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Improving the Performance of PID Controller using Fractional Elements for Heating Furnace

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Abstract

This paper is based on how the output of IOM (integer order model which is the transfer function of a system) can be made better. Moreover, we have taken the IOM form of the heating furnace (which acts as a plant) and have tried to show what kind of output we get from the complete closed loop system containing both plant and the controller with a feedback. Also, the outputs that have been achieved while the PID controller and FOPID ($PI^{\lambda}D^{\mu}$) controller were used separately are being compared and by using all the outputs of the system it has been shown that how the fractional elements affect the complete system performance.

Keywords: Ziegler-Nichols, Nelder-Mead, FOPDT, IPDT, FOIPDT, Fractional Order PID

1. INTRODUCTION

Integer order model is the model that has the order of integer that is the order of the model is in the form of whole number which can be positive or negative and the model is formed by taking the Laplace transform (either in time domain or frequency domain) of the required differential equation.

The PID controller is being applied to almost all the control systems that are in existence so as to enhance the performance of the required system [1]. The PID controller first came into existence in the year 1939 and since after that it has governed almost the 90% of the industrial sector and has remained irreplaceable since then and the reason behind this is because of the following reasons [1] [2],

(i). It gives the improved transient response [2], (ii). It gives the output that has less settling time [2], (iii). It gives the less overshoot [2], (iv). It is very reliable [2], (v). And the last is because of its simplicity [2].

The PID is basically an acronym of mathematical terms proportional (P), integral (I) and derivative (D) where proportional is the constant multiple, integral is the summation of a function over a particular interval of time and the derivative is the rate of change over a particular interval of time.

These three components help produce an output that has calculated error of the process being carried out. If the control loop functions properly then the PID eliminates the error quickly using the three factors P, I and D.

The PID controller is mathematically defined as,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt} \quad (1)$$

Where,

K_p – Proportional gain.

K_i – Integral gain.

K_d – Derivative gain.

e – Error (SP-PV).

t – Instantaneous time.

τ – Variable of integration that takes on the values from time 0 to the present t.

The Laplace transform of the PID controller equation (1) is,

$$L(s) = K_p + \frac{K_i}{s} + K_d s \quad (2)$$

Where,

s – Complex number frequency

K_p – Proportional gain.

K_i – Integral gain.

K_d – Derivative gain

In this paper we have used fractional order PID, whose equation in Laplace domain is given as,

$$L_f(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{3}$$

Where,

K_p – Proportional gain.

K_i – Integral gain.

K_d – Derivative gain.

λ, μ – Differential-integral's order for FOPID controller.

The reasons behind the use of fractional order PID are,

(i.) It does not have any steady state error, (ii). It has gain margin and phase margin, (iii). It has gain cross over frequency and phase cross over frequency, (iv). Provides robustness for the variations in the gain of the plant, (v). And it also provides robustness from the high frequency noise.

There are various types of industrial processes an about the process transfer function we know that it is very difficult to find out the real value for it, so the transfer function that is being found out for the plant is approximated for the further modeling by some definite transfer function, some of which are the FOPDT (first order plus delay time), IPDT (integral plus delay time) and FOIPDT (first order plus lag and integral delay time).

Moreover, the various tuning methods that can be used to model the PID and FOPID controllers are Ziegler-Nichols method, Cohen-Coon method, Astrum–Hagglund(AMIGO) method, Chien-Hrone-Reswick 1 (set point regulation), Chien–Hrone–Reswick 2 (disturbance rejection), etcetera.

2. DYNAMIC MODEL OF HEATING FURNACE

The approximate modeling of heating furnace includes quantity of input that varies with time and is actually the fuel mass gas flow rate and also the pressure inside the furnace which is the output value [3].

The dynamic modeling of heating furnace includes the mass, energy and the momentum balances [3]. It also includes the transfer of heat from the hot flue hot gas to water, flue gas flow from the boiler model and steam model [3].

As we know for any physical system the total force is equal to the summation of individual forces exerted by mass (m), damping (b) and spring (k) element.

Mathematically we can state the same as,

$$F = ma + bv + kx \quad (4)$$

In the equation (4) a is the acceleration, v is the velocity and x is the displacement.

Therefore the differential equation of equation (4) is,

$$F = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \quad (5)$$

It must be noted that, for designing a network based PID the above equation or model is a rough process behavior description [1].

Therefore, the differential equation of the heating furnace using the above equation becomes [3] [4],

$$F = 73043\frac{d^2x}{dt^2} + 4893\frac{dx}{dt} + 1.93x \quad (6)$$

The Laplace transfer function which gives the Integer order model (IOM) as [4] [14],

$$G_I(s) = \frac{1}{73043s^2+4893s+1.93} \quad (7)$$

S is the Laplace operator.

3. A CONCISE INTRODUCTION ON FRACTIONAL

Order calculus

Fractional order calculus is a mathematical concept that has been in existence from 300 years ago [5]. It is the mathematical concept that has proved itself better as compared to the integer order methods [5].

The definition of the fractional order calculus is as follows,

According to Lacroix [5],

$$\frac{d^n}{dx^n}x^m = \frac{m!}{(m-n)!}(x)^{(m-n)} = \frac{\Gamma(\mathrm{m}+1)}{\Gamma(m-n+1)}\ (x)^{(m-n)} \quad (8)$$

According to Liouville [5],

$$D^{\frac{-1}{2}}f = \frac{d^{\frac{-1}{2}}f}{(d(x-a))^{\frac{-1}{2}}} = \frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_{u=a}^{u=x}(x-u)^{\frac{-1}{2}}f(u)du = F^{\frac{-1}{2}}(x) \quad (9)$$

According to Riemann-Liouville [5] [6],

$${}_aD_t^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{d}{dt}\right)^m\int_a^t\frac{f(\tau)}{(t-\tau)^{1-(m-a)}}dt \quad (10)$$

According to Grunwald-Letnikov, which is being used widely is [5] [6],

$$ {}_aD_t^{\alpha} f(t) = lim \frac{1}{\Gamma(\alpha)h^{\alpha}} \sum_{k=0}^{\frac{(t-a)}{h}} \{\frac{\Gamma(\alpha+k)}{\Gamma(k+1)}\} f(t-kh) \quad (11) $$

Where,

$$ \Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx \quad (12) $$

which is called the Euler's gamma function [7].

