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Tuning of FOPID ($PI^{\lambda}D^{\mu}$) Controller for Heating Furnace

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Abstract

The paper is based on the fractional order Proportional-Integral-Derivative (fractional order PID) which is responsible for the controlling of the output of the heating furnace. The main objective is to design fractional order PID using various tuning techniques like Ziegler-Nichols and Cohen-Coon method using Nelder-Mead optimization algorithm so as to study which technique gives better results. In this paper, we have also illustrated about the Ziegler-Nichols method, Cohen-Coon method and also the Nelder-Mead optimization algorithm as before studying the results obtained from the methods and algorithms we need to know how the mentioned methods work.

Keywords: Ziegler-Nichols, Cohen-Coon, Nelder-Mead, Fractional order PID, Heating furnace, Fractional calculus

1. INTRODUCTION

The Proportional-Integral-Derivative (PID) controller is an across-the-board control system with a feedback. In industrial control system, almost 90% of the automatic controllers have the PID. In the market, it first came into existence in year 1939 and has remained irreplaceable till today [1]. The same is because of its simplicity, reliability, less overshoot, less settling time and gives an improved transient response.

The equation for Proportional-Integral-Derivative (PID) controller is,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt} \quad (1)$$

Where, K_p stands for proportional gain, K_i stands for integral gain and K_d stands for derivative gain and all of them are termed as the tuning parameters. Error is signified as the e. instantaneous (immediate) time is denoted as t and the variable of integration is meant as τ that can take the values from time 0 to the present t [2].

The Laplace domain transfer function of the PID controller is,

$$L(s) = K_p + \frac{K_i}{s} + K_d s \quad (2)$$

Where, s is the complex number frequency.

In this paper we have used fractional order PID, whose equation in Laplace domain is given as [3],

$$L(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \quad (3)$$

The reasons behind the use of fractional order PID is that it does not have steady state error, has gain margin, phase margin, gain cross over frequency and phase cross over frequency specifications. Moreover, it has robustness in the variations of the gain in the plant and also in the high frequency noise. At last it has good output disturbance desertion as it consists of five tuning parameters, K_p, K_i, K_d, λ and μ [4].

The different tuning methods used for the tuning of integer order model (IOM) are Ziegler-Nichols method, Cohen-Coon method, Astrum–Hagglund (AMIGO) method, Chien-Hrone-Reswick 1 (set point regulation), Chien–Hrone–Reswick 2 (disturbance rejection), etcetera.

In this paper section II describes the dynamic model of heating furnace, section III gives a concise introduction on fractional calculus, section IV describes about the Ziegler Nichols method, section V describes about the Cohen-Coon method, section VI gives the description about the Nelder-Mead optimization section VII gives description on the method of designing and tuning of fractional order Proportional-Integral-Derivative (PID) controller utilizing Ziegler-Nichols tuning method and Cohen-Coon tuning method.

2. HEATING FURNACE SYSTEM

A heating furnace is basically a thermal enclosure used to heat the materials below or above their melting points for several purposes.

The schematic diagram is shown in Figure 1. It has a metallic case which is the thermal enclosure which consists of a metallic shell, inside which it has refractory lining. When we talk about high temperature then metallic shell because of its high thermal conductivity and cost it is not used directly, that is this is being lined with the refractory material so that the conservation of the heat can be done and the metallic having high thermal conductivity looses all the heat being generated inside the furnace. The very vacant space in the Figure 1 is the reaction chamber.

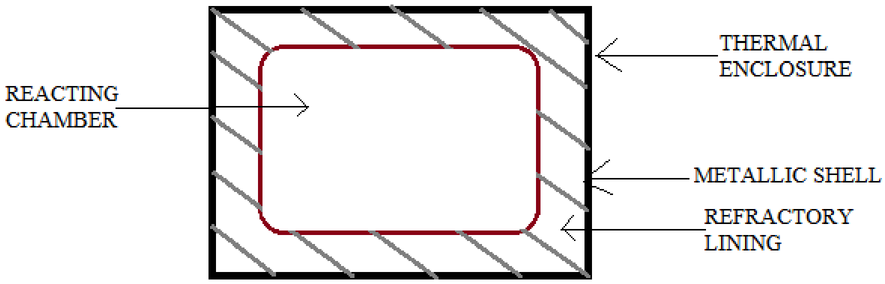

Figure 1. Schematic diagram of heating furnace [15]

The principle objectives are [4] [15],

1. Efficient utilization of heat so that the losses are minimum, which is very important particularly in the view of conservation of the energy resources are concerned.

There is a need of designing or operating of a furnace in such a way that whatever amount of thermal energy is supplied in any way is used efficiently and at the same time the losses are kept to a minimum value because whatever will be the losses, the losses are in terms of calorific value which is directly a loss of fuel.

2. To handle the distinctive stages (solid, fluid and gas) at diverse velocities for diverse times and temperatures such that wearing away and deterioration of the refractory are least.

This objective is also very important because different phases are being handled at diverse temperatures ranging from 700^{o}C to 1300^{o}C for solids, for fluids the temperature may range from 800^{o}C to 1600^{o}C depending upon the molten alloy and for gases, there could be carbon dioxide, carbon monoxide, sulphur dioxide, nitrogen et cetera which move at different speeds, velocities for different times and temperatures. Some of the thermal enclosures for certain processing may take an hour some may take half an hour some may take two hours so that is why the time and temperature are important factors. The temperature may be varying across the reaction chamber as well as at the height of the reaction chamber. The temperature for steel or liquid steel is 1600^{o}C and at the height of the reaction chamber the temperature may vary around 200^{o}C to 800^{o}C. Now what is required in this is that the wearing away (erosion) and deterioration (corrosion) should be the least.

