Implementation of Algebraic Method Based Selective Harmonic Elimination of Multilevel Inverter Using Artificial Neural Networks

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Abstract

Selective harmonics elimination (SHE) is one of the prominent technique that is used to produce required fundamental voltage of multilevel inverters with elimination of selective harmonic orders. The SHE equations are nonlinear equations and are also termed transcendental equations owing to the fact that these equations contain trigonometric terms. Attempt has been made in previous works to solve the SHE equation using broadly three techniques numeric, heuristic and algebraic. But solving SHE equations with minimum computational time and also maintaining minimum total harmonics distortion (THD) in output voltage still remains a challenging task. This is required for real time implementation of closed loop control of multilevel inverter. In this paper an artificial neural network switching control is designed for seven level multilevel inverter. The artificial neural network is trained with solutions of SHE obtained from algebraic method. The training set is collected from algebraic method because it gives best solutions of switching angles for minimum THD. The proposed neural network based switching control system is implemented on Simulink platform. The proposed neural network based switching control can easily be extended for real time closed loop control of applications like grid connected PV cells and variable frequency drives.

Keywords: Multilevel Inverter, Selective Harmonic Elimination, Heuristic Technique, Numerical Technique, Algebraic Technique, Artificial Neural Network.

1. INTRODUCTION
Multilevel inverters are being widely used in variety of fields like electrical drives, electrical vehicles, flexible AC transmission system (FACTS) etc. Also with the
Introduction of smart grids distributed energy sources especially renewable source is playing vital role where multilevel inverters have become indispensable part [1-4]. Multilevel inverters operate as interface between renewable energy source like solar power and the grid. The basic aim is to provide green energy to grid from solar power through multilevel inverter which is free from higher order harmonics except fundamental as per IEEE-519 Standards [5]. Power electronics based emerging technologies like multilevel inverter are now playing major part in green power and expected to reach US $27 billion in 2030 on the market [6]-[7]. Power electronics converters are in one way playing crucial role in reducing environmental pollution by integrating renewable with grid [8-10]. Multilevel inverters were introduced in year 1975 [11]. They have some impressive features like low dv/dt stress on switches, they can operate on both high and low switching frequencies, less electromagnetic compatibility [12-16].

Despite of remarkable features present in multilevel inverters, reducing Total Harmonics Distortion (THD) in the output ac voltage is still a challenging task. The output voltage produced by multilevel inverter can be mathematically modelled using Fourier series [17-18]. Previous works have shown that using Fourier series, the harmonic contents can be characterized in the form of set transcendental equations [19-20]. These transcendental equations are function of switching angles. One widely used strategy to reduce THD in output of multilevel inverter is to determine the value of switching angles in these sets of transcendental equations so as to eliminate harmonics we select and retain the desired. This technique is popularly known as selective harmonics elimination technique (SHE) [21].

Through literature survey it has been found that that broadly there are three methods used for solving transcendental equations of SHE. The first category is numerical method like Newton Raphson, Gauss-Newton method [22],[17],[23]-[24]. Heuristic approach or popularly known as evolutionary algorithms is second category. Few of widely used heuristic techniques in solving transcendental equations of SHE which have been reported in previous works are genetic algorithm, bee algorithm, particle swarm optimization, Grey-Wolf optimization and Bacterial foraging algorithm [25-33]. Third category is algebraic method. In this method, the set of transcendental equations of SHE are converted into algebraic form and then solved to find switching angles corresponding to minimum THD [34-39]. Applying well established techniques like resultant theory, symmetrical polynomials these algebraic equations can be solved. Although all these three types of techniques are effective in solving the nonlinear SHE equations but they have one or other short comings.

First of all, in numerical and heuristic approach are iterative techniques in which initial guess of the solution is required. Using this initial guess, these techniques find the optimum solution iteratively. Now selecting initial guess is very crucial in these methods. This is so because if initial guess is far away from the best solution then the algorithm may converge to local minima, which is not best solution.

Secondly so far there is no set method defined to select the initial guess which can guarantee convergence to best solution.
To overcome this problem algebraic method is used in which no initial guess is required. As algebraic method, with virtue of its completeness in mathematics give global minima that corresponds to best solution. But algebraic method suffers from huge computational burden since the nonlinear equations converted into algebraic form are of high degree polynomials and the degree increases with increase in level of the converter.

Thirdly, it can be observed that real time implementation of closed loop control of multilevel inverter using either of these techniques is not possible. This is because of iterative involved in numerical and heuristic approach and computational burden in algebraic method. Also to best of our knowledge no author has considered this issue also.