The fractional order derivatives and integrals properties are as follows,

a. f(t) being an analytical function of t then the fractional derivative of f(t) which is ${}_0D_t^{\alpha} f(t)$ is an analytical function of z and α [7]

b. If α = n (n is any integer) then ${}_0D_t^{\alpha} f(t)$ gives the same result as that of the classical differentiation of order n [7].

c. If α=0 then ${}_0D_t^{\alpha} f(t)$ is an identity operator [7].

$$ {}_0D_t^{\alpha} f(t) = f(t) $$

d. Fractional order differentiation and fractional order integration are said to be linear operations [7] ,

$$ {}_0D_t^{\alpha} f(t) + b_g(t) = a_0 D_t^{\alpha} f(t) b_0 D_t^{\alpha} f(t) $$

e. The semi group property or the additive index law [7],

$$ {}_0D_t^{\alpha} f(t) {}_0D_t^{\beta} f(t) = {}_0D_t^{\beta} f(t) {}_0D_t^{\alpha} f(t) = {}_0D_t^{\alpha+\beta} f(t) $$

Which is being held under some reasonable constraints on f(t) [7].

Fractional order derivative has the commutation with integer order derivative which is as follows,

$$ \frac{d^n}{dt^n} {}_aD_t^{\alpha} f(t) = {}_aD_t^{r} \left(\frac{d^n f(t)}{dt^n}\right) = {}_aD_t^{r+n} f(t) $$

where for t=a, $f^{(k)}(a)=0$ for k={0,1,…n-1}.The above equation shows that $\frac{d^n}{dt^n}$ and ${}_aD_t^{r}$ are commuted [7].

4. ZIEGLER-NICHOLS METHOD

To obtain controller parameters, in the year 1940 Ziegler and Nichols formed two empirical methods [8],

a. Non-first order plus dead time situations.

b. Involved intense manual calculations.

To calculate the tuning parameters we use the following procedure:

For feedback loop or closed loop,

a. Integral and derivative action must be removed. Integral time (T_i) must be set to 999 or larger and derivative controller (T_d) must be set to 0 [9].

b. By changing the set point create small disturbance in the loop. Until the oscillations have common amplitude keeps adjusting the proportional by increasing or decreasing the gain [9].

c. The gain value (K_u) and the period of oscillation (P_u) must be recorded [9].

d. The necessary settings of the controller must be determined by inserting the appropriate values in the Ziegler-Nichols value.

Table 1: Closed loop calculation for K_c,T_i, T_d [9]

	K_c	T_i	T_d
PID	$K_u/1.7$	$P_u/1.2$	$P_u/8$
PI	$K_u/2.2$	$P_u/2$	
P	$K_u/2$		

Advantages in this tuning process are that we only need to change the P controller which justifies that it is easy to experiment and moreover it provides a much accurate scenario of how the system is working by including the complete dynamics of the system.

Whereas the disadvantage related to the same is that the experiments being carried out are very time consuming and the other one is that it can cause the system to become uncontrollable by speculating into the unstable regions while the P controller is being tested.

For feed forward loop or open loop,

The method is also known as Process Reaction method because it has the ability of testing the open-loop reaction of the process so as to bring about the change in the control variable output [10].

The steps are as follows,

(i).Open loop step test must be performed. (ii).By studying the process reaction curve dead time or transportation lag (τ_{dead}), time for the response to change or the time constant (τ), and the value at which the system reaches the steady state (M_0) for a step change X_0.

$$K_0 = \frac{X_0}{M_u} * \frac{\tau}{\tau_{dead}} \tag{13}$$

To calculate the tuning parameters of the controller insert the values of reaction time and lag rate into the Ziegler-Nichols open loop tuning equation.

Table 2: Open loop calculation for K_c,T_i,T_d [9]

	K_c	T_i	T_d
PID	$1.2K_o$	$2\tau_{dead}$	$0.5\tau_{dead}$
PI	$0.9K_o$	$3.3\tau_{dead}$	
P	K_0		

The advantages of the above method or steps are that the method is quicker and easier to use than other methods, the method discussed above is robust and popular and the method is least disruptive and easiest to implement.

The disadvantages related to the same are the dependency on pure proportional measurements so as to estimate I and D controllers, the approximate values of K_c, T_i and T_d for different systems might not be accurate and it does not hold for I, D and PD controllers.

5. NELDER-MEAD OPTIMIZATION METHOD

Nelder-Mead optimization method is also called the Downhill simplex method or the amoeba method which is used to find the minimum and maximum of an objective function in various dimensional spaces [11]. The Nelder–Mead method is a technique which is a heuristic search method that can coincide to non-stationary points [11]. However, it is easy to use and will coincide for a large class of problems. The Nelder–Mead optimization method was proposed by John Nelder & Roger Mead in year 1965

[11]. The method uses the concept of a simplex (postulation of notion of triangle or tetrahedron to arbitrary dimensions) which is a special polytope (geometric objects having flat sides) type with $N + 1$ vertices at n dimensions. Examples of simplices are, a line segment on a line, a triangle on a plane, a tetrahedron in three-dimensional space, etcetera [11].

The different operations in Nelder-Mead optimization method are,

Taking a function f(x), $x \in R^n$ which is to be minimized in which the current points are $x_1, x_2 \ldots\ldots x_{n+1}$.

i. Order : On the basis of values at the

   Vertices, $f(x_1) \leq f(x_2) \leq \ldots\ldots\ldots\ldots \leq f(x_{n+1})$ [11].

ii. Calculate the centroid of all points (x_0) except x_{n+1} [11].

iii. Reflection: Calculate $x_r= x_0+ \alpha (x_0 - x_{n+1})$. If the reflected point is not better than the best and is better than the second worst, that is, $f(x_1) \leq f(x_r) < f(x_n)$. After this by replacing the worst point x_{n+1} with reflected point x_r to get a new simplex and go to the first step [11].

iv. Expansion: If we have the best reflected part then $f(x_r) < f(x_1)$, then solve the expanded point $x_e=x_0+\gamma(x_0-x_{n+1})$. If the reflected point is not better than expanded point, that is, $[f(x_e)<f(x_r)]$ then either by replacing the worst point x_{n+1} by expanded point x_e to get new simplex and then go to the first step or by replacing the worst point x_{n+1} by reflected point x_r to obtain or get a new simplex and then go back to the first step [11].