Heating furnace is used in different industries like iron making industries, steel making industries, non-ferrous metal extraction industries, cement factories, ceramic processing industry, glass making industry, et cetera are some of the industries that on every day basis utilize the heating furnace system in various ways [5].

In the heating furnace as shown in Figure2 when the different ores are being sent into it then it starts heating the ores at certain temperature. When the materials are being heated then the gas being formed inside the heating chamber then the gas exerts pressure on the internal wall of the furnace which can actually blast away the furnace or destroy it completely in the form of explosion which can be life threatening for the people operating the heating furnace. So, the heat that is being supplied to the heating furnace is the minimum thus the complete heat (that can be utilized for the heating purpose) is not utilized properly and because of this there is steady state error, no overshoot and also the settling time is too high.

Therefore, we use the different techniques to design the controller so that the maximum heat can be utilized for heating the material inside the furnace. Also, the steady state error can be minimized, required overshoot can be achieved and also the settling time can be improved.

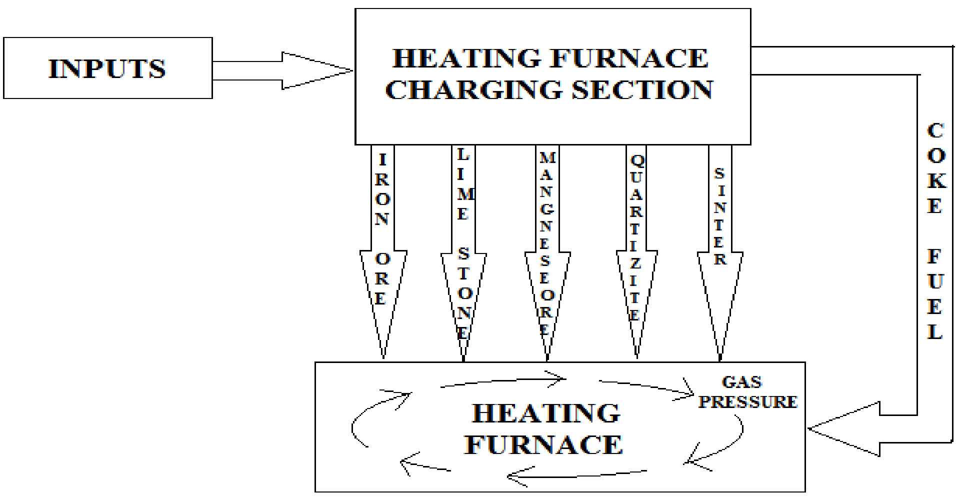

Figure 2. Heating furnace system [13]

3. DYNAMIC MODEL OF HEATING FURNACE

The approximate modeling of heating furnace includes quantity of input that varies with time and is actually the fuel mass gas flow rate and also the pressure inside the furnace which is the output value.

The dynamic modeling of heating furnace includes the mass, energy and the momentum balances. It also includes the transfer of heat from the hot flue hot gas to water, flue gas flow from the boiler model and steam model [6].

As we know for any physical system the total force is equal to the summation of individual forces exerted by mass (m), damping (b) and spring (k) element.

Mathematically we can state the same as,

$$F = ma + bv + kx \tag{4}$$

In the equation (4) acceleration is signified as a, velocity is signified as v and displacement is signified as x.

Therefore the differential equation of equation (4) is,

$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \tag{5}$$

Note: For designing a network based PID the above equation or model is a rough process behavior description [7].

Therefore, the differential equation of the heating furnace using the above equation becomes [8],

$$F = 73043\frac{d^2x}{dt^2} + 4893\frac{dx}{dt} + 1.93x \tag{6}$$

The Laplace transfer function of equation (6) which gives the Integer order model (IOM) as,

$$G_I(s) = \frac{1}{73043s^2+4893s+1.93} \tag{7}$$

s → Laplace operator

Where, the mass denoted by ‘m’ is 73043, the damping denoted by ‘d’ is 4893 and the spring denoted by ‘k’ is 1.93.

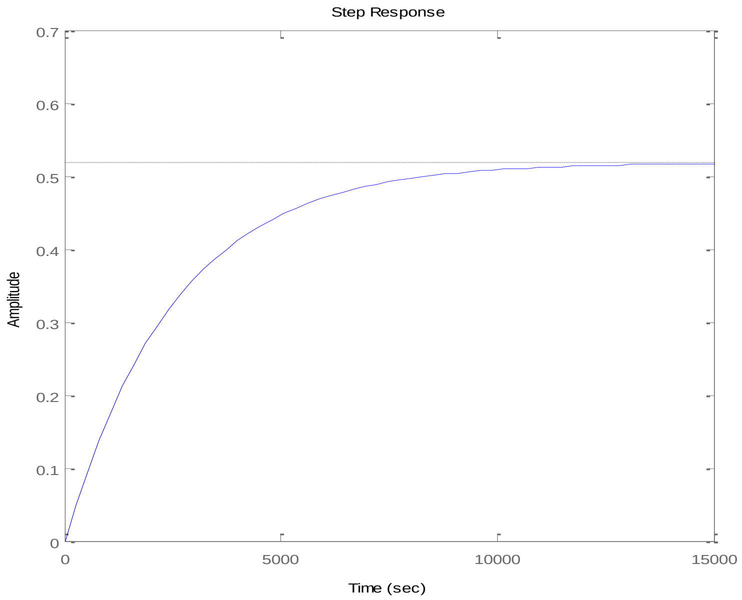

Figure 3. Step response of the equation (7) or IOM

4. A CONCISE INTRODUCTION ON FRACTIONAL ORDER CALCULUS

Fractional order calculus is a mathematical concept that has been in existence from 300 years ago. It is the mathematical concept that has proved itself better as compared to the integer order methods.

