution of the mathematical skills level, such as the first small q (2) subject, there are many problem in the process of problem solving ideas, we should be combined with the known quantity of solution to eliminate doesn't work, to further narrow the functional solutions, and specific operation procedure or reasoning step, finally complete the whole process of problem solving.

4 REVELATION

When students are faced with familiar problems or new problems that are relatively simple to them, they can form analytical thinking to solve problems and get clear answers to new problems. If the student does not immediately develop analytical thinking for the solution of the problem, he has to explore the problem. Therefore, the teacher should repeat the simple link in front of the students, explain the thinking process of exploring the way to solve the problem. Can help students remove the mystery of this insight, so that students understand the process of exploration. Show clear thinking and reasoning in the solution process, can know its source, know its root. Students can solve problems clearly and systematically, improve their ability to analyze and solve problems.

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