   Else if the reflected point is not better than second worst then move to the fifth step [11].

v. Contraction: Here we know that $f(x_r) \geq f(x_n)$, contracted point is to be calculated $x_c=x_0+\rho(x_0-x_{n+1})$, if $f(x_c) < f(x_{n+1})$ that is the contracted point is better than the worst point then by replacing the worst point x_{n+1} with contracted point x_c to obtain a new simplex and then go to first step or proceed to sixth step [11].

vi. Reduction: Replace the point with $x_i=x_1+\sigma(x_i-x_1)$ for all $i \in \{2,\ldots\ldots,n+1\}$, then go to the first step [11].

Note: Standard values for α, σ, ρ, γ are 1, ½, -1/2, 2 respectively. In reflection the highest valued vertex is x_{n+1} at the reflection of which a lower value can be found in the opposite face which is formed by all vertices x_i except x_{i+1}[11]. In expansion we can find interesting values along the direction from x_0 to x_r only if the x_r which is the reflection point is new minimum along vertices [11]. In contraction it can be expected that a better value will be inside the simplex which is being formed by the vertices x_i only if $f(x_r) > f(x_n)$ [11]. In reduction to find a simpler landscape we contract towards the lowest point when the case of contracting away from the largest point increases f occurs and which for a non-singular minimum cannot happen properly [11]. Indeed initial simplex is important as the Nelder-Mead can get easily stuck as too small initial simplex can lead to local search, therefore the simplex should be dependent on the type or nature of problem [11].

6. FOPDT, IPDT AND FOIPDT MODELING

Huge number of industrial models can be approximately modeled first order plus time delay (FOPDT) [12]. FOPDT model is actually the combination of the first order process model with dead time [12]. The FOPDT equation is given as [12] [13],

$$G_{FOPDT}(s) = \frac{K}{(1+Ts)} e^{-Ls} \tag{14}$$

Where, K is the gain, L is time delay and T is the time constant [12].

K can be found out by,

$$K = \frac{Change\ in\ output}{Change\ in\ input} \tag{15}$$

Then, by finding out the step response of the transfer function of the plant (heating furnace) we find out the value of L and T [12].

Where, $T = \frac{3(T_2 - T_1)}{2}, L = (T_2 - T_1)$ and $a = \frac{KL}{T}$, here, T_1 and T_2 are the time instances in seconds taken from the step response obtained having a particular steady state gain [12].

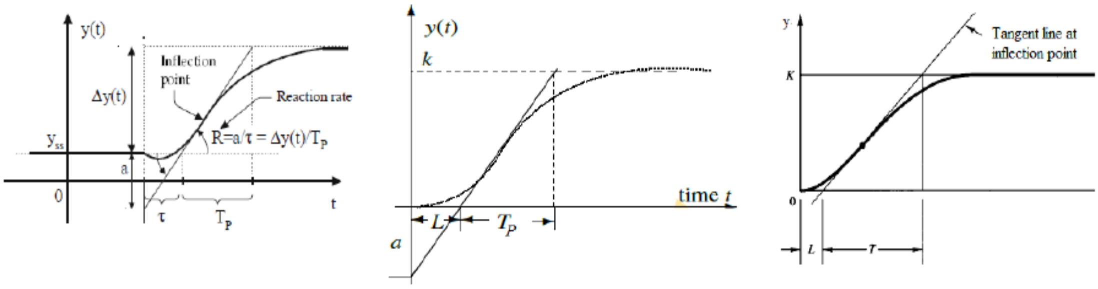

Figure 1: S-shaped curve having terminology [12]

In Figure 1 the S-shaped curve having no overshoot is known as the reaction curve which is being characterized by two constants, (i) time constant T and (ii) delay time L and by drawing the tangent line at the inflection point of the S-shaped curve and then by finding out the intersection point of the time axis and the steady state level K with the tangent line[12]. Then the transfer function of the S-shaped curve can be approximated by the first order system with time delay lag which is given by equation (14) [12].

Now, coming to the integer plus delay time (IPDT), in this the parameters can be found similarly as we find it in FOPDT which we have discussed earlier in this paper. The IPDT equation is [13],

$$G_{IPDT}(s) = \frac{K}{s}e^{-Ls} \tag{16}$$

There is no requirement of integrator so as to remove the steady state error as already there is existence of an integrator. PD controller can be used to minimize the overshoot [13].

The first order plus lag and integral delay time (FOIPDT) is also calculated using the same procedure as followed in the FOPDT process. The FOIPDT equation is,

$$G_{FOIPDT}(s) = \frac{K}{s(1+Ts)}e^{-Ls} \tag{17}$$

As in the model an integrator is contained then there is no extra integrator is compulsory so as to remove the steady state error because of set point change. Therefore, if there is steady state disturbance then PD controller can be used.

7. MODELING OF PID AND FOPID CONTROLLER FOR HEATING

Furnace using different models, tuning and optimization methods.

The integer order model (IOM) of heating furnace, using Laplace transform, which is a second order transfer function, which is given as [4] [14],

$$G_I(s) = \frac{1}{73043s^2+4893s+1.93}$$

The above equation is taken from the equation (7), where it has been discussed that how it has been derived.

The FOPDT model for the equation (7) using the equation (14) will be,

$$G_{FOPDT}(s) = \frac{0.518135}{(1+2520.22s)}e^{-15.0619s} \tag{18}$$

Where, K=0.518135, L=15.0619 and T=2520.22.

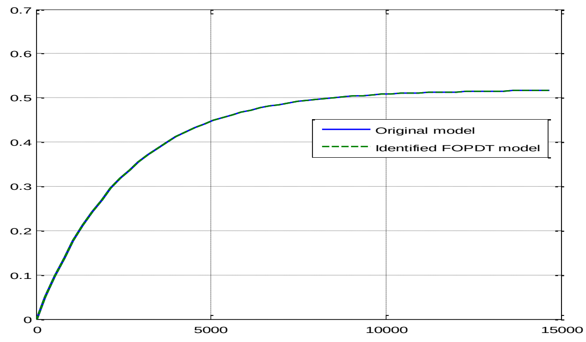

Figure 2: FOPDT identification result.

