Image Compression Algorithm based on Curvelet Transforms and Comparative Analysis with JPEG and JPEG 2000

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Abstract

This paper proposes an unique image compression model based on Curvelet transform as well as a qualitative comparative study with existing state-of-the-art image compression standards. The anisotropic scaling and better directional sensitivity of curvelets aids in the optimal representation of image edges with a fewer number of non-zero coefficients. In this study a thresholding, quantization based mechanism is employed to retain only a few significant coefficients followed by lossless Huffman encoding to reconstruct the compressed image with fewer number of bits. The proposed algorithm is tested on several standard images of different dimensions. Peak Signal to Noise Ratio, Mean Square Error, Structural Similarity Index Measure and Edge Keeping Index are used as quality metrics to compare the quality of reconstructed image with JPEG and JPEG 2000 standards. The quantitative analysis for different Compression Ratio indicates the potential of Curvelets over Wavelets and
Cosine transforms in representing prominent features of images with fewer bits.

**Index Terms:** JPEG · JPEG 2000 · Curvelet transform · Quality metrics

I. INTRODUCTION

In this age of digitization, image processing is one among the essential techniques. The basic drive to represent images in a digital form is to achieve faster transmission, minimum memory size for storage and faster image processing to enable image manipulation in new and more efficient ways. Image compression is one of the major challenges in image processing which attracts significant research effort.

Image compression[1] attributes to achieve reduction in number of bits to represent an image without losing the readability of the image at the other end beyond a certain quality level as limit for compression. For a image compression application, a suitable choice of compression technique would be the one which retains the image quality while maintaining necessary compression levels plays a deciding role.

A multitude of image compression algorithms have been proposed and standard compression models been developed in the recent years. To represent large size images with less number of bits is the major principle behind compression. Therefore, the goal of image compression is to reach for a best possible system for available transmission and storage channels with low bit rates without losing image quality. In still image compression, JPEG and JPEG 2000, modeled by ISO(International Standards Organization) and IEC(International Electro-Technical Commission) through established ‘Joint Photographic Experts Group’ respectively, are the two major compression models which have been extensively studied and gained popularity.

In JPEG[2] compression standard, its performance falls heavily at lower compression rates due to its Discrete Cosine Transform based scheme which has an underlying block-based processing; this also leads to the presence of blocking artifacts in the reconstructed images thereby degrading the reconstructed image quality.

Recently, multi-scale representations pervade all areas of signal processing wavelet are one among such. The wavelet transform-based JPEG 2000[3] standard was proposed for image compression. This scheme provides considerable improvements in quality of reconstructed image at higher compression ratio. The reason for the success of wavelets is the fact that wavelet bases represent well a large class of signals. Wavelets are less ‘sparse’ in image representation, to put it simply they suffer from lack of visibility at smoothness along the edges of images.

More recently, the curvelet[4] based transforms studied for multi-resolution analysis as an alternative to wavelets. Most of its studies point out that they are more ‘sparse,’ gives visually sharper and provides more visibility in image representation along with producing better quality metrics.
The objective of this paper is to examine the proposed curvelet compression model and compare it with JPEG and JPEG 2000[5] compression standards. This investigation is carried out by performing compression on a set of standard images and by calculation of certain quality metrics[6] such as Compression Ratio(CR), Peak Signal to Noise Ratio(PSNR), Mean Square Error(MSE), Structural Similarity Index(SSIM), and Edge Keeping Index(EKI) for proposed curvelet method, JPEG and JPEG 2000 standards.

II. PRINCIPLE BEHIND TRANSFROM BASED IMAGE COMPRESSION TECHNIQUES.

Image compression basically involves the use of new and different fields of applied mathematics called as Fourier, Cosine[7], Wavelet and Curvelet transformations. All of these transforms together fall under a larger class of family known as ‘transform-based image compression.’