The definition of the fractional order calculus is as follows [9],

According to Lacroix,

$$\frac{d^n}{dx^n}x^m = \frac{m!}{(m-n)!}(x)^{(m-n)} = \frac{\Gamma(\mathrm{m}+1)}{\Gamma(m-n+1)}\,(x)^{(m-n)} \tag{8}$$

According to Liouville,

$$D^{\frac{-1}{2}}f = \frac{d^{\frac{-1}{2}}f}{(d(x-a))^{\frac{-1}{2}}} = \frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_{u=a}^{u=x}(x-u)^{\frac{-1}{2}}f(u)du = F^{\frac{-1}{2}}(x) \tag{9}$$

According to Riemann-Liouville,

$$_{\mathrm{a}}D_t^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{d}{dt}\right)^m\int_a^t \frac{f(\mathcal{T})}{(t-\mathcal{T})^{1-(m-\alpha)}}dt \tag{10}$$

According to Grunwald-Letnikov, which is being used widely is [10],

$$ {}_{a}D_t^{\alpha} f(t) = lim \frac{1}{\Gamma(\alpha)h^{\alpha}} \sum_{k=0}^{\frac{(t-a)}{h}} \{\frac{\Gamma(\alpha+k)}{\Gamma(k+1)}\} f(t-kh) \tag{11} $$

Where,

$$ \Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx \tag{12} $$

which is called the Euler's gamma function.

The fractional order derivatives and integrals properties are as follows [11],

a. f(t) being a logical function of t then the fractional derivative of f(t) which is ${}_{0}D_t^{\alpha} f(t)$ is an analytical function of z and α.

b. If α = n (n is any integer) then ${}_{0}D_t^{\alpha} f(t)$ produces the similar result as that of the traditional differentiation having order of n.

c. If α=0 then ${}_{0}D_t^{\alpha} f(t)$ is an identity operator.

$$ {}_{0}D_t^{\alpha} f(t) = f(t) $$

d. The differentiation and integration of fractional order are said to be linear operations,

$$ {}_{0}D_t^{\alpha} f(t) + b_g(t) = a_0 D_t^{\alpha} f(t) b_0 D_t^{\alpha} f(t) $$

e. The semi group property or the additive index law,

$$ {}_{0}D_t^{\alpha} f(t) {}_{0}D_t^{\beta} f(t) = {}_{0}D_t^{\beta} f(t) {}_{0}D_t^{\alpha} f(t) = {}_{0}D_t^{\alpha+\beta} f(t) $$

Which is being held under some sensible limitations on f(t).

Derivatives which are of fractional order has the commutation with derivative of integer order which is as follows,

$$ \frac{d^n}{dt^n} {}_{a}D_t^{\alpha} f(t) = {}_{a}D_t^{r} \left(\frac{d^n f(t)}{dt^n}\right) = {}_{a}D_t^{r+n} f(t) $$

where for t=a, $f^{(k)}(a)=0$ for k={0,1,…n-1}.The given equation shows that $\frac{d^n}{dt^n}$ and ${}_{a}D_t^{r}$ are commuted.

5. ZIEGLER-NICHOLS METHOD

To acquire controller parameters, in the year 1940 Ziegler and Nichols framed two exact techniques [12],

1. Non-first order plus dead time circumstances.

2. Involved exquisite manual calculations.

To compute the tuning parameters we apply the following procedure:

For feedback loop or closed loop,

a. Integral and derivative action must be removed. Integral time (T_i) must be set to 999 or larger and derivative controller (T_d) must be set to 0.
b. By changing the set point create small disturbance in the loop. Until the oscillations have common amplitude keeps adjusting the proportional by increasing or decreasing the gain.
c. The gain value (K_u) and the period of oscillation (P_u) must be recorded.
d. The necessary settings of the controller must be determined by inserting the appropriate values in the Ziegler-Nichols value.

Table 1: Closed loop calculation for K_p,T_i, T_d

	K_p	T_i	T_d
PID	$K_u/1.7$	$P_u/1.2$	$P_u/8$
PI	$K_u/2.2$	$P_u/2$	
P	$K_u/2$		

Advantages in this tuning process are that the P controller is required to be changed which justifies that it is easy to experiment and moreover it provides a much accurate scenario of how the system is working by including the complete dynamics of the system.

Whereas the disadvantage related to the same is that the experiments being carried out are very time consuming and the other one is that it can cause the system to become uncontrollable by speculating into the unstable regions while the P controller is being tested.

For feed forward loop or open loop,

The procedure is also known as Process Reaction procedure because it has the capacity of testing the open-loop response of the procedure so as to bring about the change in the control variable yield.

The steps are as follows,

1. Open loop step test must be performed.
2. By studying the process reaction curve dead time or transportation lag (τ_{dead}), time

for the response to change or the time constant (τ), and the value at which the system reaches the steady state (M_0) for a change of step X_0.

$$K_0 = \frac{X_0}{M_u} * \frac{\tau}{\tau_{dead}} \tag{13}$$

3. To calculate the tuning parameters of the controller insert the values of reaction time and lag rate into the Ziegler-Nichols open loop tuning equation.

Table 2: Open loop calculation for K_c,T_i,T_d

	K_p	T_i	T_d
PID	$1.2K_o$	$2\tau_{dead}$	$0.5\tau_{dead}$
PI	$0.9K_o$	$3.3\tau_{dead}$	
P	K_0		

The advantages of the above method or steps are that the method is quicker and easier to use than other methods, the method discussed above is robust and popular and the method is least disruptive and easiest to implement.