The IPDT model for the equation (7) using equation (16) will be,

$$G_{IPDT}(s) = \frac{4.94476e^{-5}}{s} e^{-3.89122e^{-5}s} \tag{19}$$

Where, K= $4.94476e^{-5}$ and L= $3.89122e^{-5}$

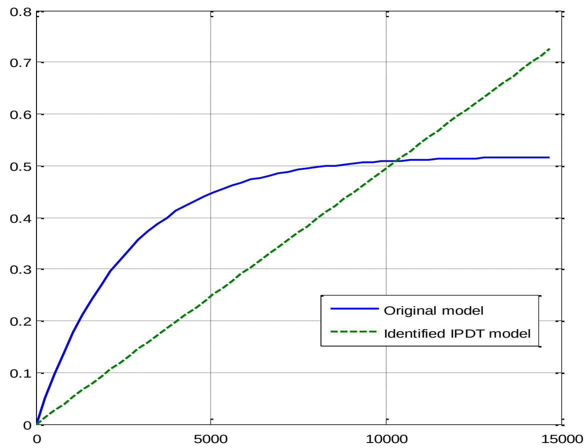

Figure 3: IPDT identification result

The FOIPDT model for the equation (7) using equation (17) will be,

$$G_{FOIPDT}(s) = \frac{4.94476e^{-5}}{s(1+2.01299e^{-10}s)} e^{-1.99743e^{-10}s} \tag{20}$$

Where K=$4.94476e^{-5}$, L=$1.99743e^{-10}$ and T = $2.01299e^{-10}$

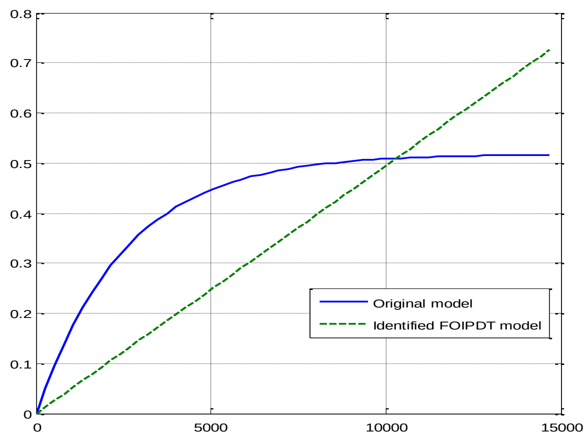

Figure 4: FOIPDT identification result

Now, we find out the equation or design of PID controllers using different modeling methods,

In case of FOPDT modeling, using the Ziegler-Nichols method we find out the values of K_p, K_i and K_d, the values of which are found out to be 387.523, 12.8643 and 2918.41 respectively and put them in equation (2), then we get,

$$L(s)_{FOPDT} = 387.523 + \frac{12.8643}{s} + 2918.41s \quad (21)$$

In case of IPDT modeling, using the IPDT-ISE method we find out the values of K_p, K_i and K_d, the values of which are found out to be 7.12016e^8, 1.22805e^{13} and 16346.6 respectively and put them in equation (2), then we get,

$$L(s)_{IPDT} = 7.12016e^8 + \frac{1.22805e^{13}}{s} + 16346.6s \quad (22)$$

In case of FOIPDT modeling, using the FOIPDT tuning method we find out the values of K_p, K_i and K_d, the values of which are found out to be 5.55164e^{13}, 6.91215e^{22} and 11147.3 respectively and put them in equation (2), then we get,

$$L(s)_{FOIPDT} = 5.55164e^{13} + \frac{6.91215e^{22}}{s} + 11147.3s \quad (23)$$

Now, we find out the equation and design of the FOPID controller,

We use the values of K_p, K_i and K_d (which we found out during the design of PID controllers using the Ziegler-Nichols, IPDT-ISE and FOIPDT tuning method) so as to optimize them using Nelder-Mead optimization algorithm so as to get the proper value of the 5 tuning parameters of FOPID.

In case of FOPDT modeling, using the Nelder-Mead optimization algorithm we find out the values of K_p, K_i ,K_d, μ and λ, the values of which are found out to be 99.99, 5.5042, 99.995, 0.010078 and 0.28268 respectively and put them in equation (3), then we get,

$$L_f(s)_{FOPDT} = 99.99 + \frac{5.5042}{s^{0.28268}} + 99.995s^{0.010078} \tag{24}$$

In case of IPDT modeling, using the Nelder-Mead optimization algorithm we find out the values of K_p, K_i ,K_d, μ and λ, the values of which are found out to be 69.36, 3.801, 99.982, 0.89611 and 0.56756 respectively and put them in equation (3), then we get,

$$L_f(s)_{IPDT} = 69.36 + \frac{3.801}{s^{0.56756}} + 99.982s^{0.89611} \tag{25}$$

In case of FOIPDT modeling, using the Nelder-Mead optimization algorithm we find out the values of K_p, K_i ,K_d, μ and λ, the values of which are found out to be $5.55164e^{13}$, $6.91215e^{22}$, 11147.3, 0.50872 and 0.49964 respectively and put them in equation (3), then we get,

$$L_f(s)_{FOIPDT} = 5.55164e^{13} + \frac{6.91215e^{22}}{s^{0.49964}} + 11147.3s^{0.50872} \tag{26}$$

Now, we put the equation (7) in the plant block and equation (21) in the plant block of the closed loop system which is given below,

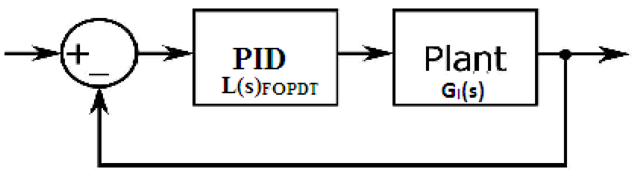

Figure 5: Closed loop PID plant having equation (7) at the plant block and equation (21) in the PID block.

The output of the Figure 5 when the equation (7) and equation (21) are fed in the plant and the PID respectively and the output equation comes out to be,

$$G_{O1}(s) = \frac{2918.4s^2+387.52s+12.864}{73043s^3+7811.4s^2+389.45s+12.864} \tag{27}$$

Now, we put the equation (7) in the plant block and equation (22) in the plant block of the closed loop system which is given below,

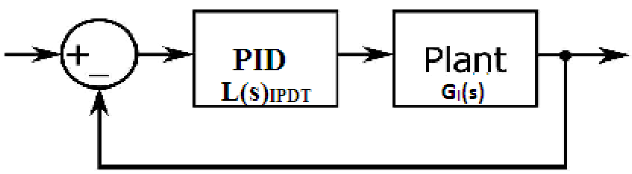

Figure 6: Closed loop PID plant having equation (7) at the plant block and equation (22) in the PID block.