In this paper, we have proposed switching control of seven level multilevel inverter using multi-layered feed forward neural network. Using algebraic method, around 500 data sets have been calculated which consists of switching angles for different modulation index \((m)\) which ranges from \(0 < m < 1\). This training set is then used to train the multi-layered feed forward neural network. The trained neural network can be implemented with different types of loads in closed loop control of the converter. It is shown in the paper that once the neural network is trained then the switching angles can be calculated for given modulation index within bear minimum time which makes real time implementation of converter possible.

The proposed neural networks based switching control of multilevel inverter is implemented on Simulink platform.

The paper is divided into following sections: In section II mathematical model of SHE equations for seven level multilevel inverter is derived and then equivalent algebraic equations are obtained. A comparative analysis of numerical technique, heuristic technique and algebraic technique is done in terms of THD and computational time for solving SHE equations in section III. In section IV neural network based switching control of seven level multilevel inverter is proposed and implemented on Simulink platform.

2. MATHEMATICAL MODEL OF SHE

In this section mathematical model SHE equations of seven level inverter have been derived. A seven level cascaded multilevel inverter is shown in Figure 1. The staircase output voltage of seven level inverter is shown in Figure 2.
Since the SHE staircase waveform shown in Figure 2 has half wave and odd symmetry hence the Fourier series coefficient can be expressed as

\[ V_n = \frac{8}{T} \int_{0}^{T/4} f(t) \sin(n\omega_0 t)\,d(\omega_0 t) \].

For odd \( n \), all other values are zero. (1)

Considering period \( T = 2\pi \), equation (1) can be written as

\[ V_n = \frac{8}{2\pi} \int_{0}^{\pi/2} f(t) \sin(n\omega_0 t)\,d(\omega_0 t) \]. (2)
Substituting switching angles $\alpha_1, \alpha_2, \alpha_3$ from waveform shown in Figure 1(b), one can write equation (2) as

$$V_n = \frac{4V_{dc}}{\pi} \left\{ \left[ -\frac{\cos(n\omega_0 t)}{n} \right]^{\alpha_1}_{\alpha_2} + 2\left[ -\frac{\cos(n\omega_0 t)}{n} \right]^{\alpha_2}_{\alpha_3} + 3\left[ -\frac{\cos(n\omega_0 t)}{n} \right]^{\alpha_3}_{\alpha_3} \right\}$$

$$V_n = \frac{4V_{dc}}{\pi n} \left\{ -\cos(n\alpha_2) + \cos(n\alpha_1) - 2\cos(n\alpha_3) + 2\cos(n\alpha_2) - 3\cos\left(\frac{n\pi}{2}\right) + 3\cos(n\alpha_3) \right\}$$

Since only odd harmonics will be there

Hence $-3\cos\left(\frac{n\pi}{2}\right) = 0$

$$V_n = \frac{4V_{dc}}{\pi n} \left\{ \cos(n\alpha_1) + \cos(n\alpha_2) + \cos(n\alpha_3) \right\}$$

(3)

In equation (3), fundamental harmonic is for $n=1$ which in turn is $V_1$. Maximum fundamental voltage $V_{1\text{max}}$ is obtained when $\alpha_1 = \alpha_2 = \alpha_3 = 0$ which gives value $\frac{4V_{dc}}{\pi}$.

The modulation index is defined as the ratio of fundamental voltage ($V_1$) to the maximum obtainable fundamental voltage($V_{1\text{max}}$).

$$m = \frac{\pi V_1}{4*3V_{dc}} => 3m = \frac{\pi V_1}{4V_{dc}} => 3m = \frac{\pi V_1}{4V_{dc}}$$

The SHE equation for 7 level inverter can be obtained from equation (3) in terms of modulation index as follows

$$\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) = 3m$$

$$\cos(3\alpha_1) + \cos(3\alpha_2) + \cos(3\alpha_3) = 0$$

$$\cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3) = 0$$

$$\cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3) = 0$$

(4)

Condition required to be satisfy $0 < \alpha_1 < \alpha_2 < \alpha_3 < \frac{\pi}{2}$

Also in equation (4), $m$ is modulation index and is given as $m = \frac{\pi V_1}{4*3V_{dc}}$.

Equation (4) contains sets of transcendental equations with three unknowns switching angles $\alpha_1, \alpha_2$ and $\alpha_3$. With three phase delta connection, 3rd order harmonics can be eliminated. Considering this, equation (4) is reduced to...
\[ \cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) = 3m \]
\[ \cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3) = 0 \]
\[ \cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3) = 0 \]  \hspace{1cm} (5)

The task is to determine switching angles \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) such that fundamental harmonic is retained and 5\(^{th}\) and 7\(^{th}\) harmonics are eliminated. Other higher order harmonics are neglected.