The disadvantages related to the same are the dependency on pure proportional measurements so as to estimate I and D controllers, the approximate values of K_c, T_i and T_d for different systems might not be accurate and it does not support for I, D and PD controllers.

6. COHEN-COON METHOD

Cohen-Coon tuning method is mainly used to overcome the slow, steady state response which occurs in the Ziegler-Nichols tuning method. This method is generally utilized for the first order systems or models having time delay as the controller does not spontaneously responds to the disturbances.

It is an offline method that is when a it is at steady state then a step change can be introduced at the input. After this based on the time constant and the time delay the output can be calculated and the initial control parameters can be found out using the response.

To get minimum offset and standard decay ratio there are an arrangement of pre-decided settings for the Cohen-Coon method,

Where, P is the percentage in the input, N is percentage change of output/ τ, L is τ_{dead} and R is (τ_{dead}/τ). We can use K_o in place of $(P/(NL))$.

The procedure of the method is as follows [13],

a. Wait for the complete procedure to achieve the steady state.

b. Step change is to be introduced at the input.

c. Approximate first order constant with time constant τ which is delayed by τ_{dead} units which is based on the output, from the time the step input was introduced.

   By recording the following time instances the value of τ and τ_{dead} can be found,

   t_0=input step start up point, t_2=half point time and t_3= time at 63.2%.

d. Calculate the process parameters τ, τ_{dead} and K_o by utilizing the assessment done at t_0, t_2, t_3, A and B.

e. On the basis of τ, τ_{dead} and K_0 the parameters of controller can be found.

The advantages of the Cohen-Coon method are that the time of reaction of the closed loop is quick or fast and this method can be used in the systems with time delay.

Whereas the disadvantages of this method are that it can only be utilized for the first order systems which include large process delay, it is an offline method, closed loop systems are unstable and the approximated value of τ, τ_{dead} and K_0 might not be compulsorily accurate for different systems.

Table 3: Calculation of K_p, K_i and K_d

	K_p	T_i	T_d
PID	(P/NL)*(1.33+(R/4))	L*(30+3R)/(9+20R)	$0.5\tau_{dead}$
PI	(P/NL)*(0.9+(R/12))	L*(30+3R)/(9+20R)	4L/(11+2R)
P	(P/NL)*(1+(R/3))		

7. NELDER-MEAD OPTIMIZATION METHOD

Nelder-Mead optimization method is also called the Downhill simplex method or the amoeba method which is used to find the minimum and maximum of an objective function in various dimensional spaces. The Nelder–Mead method is a technique which is a heuristic search method that can coincide to non-stationary points. However, it is easy to use and will coincide for a large class of problems. The Nelder–Mead optimization method was proposed by John Nelder & Roger Mead in year 1965. The procedure uses the concept of a simplex (postulation of notion of triangle or tetrahedron to arbitrary dimensions) which is a special polytope (geometric objects having flat sides) type with $N + 1$ vertices at n dimensions. Illustrations of simplices are, a tetrahedron in three-dimensional space, a triangle on a plane, a line segment on a line, etcetera.

The different operations in Nelder-Mead optimization method are as in [14],

Taking a function f(x), $x \in R^n$ which is to be minimized in which the current points are $x_1, x_2 \ldots\ldots x_{n+1}$.

i. Order : On the basis of values at the

   Vertices, $f(x_1) \leq f(x_2) \leq \ldots\ldots\ldots\ldots \leq f(x_{n+1})$

ii. Calculate the centroid of all points (x_0) except x_{n+1}.

iii. Reflection: Calculate $x_r = x_0 + \alpha (x_0 - x_{n+1})$. If the reflected point is not better than the best and is better than the second worst, that is, $f(x_1) \leq f(x_r) < f(x_n)$. After this by replacing the worst point x_{n+1} with reflected point x_r to get a new simplex and go to the first step.

iv. Expansion: If we have the best reflected part then $f(x_r) < f(x_1)$, then solve the expanded point $x_e = x_0 + \gamma(x_0 - x_{n+1})$. If the reflected point is not better than expanded point, that is, $[f(x_e) < f(x_r)]$ then either by substituting the worst point x_{n+1} by expanded point x_e to get new simplex and then go to the first step or by replacing the worst point x_{n+1} by reflected point x_r to obtain or get a new simplex and then go back to the first step.

   Else if the reflected point is not better than second worst then move to the fifth step.

v. Contraction: Here we know that $f(x_r) \geq f(x_n)$, contracted point is to be calculated $x_c = x_0 + \rho(x_0 - x_{n+1})$, if $f(x_c) < f(x_{n+1})$ that is the contracted point is better than the worst point then by substituting the worst point x_{n+1} with contracted point x_c to procure a new simplex and then go to first step or proceed to sixth step.

vi. Reduction: substitute the point with $x_i = x_1 + \sigma(x_i - x_1)$ for all $i \in \{2, \ldots\ldots, n+1\}$, then go to the first step.

Note: Standard values for α, σ, ρ, γ are 1, ½, -1/2, 2 respectively. In reflection the highest valued vertex is x_{n+1} at the reflection of which a lower value can be found in the opposite face which is formed by all vertices x_i except x_{i+1}. In expansion we can find fascinating values along the direction from x_0 to x_r only if the x_r which is the reflection point is new nadir along vertices. In contraction it can be expected that a superior value will be inside the simplex which is being formed by the vertices x_i only if $f(x_r) > f(x_n)$. In reduction to find a simpler landscape we contract towards the lowest point when the case of contracting away from the largest point increases f occurs and which for a non-singular minimum cannot happen properly. Indeed initial simplex is important as the Nelder-Mead can get easily stuck as too small inceptive simplex can escort to local search, therefore the simplex should be dependent on the type or nature of problem.