The output of the Figure 6 when the equation (7) and equation (22) are fed in the plant and the PID respectively and the output equation come out to be,

$$G_{O2}(s) = \frac{16347s^2+7.1202e^8 s+1.2281e^{13}}{73043s^3+21240s^2+7.1202e^8 s+1.2281e^{13}} \quad (28)$$

Now, we put the equation (7) in the plant block and equation (23) in the plant block of the closed loop system which is given below,

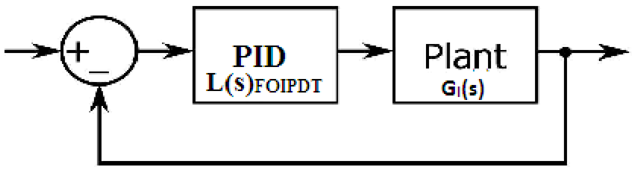

Figure 7: Closed loop PID plant having equation (7) at the plant block and equation (23) in the PID block.

The output of the Figure 7 when the equation (7) and equation (23) are fed in the plant and the PID respectively and the output equation comes out to be,

$$G_{O3}(s) = \frac{1147s^2+5.5516e^{13}s+6.1921e^{22}}{73043s^3+16040s^2+5.551e^{13}s+6.9121e^{22}} \quad (28)$$

Again we take the closed loop system that consists FOPID block instead of PID block as shown earlier along with a plant bock.

Now, we put the equation (7) in the plant block and equation (24) in the plant block of the closed loop system which is given below,

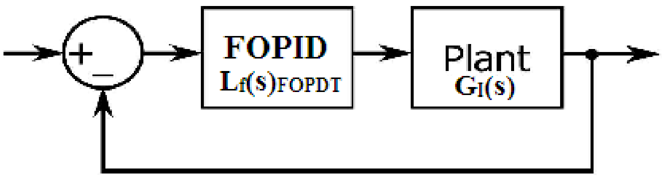

Figure 8: Closed loop FOPID plant having equation (7) at the plant block and equation (24) in the FOPID block.

The output of the Figure 8 when the equation (7) and equation (24) are fed in the plant and the PID respectively and the output equation comes out to be,

$$G_{O4}(s) = \frac{99.995s^{0.29276}+99.99s^{0.28268}+5.5042}{73043s^{2.2827}+4893s^{1.2827}+99.995s^{0.29276}+101.92s^{0.28268}+5.5042} \quad (29)$$

Now, we put the equation (7) in the plant block and equation (25) in the plant block of the closed loop system which is given below,

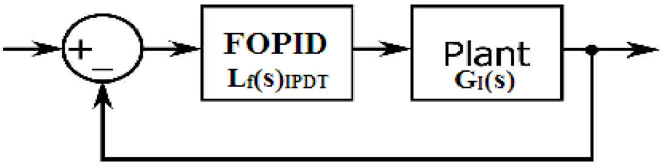

Figure 9: Closed loop FOPID plant having equation (7) at the plant block and equation (25) in the FOPID block.

The output of the Figure 9 when the equation (7) and equation (25) are fed in the plant and the PID respectively and the output equation comes out to be,

$$G_{O5}(s) = \frac{99.982s^{2}+69.36s^{0.56756}+3.801}{73043s^{2.5676}+4893s^{1.5676}+99.982s^{1.4637}+71.29s^{0.56756}} \quad (30)$$

Now, we put the equation (7) in the plant block and equation (26) in the plant block of the closed loop system which is given below,

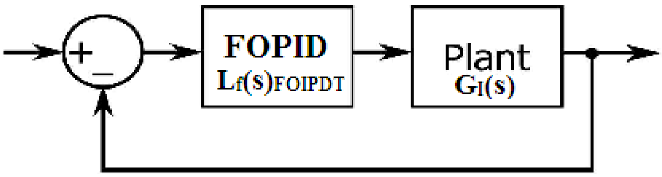

Figure 10: Closed loop FOPID plant having equation (7) at the plant block and equation (26) in the FOPID block.

The output of the Figure 10 when the equation (7) and equation (26) are fed in the plant and the PID respectively and the output equation comes out to be,

$$G_{O6} = \frac{11147s^{1.0084}+5.5516e+13s^{0.49964}+6.9121e^{22}}{73043s^{2.4996}+4893s^{1.4996}+11147s^{1.0084}+5.5516e+0.31s^{0.49964}+6.1921e^{22}} \quad (31)$$