Using trigonometric identities, the transcendental equations shown in (5) can be converted into algebraic as follows

Let, \( x_i = \cos(\alpha_i) \) for \( i=1, 2, 3 \)

\[ \cos(5\alpha) = 5 \cos(\alpha) - 20\cos^3(\alpha) + 16\cos^5(\alpha) \]
\[ \cos(7\alpha) = -7 \cos(\alpha) + 56\cos^3(\alpha) - 112\cos^5(\alpha) + 64\cos^7(\alpha) \]

Now substituting the identities in equation (5), the corresponding polynomial equations obtained are as follows

\[ p_1(x) = \sum_{i=1}^{3} x_i^3 - 3x_i = 0 \]
\[ p_2(x) = \sum_{i=1}^{3} 16x_i^5 - 20x_i^3 + 5x_i = 0 \]
\[ p_3(x) = \sum_{i=1}^{3} -112x_i^5 + 64x_i^7 + 56x_i^3 - 7x_i = 0 \]  \hspace{1cm} (6)

Equation (6) is the algebraic form corresponding to equation (5).

The equation (6) is solved using algebraic method. While solving equation (6) there can be multiple solutions which can be obtained, only those solutions \( (x_i) \) should be selected which satisfy following criterion

(i) All \( x_i \) should be real and not equal to each other.
(ii) All \( x_i \) are in the range \( 0 < x_i < 1 \).
(iii) The \( x_i \) obtained must satisfy condition \( 0 < x_3 < x_2 < x_1 < 1 \).

3. COMPARATIVE ANALYSIS AMONG NUMERICAL, HEURISTIC AND ALGEBRAIC TECHNIQUE IN TERMS OF THD AND COMPUTATIONAL TIME

In this section a comparative analysis of numerical, heuristic and algebraic method has been done in terms of THD and computational time for solving SHE equations. In numerical technique Newton Raphson (NR) has been used, Genetic Algorithm (GA) has been used for solving SHE equations under heuristic approach category, and lastly polynomial equations obtained in (3) are solved using Algebraic Method (AM).

The switching angles obtained are fed to the simulation model of 7 level multilevel inverter obtained on MATLAB 2010 platform and THD is obtained. Table-1 shows the switching angles \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) obtained for given modulation index using NR, GA and AM approach. We have considered ten different values of modulation index \( (m) \) for comparative study in the range of \( 0 < m < 1 \), where the values of \( m \) are considered randomly. From Table 1, one can observe that AM gives minimum THD for all ranges of modulation index considered. However the computational time is
high. Hence one can conclude that AM ensures global minima so as to get switching angles which give minimum THD. The switching angles obtained from different methods at modulation index 0.6 shown in Table 1 are fed into Simulink model shown in Figure 2 to calculate THD.

**Table 1.** Comparative Analysis of different approaches in terms of computational time and THD

<table>
<thead>
<tr>
<th>S.NO.</th>
<th>M</th>
<th>Method</th>
<th>Alpha1</th>
<th>Alpha2</th>
<th>Alpha3</th>
<th>THD</th>
<th>CT(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.5</td>
<td>NR</td>
<td>39.4662</td>
<td>56.2204</td>
<td>80.0959</td>
<td>47.64</td>
<td>0.000032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GA</td>
<td>25.7102</td>
<td>29.1908</td>
<td>29.8881</td>
<td>27.46</td>
<td>0.000004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>20.45</td>
<td>56.1237</td>
<td>89.6768</td>
<td>22.96</td>
<td>0.144464</td>
</tr>
<tr>
<td>2.</td>
<td>0.6</td>
<td>NR</td>
<td>35.4881</td>
<td>54.7602</td>
<td>67.1077</td>
<td>41.35</td>
<td>0.000044</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GA</td>
<td>25.8419</td>
<td>29.4249</td>
<td>31.3506</td>
<td>26.94</td>
<td>0.000004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>11.8257</td>
<td>41.7108</td>
<td>85.7153</td>
<td>17.88</td>
<td>0.145041</td>
</tr>
<tr>
<td>3.</td>
<td>0.73</td>
<td>NR</td>
<td>13.5905</td>
<td>36.5825</td>
<td>61.6406</td>
<td>15.74</td>
<td>0.000045</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GA</td>
<td>24.0767</td>
<td>29.6574</td>
<td>31.0186</td>
<td>25.8</td>
<td>0.000004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>15.1665</td>
<td>39.5716</td>
<td>62.9993</td>
<td>17.91</td>
<td>0.145121</td>
</tr>
<tr>
<td>4.</td>
<td>0.80</td>
<td>NR</td>
<td>12.5781</td>
<td>23.7943</td>
<td>54.2676</td>
<td>13.22</td>
<td>0.000054</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GA</td>
<td>25.7102</td>
<td>28.8365</td>
<td>30.9072</td>
<td>26.91</td>
<td>0.000005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>11.5042</td>
<td>28.7167</td>
<td>57.106</td>
<td>12.54</td>
<td>0.145922</td>
</tr>
<tr>
<td>5.</td>
<td>0.919</td>
<td>NR</td>
<td>16.8632</td>
<td>17.05986</td>
<td>51.3912</td>
<td>18.16</td>
<td>0.000042</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GA</td>
<td>24.3561</td>
<td>30.2313</td>
<td>31.0186</td>
<td>26.12</td>
<td>0.000004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>10.1596</td>
<td>13.7092</td>
<td>36.7582</td>
<td>19.17</td>
<td>0.146689</td>
</tr>
</tbody>
</table>