![Generalized Encoder-Decoder structure of transform based image compression](image)

**Fig. 1.** Generalized Encoder-Decoder structure of transform based image compression.

All transform-based image compression techniques, follow more or less a similar procedure to achieve the required rate of compression. The generalized working principle can be summarized in the following three steps as shown in the figure 1.

a) In general the first step is the lossless mathematical transformation of the image which provides a sparse representation of input image in frequency domain.

b) The transformed coefficients are then quantized. Quantization is the step where maximum compression takes place due to permanent loss in the transformed coefficients. The greater the quantization, the higher is the compression ratio achieved and larger is the error introduced due to precision loss.
The final step is frequently known as ‘entropy coding’ which involves the application of lossless compression techniques like Run Length Encoding (RLE), Huffman coding, arithmetic coding, etc., which help to reduce the overall number of bits to represent the image.

III. ADVANTAGES OF CURVELETS.

Wavelet transforms reveal a large number of wavelet coefficients along the edges of the image. These edges are repeated for every scale in wavelet decomposition, resulting a large number of wavelet coefficients for compression as shown in the figure 2(a).

![Wavelet Coefficients](image1)

![Curvelet Coefficients](image2)

**Fig. 2.** (a) Representation of wavelet coefficients.  (b) Representation of curvelet coefficients.

These large numbers of wavelet coefficients for compression would in turn require huge number of coefficients to reconstruct the image. This contributes to a significant amount of errors due to iterative approximation. This is a limitation on wavelet-based compression techniques. Also Wavelets though best suited for multi-resolution analysis but being nongeometrical, they can move only in horizontal and vertical directions and not along the curves[8]. Wavelets miss to take advantage of the geometry of the edge and cannot achieve the optimal rate of convergence.

On the other side, curvelet transform has a tight frame having both the multiscale resolution[9] analysis and sensitive to geometrical directions as shown in the figure 2(b), it can achieve finest convergence levels with mere thresholding and quantization.

IV. THEORY OF CURVELETS.

Curvelet transform is a multiresolution transform[10] with a strong directional feature in which at finer scales its elements are anistropic. The frequency plane in curvelets is partitioned into dyadic annuli, which in turn are subpartitioned into angular wedges displaying parabolic scaling aspect of \( length^2 \sim width \) as shown in following Fig. 3(a).
It represents the plurality of every pixel data of an image to frequency space using fourier transform. Then divides the plurality of fourier transform of each pixels into dyadic coronae based on concentric squares in the case of two-dimensional data or image which is shown in fig. 3(b).

Since the theoretical study of continuous curvelet transform assures improvement in order of magnitude over the wavelets and cosine transforms in numerous image processing applications, many implementations of Discrete curvelet Transforms have been emerging to operate on digital data. In recent times two different algorithms for discrete curvelet transform are introduced and gained importance

- Un-Equispaced Fast Fourier Transform (USFFT).
- Fast Wrapping Transform (FWT).

In USFFT the curvelet coefficients are produced by first finding the fourier transform coefficients of an image and then sampling them irregularly. Whereas in FWT, curvelet coefficients are the result of applying a series of translations across the scales and then wrapping them around for each scale. Both the algorithm are found to produce similar results, but FWT is more intuitive and faster in comparison with USFFT. In this work, FWT algorithm is used to obtain curvelet coefficients.

V. IMPLEMENTATION OF FAST DISCRETE CURVELET TRANSFORM.

The implementation of Fast Discrete Curvelet Transform (FDCT)\cite{11} algorithm for forward and inverse wrapping is as follows.

- Convert the image data into frequency space using forward Fast Fourier transform (FFT).
- The transformed data is multiplied using a set of rectangular or trapezoidal window functions. The shape is dependant on the needs of
ideal curvelet transformations like parabolic scaling rule.

- Then Inverse Fast Fourier Transform (IFFT) is applied to windowing data to obtain curvelet coefficients at each and angle. The advantage here is the number of regions that need to be transformed by IFFT is very much less than the original data since the window functions are non-zero only on support areas of elongated wedges.

The block diagram of Fast Discrete Curvelet Transform (FDCT) for forward and inverse wrapping is shown in the following diagram.