8. DESIGNING AND TUNING OF FOPID FOR THE HEATING FURNACE

The integer order model (IOM) of heating furnace, using Laplace transform, which is a second order transfer function, which is given as [15],

$$G_I(s) = \frac{1}{73043s^2+4893s+1.93} \tag{14}$$

Now, by using the Grunwald-Letnikov equation (11) for fractional calculus which is being given as,

$${}_aD_t^\alpha f(t) = lim \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{\frac{(t-a)}{h}} \{\frac{\Gamma(\alpha+k)}{\Gamma(k+1)}\} f(t-kh)$$

When the integer order model (IOM) is being solved using the Grunwald-Letnikov equation given above then we get the fractional order model (FOM) of heating furnace which comes out to be [16],

$$G_F(s) = \frac{1}{14494s^{1.31}+6009.5s^{0.97}+1.69} \tag{15}$$

The equation for FOPDT (first order plus dead time) is given as [17],

$$G_{FOPDT}(s) = \frac{K}{(1+Ts)} e^{-Ls} \tag{16}$$

Where, K is referred to as the gain, L is referred as time delay and T is referred as the time constant.

Then, by finding out the step response of the transfer function of the plant (heating furnace) we find out the value of K, L and T [18] [19],

Where, $T = \frac{3(T_2-T_1)}{2}, L = (T_2 - T_1)$ and $a = \frac{KL}{T}$

Where, T_1 and T_2 are the time instances in seconds taken from the step response obtained having a particular steady state gain.

So, the FOPDT model for the plant which is the heating furnace comes out to be,

$$G_{FOPDT}(s) = \frac{0.404272}{1 + 3421.93s} e^{-72.464} \tag{17}$$

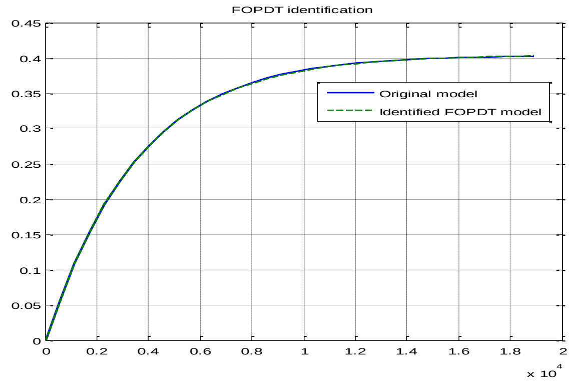

Figure 4. FOPDT identification, comparison between the original and the identified one which appears to be perfect.

Now, on applying Ziegler-Nichols method and Cohen- Coon method,

Table 4: Values of K_p, K_i and K_d using ZN and CC methods

	Ziegler-Nichols method	Cohen- Coon method
K_p	140.17	151.412
K_i	0.967173	0.803807
K_d	5078.65	4241.46

The value of λ and μ is being calculated by the Nelder-Mead optimization algorithm separately for both the methods with phase margin = 60° and gain margin = 10dB,

Table 5: Values of λ and μ for ZN and CC methods

	For Ziegler-Nichols Method	For Cohen-Coon method
λ	0.56202	0.15932
μ	0.010084	0.51597

Now, the FOPID ($PI^{\lambda}D^{\mu}$) model using the values obtained from and for the Ziegler-Nichols method is,

$$G_{ZN}(s) = 140.17 + \frac{0.967173}{s^{0.56202}} 5078.65s^{0.010084} \quad (18)$$

and, the FOPID ($PI^{\lambda}D^{\mu}$) model using the values obtained from and for the Cohen-Coon method is,

$$G_{cc}(s) = 151.412 + \frac{0.803807}{s^{015932}} + 4241.46s^{0.51597} \quad (19)$$

Now, the equations are to be put into the below given closed loop,

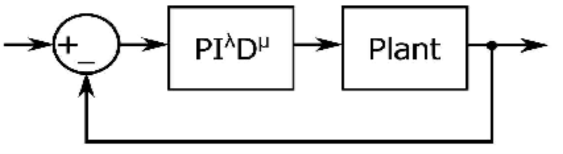

Figure 5. Fractional order PID with Plant (system)

Feeding the equation obtained from Ziegler-Nichols method in the closed loop shown in Figure5, then,

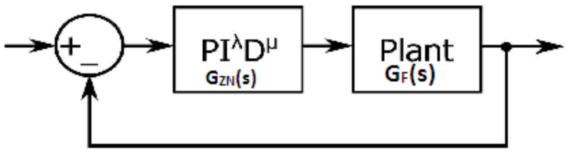

Figure 6. Closed loop with the plant $G_f(s)$ and FOPID $G_{ZN}(s)$

The output obtained after solving the Figure6 is, that is the value of $G_{ZNO}(s)$ is,

$$\frac{5078.6s^{0.5721}+140.17s^{0.56202}+0.96717}{14494s^{1.872}+6009.5s^{1.532}+5078.6s^{0.572}+141.86s^{0.562}+0.96} \quad (20)$$

Feeding the equation obtained from Cohen-Coon method in the closed loop shown in Figure5, then,