8. RESULT

8.1. The different response graphs we got from figure 5 are,

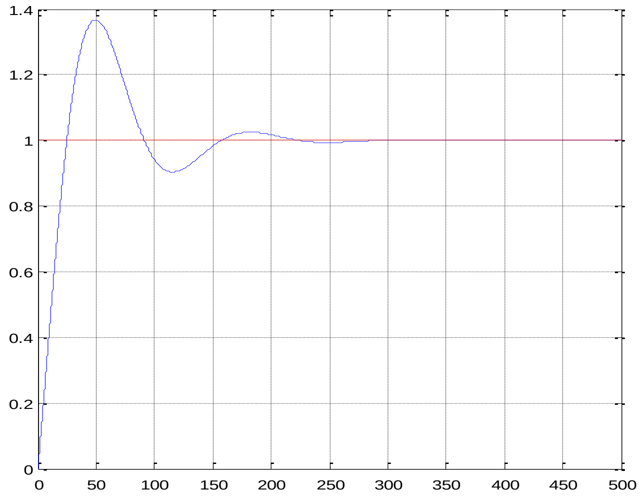

Figure 11: Time response produced by the figure 5

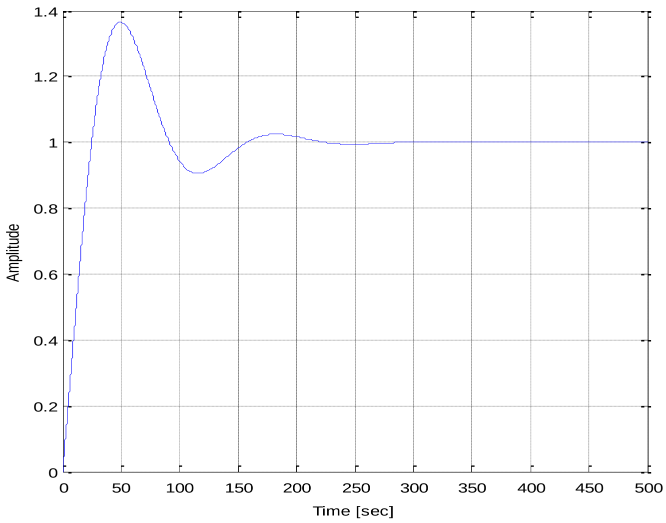

Figure 12: Step response produced by the figure 5

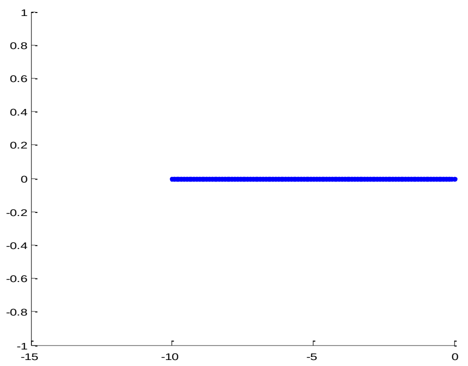

Figure 13: Root locus produced by the figure 5

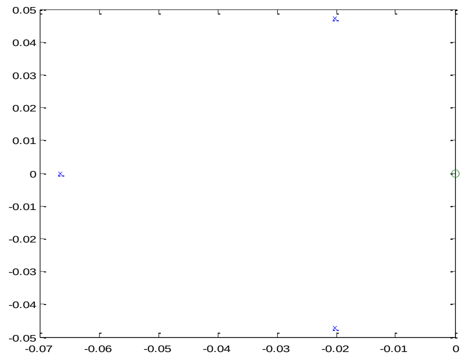

Figure 14: Stability graph produced by the figure 5 is stable with order q=1, K=1, err = $1.677e^{-14}$ and apol= 1.9739.

8.2. The different response graphs we got from figure 6 are,

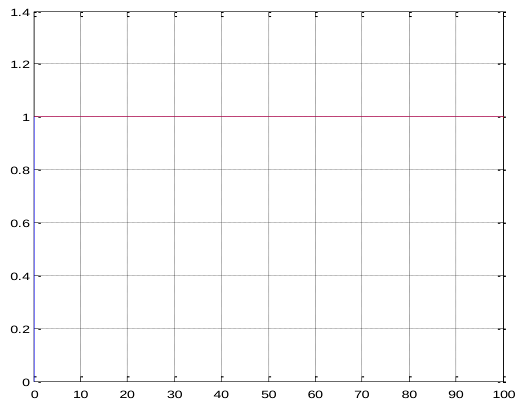

Figure 15: Time response produced by the figure 6

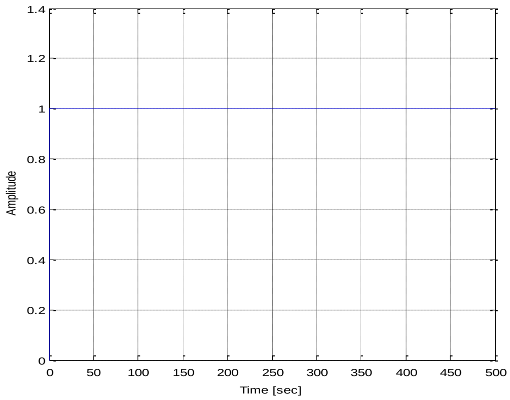

Figure 16: Step response produced by the figure 6

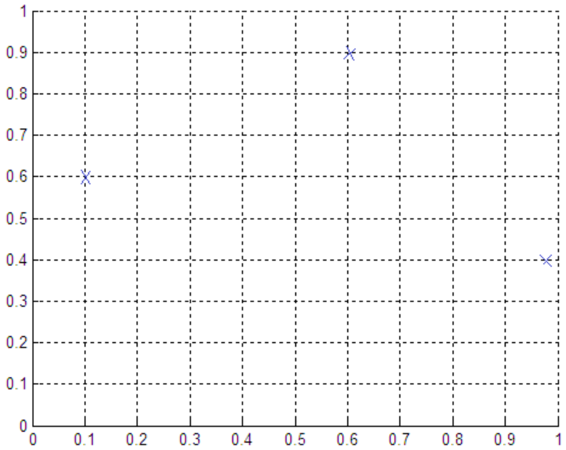

Figure 17: Root locus produced by the figure 6

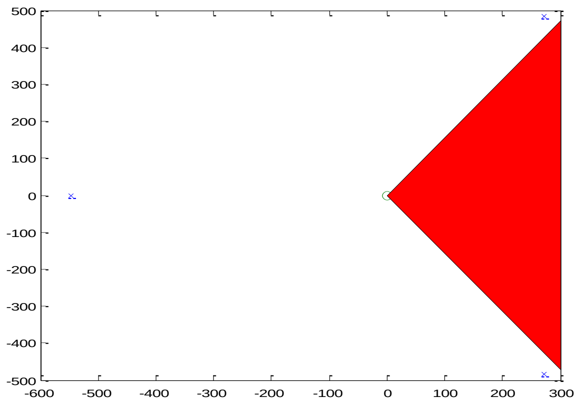

Figure 18: Stability graph produced by the figure 5 is unstable with order q=1, K=0, err = 0.0379 and apol= 1.0565.