![Block Diagram of FDCT forward and inverse transforms.](image)

**Fig. 4.** Block Diagram of FDCT forward and inverse transforms.

In wrapping FDCT algorithm the obtained FFT coefficients on the wedge-shaped regions are ‘wrapped’ as in folded into rectangular shapes then IFFT is performed to get final curvelet coefficients. This whole process is equivalent to performing filtering and subsampling on curvelet sub-bands by rational numbers in two dimensions.
VI. FLOW CHART FOR THE PROPOSED CURVELET BASED IMAGE COMPRESSION MODEL.

VII. ALGORITHM FOR THE PROPOSED CURVELET BASED IMAGE COMPRESSION MODEL.

1. Preprocess the image by level shifting such that input image pixel values shifts equally around zero.

2. Calculate the curvelet coefficient of the image planes using following equations, which apply FFT and IFFT back to back to obtain curvelet coefficients $C(i,j)$.

   \[ C(i,j) = \int f(x).\psi_{j,k}(x) \, dx \quad R^2; \]

   \textit{where} \( R \) \textit{denotes, real – line.}

3. Rearrange the transformed coefficients to get an array consisting of curvelet coefficients in descending order $C_0(i,j)$, thereby obtaining the maximum coefficient value $C_{\text{max}}$.

4. Calculate the threshold value as cut-off to eliminate the low frequency coefficients and retaining the important high frequency coefficients. Then apply rate control to get the threshold coefficient values with rate control.
\[ T = C_{\text{max}}(i,j)/2; \]

\[ \text{if, } C(i,j) < T, \text{ Set } C(i,j) = 0; \]

\[ \text{else, set } C_T(i,j) = \text{Rate} \ast C(i,j). \]

5. Apply quantization\[12\] to the rate controlled thresholding\[13\] coefficients \( C_T(i,j) \), using maximum value \( C_{\text{max}} \) to further simplify the coefficients for compression. This affects more number of coefficients to attain zero or near zero values to achieve high compression.

\[ C_Q(i,j) = ([C_T(i,j)/T] + 0.5) \ast T; \]

6. Apply entropy coding to the quantized coefficients \( C_Q(i,j) \) using Huffman coding\[14\], which is a lossless compression code. This minimizes the overall number of bits to represent the image. The output from entropy step is the compressed image.

7. Perform inverse of the above mentioned steps starting from entropy decoding, inverse quantization, inverse curvelet transform IFDCT and finally inverse the level shifting to get the reconstructed image in the sequence mentioned.

**VIII. CALCULATION OF QUALITY METRICS**

It is always a critical choice to choose the highest compression ratio possible while keeping the quality of image at stake. The maximum compression ratio must be chosen such that image will not lose its pictorial information to serve a particular application.

Here we calculated certain quality metrics which help to reach a trade-off between achievable compression ratios by not losing quality of the image. For each compressed and reconstructed image, an error image was calculated. From the error data PSNR, MSE, SSIM, and EKI are calculated.

**A. Compression Ratio**

Compression ratio is, the ratio of an original image size to that of a compressed image size as given by the following expression

\[ \text{Compression Ratio} = \frac{\text{Original Image Size}}{\text{Compressed Image Size}} \]

The less the Compression ratio\[15\], the more is the quality retained in the original image. A high value of compression ratio signifies more compression and least quality in the final reconstructed image.
B. Peak Signal to Noise Ratio

PSNR value is calculated with the help of mean squared error present in the reconstructed image, it is calculated by the following equation,

$$P.S.N.R = 10 \cdot \log \left( \frac{(255)^2}{M.S.E} \right)$$

A lower value for MSE means minimum error, and from the existing inverse relationship between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher.

Here, the 'signal' is the original image, and the 'noise' is the error in reconstruction. So, a compression scheme having a lower MSE (and a high PSNR), can be recognized as a better one. The higher the PSNR value, the less is the error, and greater is the quality. PSNR is measured in the scale of logarithmic decibels (dB). Typical PSNR values fall in the range of 25 to 35dB.