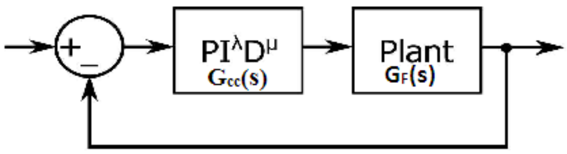

Figure 7. Closed loop with the plant $G_f(s)$ and FOPID $G_{cc}(s)$

The output obtained after solving the Figure7 is, that is the value of $G_{cco}(s)$ is,

$$\frac{4241.5s^{0.67529}+151.41s^{0.15932}+0.80381}{14994s^{1.469}+6009.5s^{1.129}+4241.5s^{0.675}+153.1s^{0.15}+0.803} \tag{21}$$

9. RESULTS

9.1. For Ziegler-Nichols method,

9.1.1. Time response

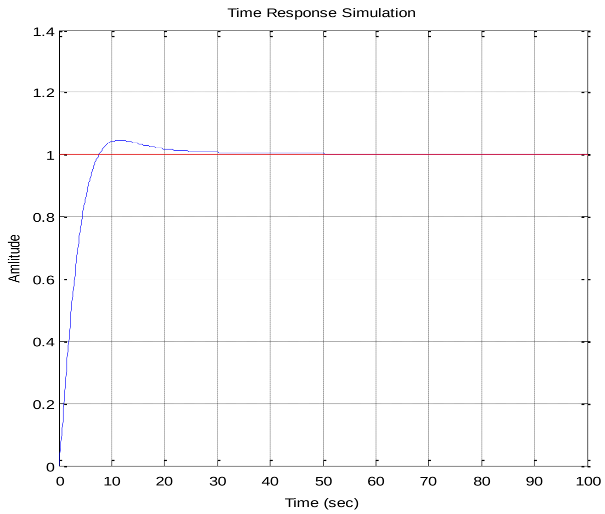

Figure 8. Time response of the output given by Figure6

9.1.2. Step Response

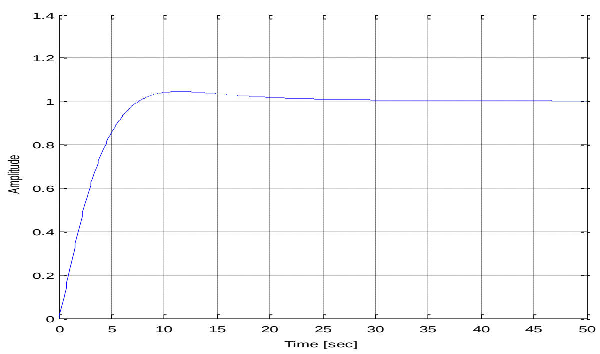

Figure 9. Step response of the output of the Figure6

9.1.3 Root Locus

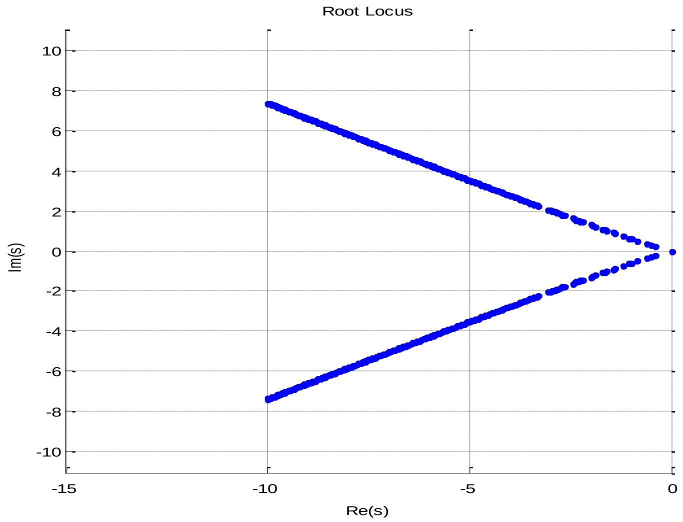

Figure 10. Root locus of the output of the Figure6

9.1.4. Open loop Bode

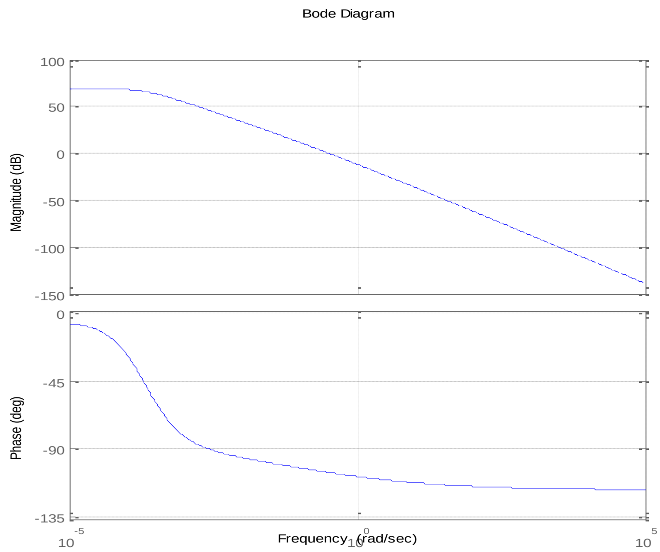

Figure 11: Open loop Bode plot of the system without feedback

9.1.5. Bode plot closed loop

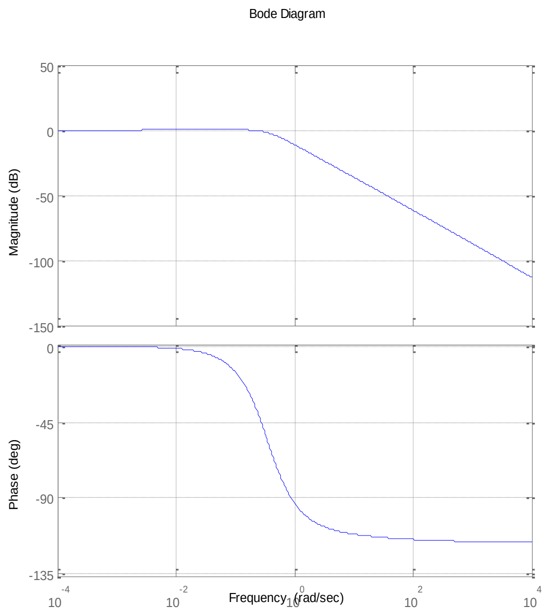

Figure 12. Bode plot of the output of the Figure6

9.1.6. Stability

K = 1, q = 0.0100, err = 1.6024e^{-8}, apol = 0.0267

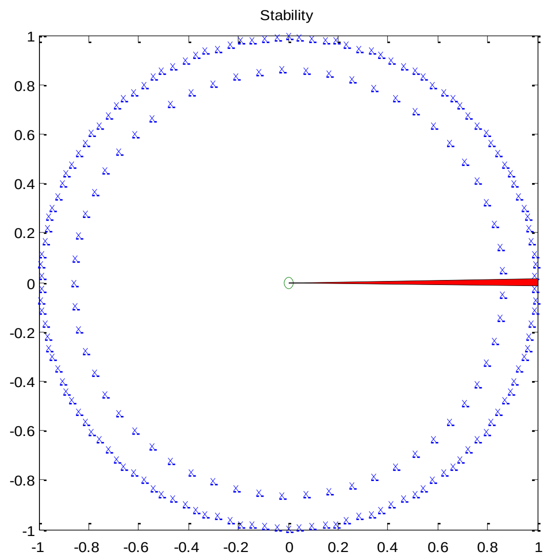

Figure 13: Stability of the output of Figure2, which appears to be stable with order q=0.01