8.3. The different response graphs we got from figure 7 are,

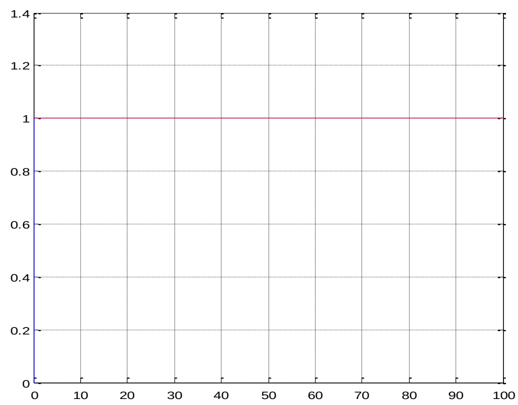

Figure 19: Time response produced by the figure 7

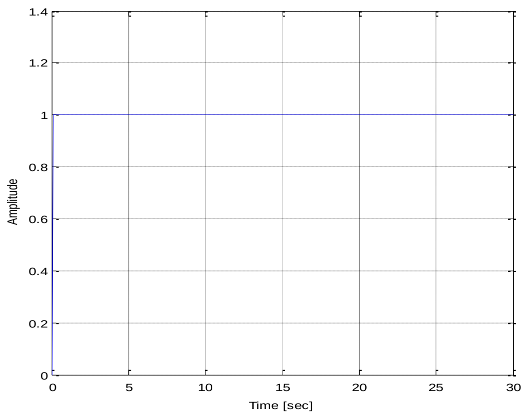

Figure 20: Step response produced by the figure 7

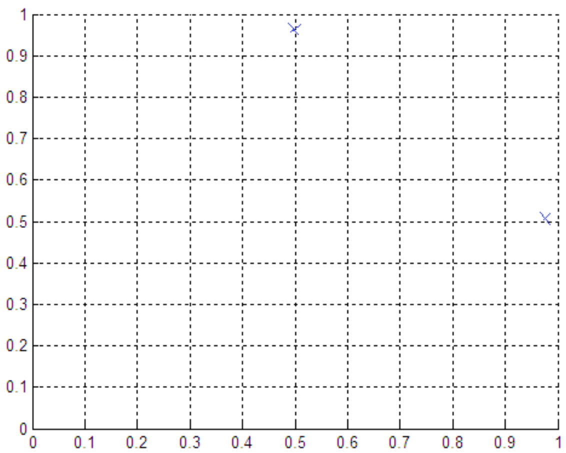

Figure 21: Root locus produced by the figure 7

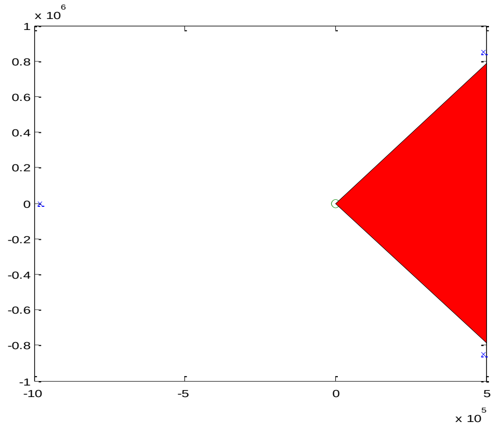

Figure 22: Stability graph produced by the figure 7 is unstable with order q=1, K=0, err = $2.1379e^8$ and apol= 1.0474

8.4. The different response graphs we got from figure 8 are,

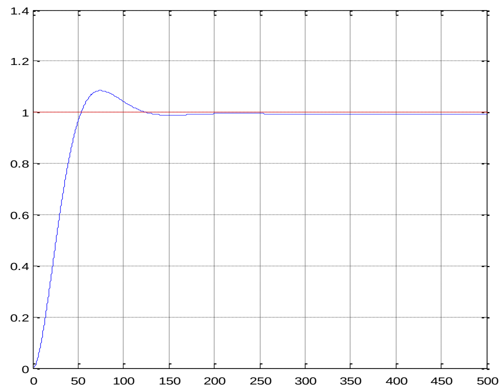

Figure 23: Time response produced by the figure 8

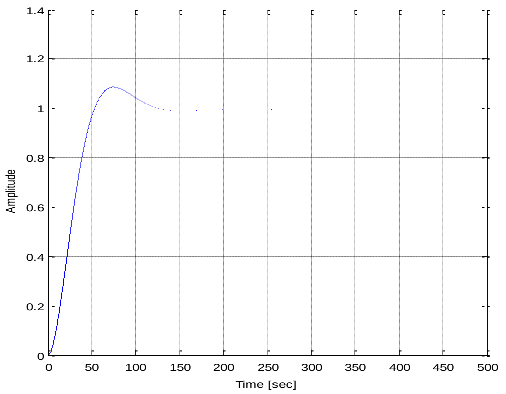

Figure 24: Step response produced by the figure 8

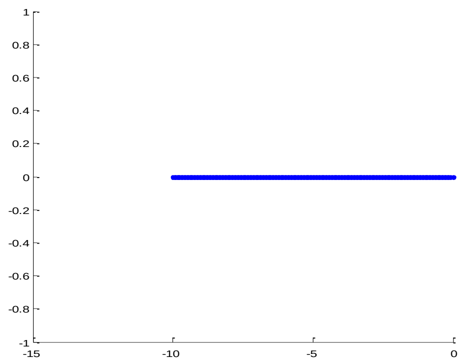

Figure 25: Root locus produced by the figure 8

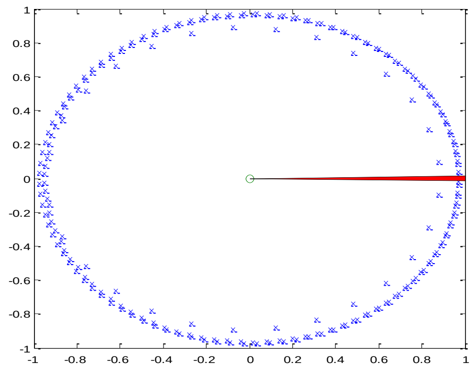

Figure 26: Stability graph produced by the figure 8 is unstable with order q=0.01, K=1, err = $9.9159e^{-10}$ and apol= 0.0223