C. Mean Square Error

It is one of the most used quality measures for compression analysis for a reconstructed image of m x n size input image, is given by the following equation.

$$MSE = \frac{1}{m,n} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} \left[ |a(i,j) - a^*(i,j)|^2 \right]$$

Where, $a(i,j)$ is Original image data; $a^*(i,j)$ is Compressed data.

The lesser the value, the less is the error between the original image and reconstructed image and greater is the quality.

D. Structural Similarity Index

SSIM assess the quality of image by calculating three characteristics of image namely the contrast, the luminance and the structure of original image to that of reconstructed image; it is given by the following equation

$$SSIM(x,y) = \left[I(i,j)\right]^α \cdot \left[c(i,j)\right]^β \cdot \left[s(i,j)\right]^γ$$

where,

$$I(i,j) = \frac{2 \mu_x \mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1},$$

$$c(i,j) = \frac{2 \sigma_x \sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2},$$

$$s(i,j) = \frac{\mu_x - \mu_y}{\sigma_x + \sigma_y + C_3},$$

where $\mu_x$, $\mu_y$, $\sigma_x$, $\sigma_y$ are the means and standard deviations of $I$ and $J$.
\[
C(i, j) = \frac{2 \sigma_x \sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2},
\]
\[
S(i, j) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3}.
\]
Here, \(\mu_x, \mu_y\) are local mean values; \(\sigma_x, \sigma_y\) and \(\sigma_{xy}\) are standard deviations and cross covariance for images \(x, y\).

The SSIM value for the reconstructed image in comparison with original image gives a value ranging from 0 to 1. Index value closer to one means that structural similarity of the reconstructed image is almost same as the original image and possess less error with the great quality.

**E. Edge Keeping Index**

This index measure the edge preservation in the reconstructed image compared to the original image. The equation for calculating EKI is given by the following expression.

\[
EKI = \frac{\sum_{i=1}^{m} G'(w_i)}{\sum_{i=1}^{m} G(w_i)}
\]

Where \(G(w_i)\) – is maximum grey value gradient of original and \(G'(w_i)\) – is maximum grey value gradient of reconstructed image.

EKI index value varies from 0 to 1, value approaching to ‘1’ means more edge preservation.

**IX. EXPERIMENTATION**

Experiment is conducted on a set of standard images with each image considered in three different sizes 512X512, 256X256, and 128X128. Image compression is performed for every size using JPEG with Huffman encoding, JPEG 2000 with Embedded Block Coding with Optimized Truncation(EBCOT)[16][17] encoding and in Proposed Curvelet method with Huffman encoding.

For every image considered quality metrics PSNR, MSE, SSIM and EKI are calculated for different Compression ratios. The experimentation is conducted with an un-optimized MATLAB code running on a machine with Intel core processor with 4 GB of RAM running on Windows 10.
Experimental results obtained for standard image ‘Lena’ in the above mentioned procedure are presented in the following tables and graphs.

The performance of proposed compression method is evaluated and compared with JPEG and JPEG 2000 compression schemes. The quality metrics PSNR, MSE, SSIM and EKI are obtained, tabled and plotted against CR.

Test result for standard image ‘Lena’ in different sizes are provided in the following tables and graphs as given in the following.

Image: Lena.bmp
Size: 512 X 512
Image : Lena.bmp
Size : 256 X 256

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Compression Standard : JPEG 2000

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Compression Standard : Curvelet Method

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XI. CONCLUSION

In this work, a new image compression technique based on curvelets with thresholding and quantization of curvelet coefficients is proposed. The reconstructed images depicted numerically and visually better preserving at terms of edges and details. Curvelets utilizing their edge of being geometrical and align themselves along the edges when compared to the wavelet based JPEG 2000 and cosine transform based JPEG techniques. Further works can extend the propose method using EBCOT compression to achieve better quality metrics of compression, also it can extend to images with higher resolutions.
REFERENCES