9.2. For Cohen-Coon method:

9.2.1. Time Response

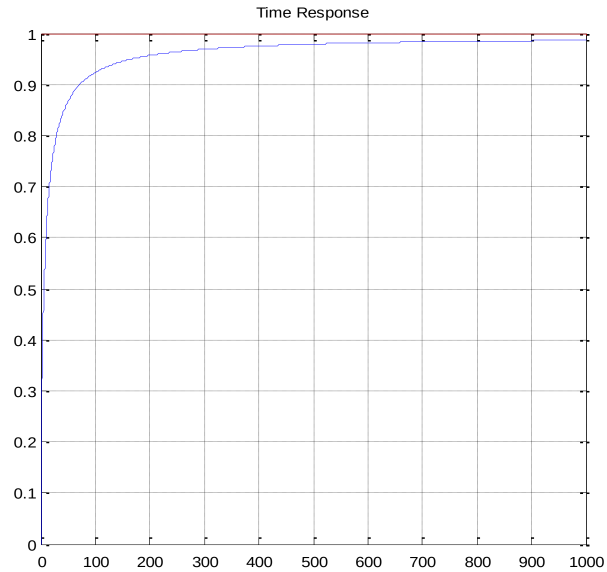

Figure 14. Time response of the output of Figure7

9.2.2. Step Response

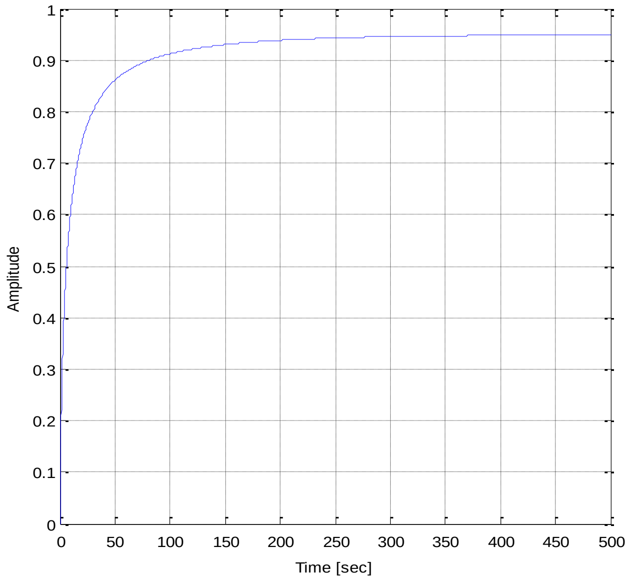

Figure 15. Step response of the output of Figure7

9.2.3. Root Locus

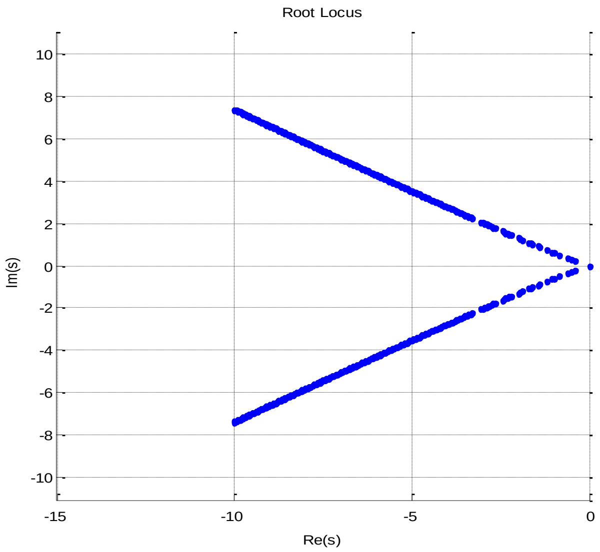

Figure 16. Root locus of the output of Figure7

9.2.4. Open loop Bode Plot

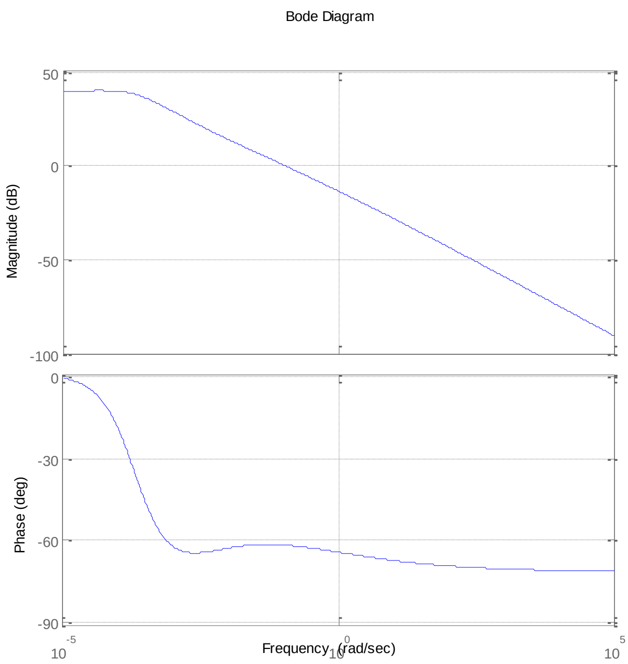

Figure 17. Open loop Bode plot of the system without feedback

9.2.5. Bode plot closed loop

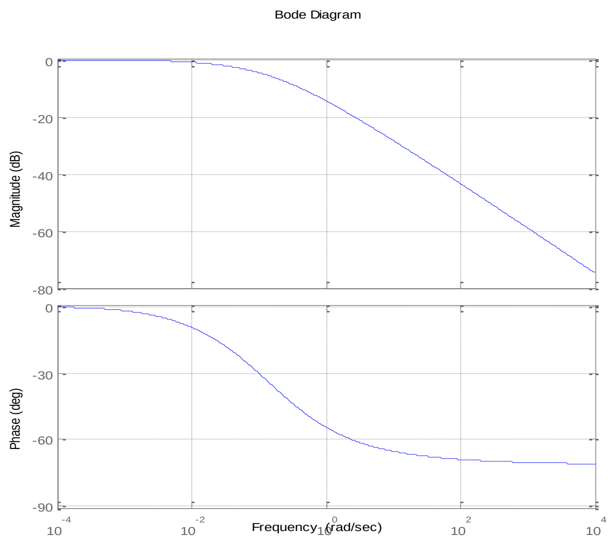

Figure 18. Bode plot of the output of Figure7

9.2.6. Stability

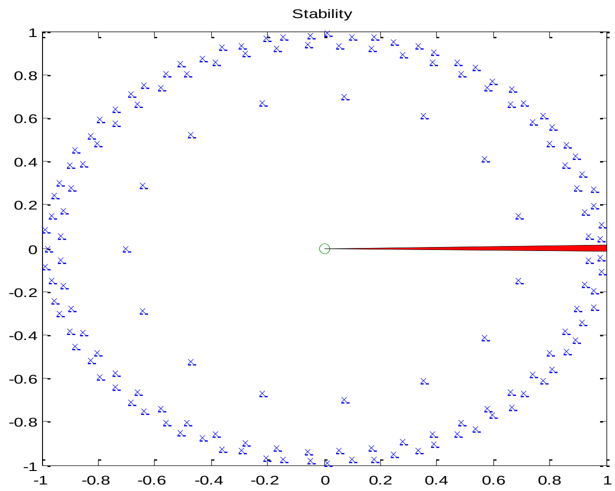

Figure 19: Stability of the output of Figure3, which appears to be stable with order q=0.01

With following values:

K = 1, q =0.0100, err = 2.9453e^{-9}, apol = 0.0475

10. DISCUSSION

From the various results, it is obvious that system is stable with both the PIDs. The optimization technique we have used is Nelder-Mead optimization method. We have approximated a Phase Margin of 60^0 & Gain Margin of 10dB as per the Bode Plot given. Various other optimization techniques are also there for tuning techniques. Like interior point optimization, SQP (sequential quadratic programming) optimization, active-set optimization etcetera. However, Nelder-Mead optimization technique gives a good response as compared to others. But comparing both the methods we can check that there is a slight overshoot of 4.5% when the Ziegler-Nichols method is used for FOPID design and the settling time is 31.6 seconds and when the Cohen-Coon method is used for the designing of FOPID then the overshoot is 0% and the settling time is 350.7 seconds. Therefore from the observations we can deduce that when Ziegler-Nichols method is used then there is a slight overshoot but the system achieves the steady state very swiftly and when the Cohen-Coon method is used for the designing of FOPID then the overshoot is completely altered but the settling time of the system increases significantly as compared to the result of Ziegler-Nichols method for the settling time.

CONCLUSION

Thus, we have successfully tuned and designed the FOPID using the Ziegler-Nichols method and Cohen-Coon method and simulated the different responses and plots of the system.

Also, we have successfully differentiated between the Ziegler-Nichols and Cohen-Coon method. The overshoot is the least in Cohen-Coon method and the settling time is better in the Ziegler-Nichols method. Also when the step response was studied for the IOM model of the heating furnace then the settling time of the same came out to be too high and when we used its fraction order model along with the FOPID ($PI^{\lambda}D^{\mu}$) controller then the settling time reduced significantly using the both tuning method mentioned. Various other tuning rules are also available which can be applied to a fractional order model.

Moreover, from the obtained results it can be deduced that using the fractional order PID controller being designed using various techniques, a heating furnace system with maximum utilization of the heat and minimum risk of explosion of the system can be achieved.

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