8.5. The different response graphs we got from figure 9 are,

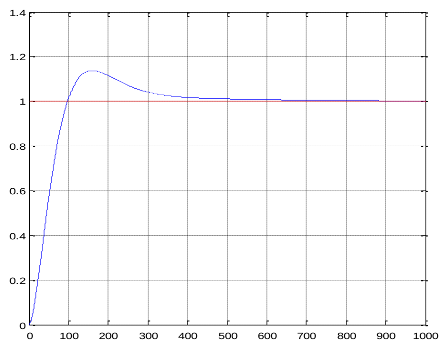

Figure 27: Time response produced by the figure 9

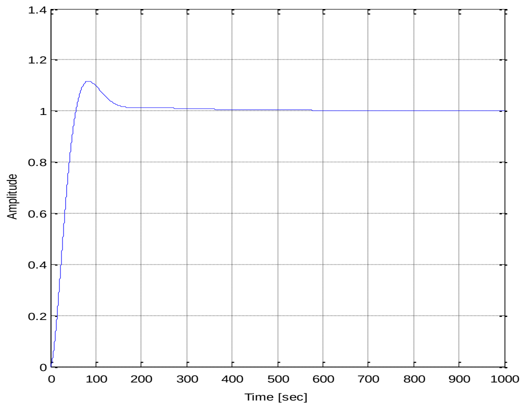

Figure 28: Step response produced by the figure 9

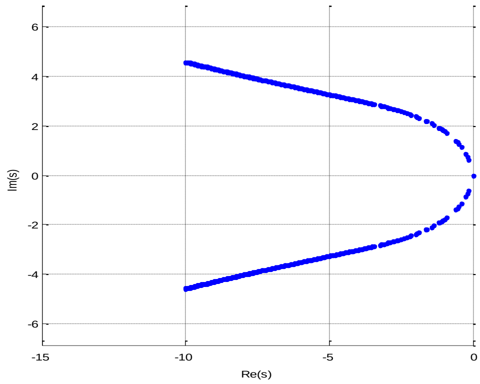

Figure 29: Root locus produced by the figure 9

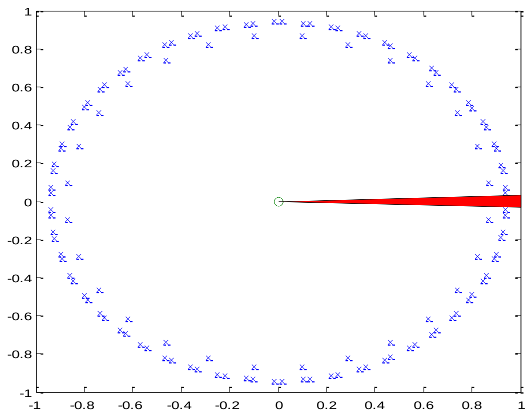

Figure 30: Stability graph produced by the figure 9 is unstable with order q=0.02, K=1, err = $1.8128e^{-10}$ and apol= 0.0456

8.6. The different response graphs we got from figure 10 are,

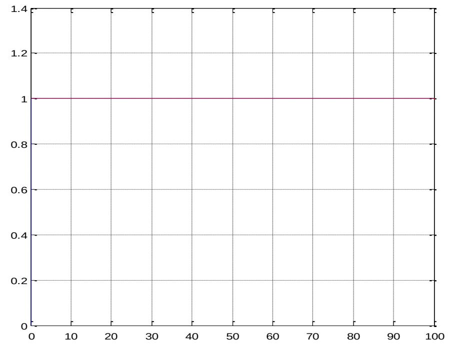

Figure 31: Time response produced by the figure 10

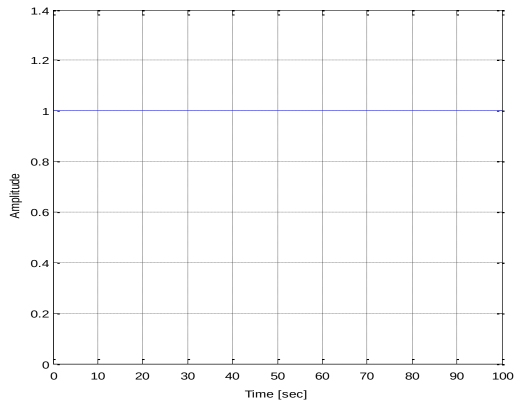

Figure 32: Step response produced by the figure 10

Figure 33: Root locus produced by the figure 10

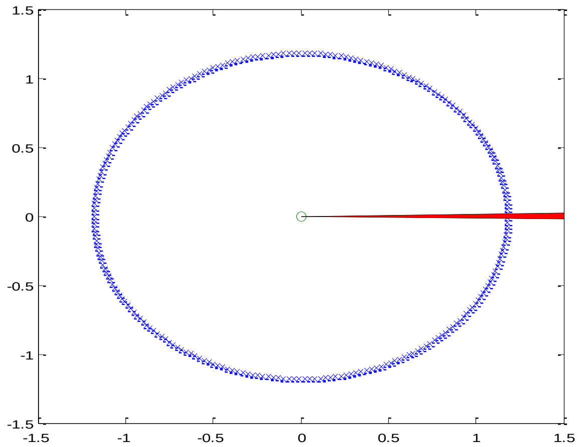

Figure 34: Stability graph produced by the figure 10 is unstable with order q=0.01, K=0, err = $1.1643e^{23}$ and apol= 0.0129

9. DISCUSSION

It is clear that the transfer function of heating furnace exhibits almost unstable response. So a classical PID of integer order was designed which was also unable to improve the response. In this new design method the integer order model (IOM) of heating furnace was first converted into a fractional order model. When the same classical IPDT method was applied to FOM for tuning the parameters, the final system became almost stable, although an overshoot of 15% was still present. The settling time was also high with almost 1000 seconds. Therefore to decrease the settling time up to a significant low value Ziegler-Nichols method was used to tune the parameters. Ziegler-Nichols method yielded a very nice settling time of 300 seconds but the overshoot increased significantly to a level of 36%. Without using these classical methods to tune the K_p, K_i and K_d at last we optimized the fractional parameters of FOPID only, i.e. λ and μ using Nelder-Mead optimization. Due to this the overshoot of the system decreased to 9% and settling time became 415 seconds, without any steady state error.

10. CONCLUSION

Thus, we successfully deigned the PID controller with fractional elements, i.e. FOPID controller for heating furnace. The plots of time response characteristics made it very clear that the fractional order model of furnace gave good response by using traditional tuning methods also. Although it was pretty clear that use of Ziegler-Nichols in FOM created a high overshoot in spite of a very quick response. The overshoot in furnace creates a sudden high pressure which may endanger the life of workers and properties, therefore this method was avoided. Whereas by using the classical IPDT method on FOM for tuning the PID, overshoot diminished significantly in spite of a sluggish response. So the furnace takes a long time to exhibit a steady state pressure, which results in unnecessary heat loss. But when only fractional elements of PID were optimized using Nelder-Mead optimization, the system established a very low overshoot and also a comparatively low settling time. Therefore, it can be concluded that more tuned the fractional elements are, more the response will be smooth and rapid. Various other tuning methods are also there to tune the proportional, derivative and integral coefficients of PID for FOM. These tuning methods can be further used along with various other optimization algorithms to tune the fractional elements λ and μ. A good optimization of λ and μ always yields a better response for heating furnace.

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