**4. SWITC**

**H. CONTROL OF MULTILEVEL INVERTER USING ARTIFICIAL NEURAL NETWORK**

In this section we implement the switching control of a seven level multilevel inverter using multi-layered feed forward neural network. The architecture of the neural network is shown in Figure 3. It consists of one neuron in input layer which takes modulation index as input. Three neurons in output layer which give switching angles $\alpha_1, \alpha_2, \alpha_3$ as output. The number of neurons required in hidden layer are obtained after training which can give minimum error. To train the neural network first training data was collected in which switching angles of different modulation index were calculated using algebraic method.
Algebraic method was chosen for collecting training data because this method gives switching angles corresponding to minimum THD as compared to numerical method and heuristic approach. This has already been verified in previous section. The modulation index was varied within the range $0 < m < 1$ with step size of 0.001 and the switching angles obtained using algebraic method are plotted in Figure 4.

One can observe from Figure 4 that for some values of modulation index no switching angles are plotted. This signifies that no solution exists for such values of modulation index which could satisfy conditions mentioned for viable solution in section II. Also
Implementation of Algebraic Method Based Selective Harmonic Elimination.

for some values of modulation index, multiple solutions exist which satisfy the viable solution condition mentioned in section II as shown in Table 2. In such cases only those solutions were selected as training data for neural network which gave minimum THD. For example in Table 2, at $m=0.497$, there are two viable solutions and in this case we have selected the solution as training data which gives minimum THD. For range of modulation index where no viable solution exist in algebraic method, the data set is taken from solutions obtained using genetic algorithm for training.

Table 2. Modulation index with multiple viable solutions

<table>
<thead>
<tr>
<th>S. No.</th>
<th>$m$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.497</td>
<td>20.53756696</td>
<td>56.43731575</td>
<td>89.90210055</td>
<td>22.93</td>
</tr>
<tr>
<td>2.</td>
<td>0.520</td>
<td>19.70028086</td>
<td>53.96616168</td>
<td>88.26549724</td>
<td>23.18</td>
</tr>
</tbody>
</table>

The neural network was presented with training data and trained using backpropagation algorithm. During training, the optimal number of neurons in hidden layer which gave minimum error in training were found to be 280. The performance of trained artificial neural network (ANN) has been tested with unseen modulation i.e. that value of $m$ for which it is not trained. The switching angles calculated for unseen modulation index by neural network and actual values calculated by algebraic method (AM) is shown in Table 3.

Table 3. Performance of ANN for unseen data

<table>
<thead>
<tr>
<th>S. No.</th>
<th>$m$</th>
<th>Methods</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.2735</td>
<td>ANN</td>
<td>46.36197474</td>
<td>82.952047</td>
<td>89.54162887</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>46.37817884</td>
<td>82.99475706</td>
<td>89.50469833</td>
</tr>
<tr>
<td>2.</td>
<td>0.4528</td>
<td>ANN</td>
<td>39.51122072</td>
<td>60.21808714</td>
<td>84.84214581</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>39.51419886</td>
<td>60.22078187</td>
<td>84.82063592</td>
</tr>
<tr>
<td>3.</td>
<td>0.5956</td>
<td>ANN</td>
<td>12.76098228</td>
<td>42.85308853</td>
<td>85.50914364</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>12.68134985</td>
<td>42.83189465</td>
<td>85.53544144</td>
</tr>
<tr>
<td>4.</td>
<td>0.7854</td>
<td>ANN</td>
<td>11.76278709</td>
<td>31.36157683</td>
<td>58.59393372</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>11.68167625</td>
<td>31.17796551</td>
<td>58.57721874</td>
</tr>
<tr>
<td>5.</td>
<td>0.8356</td>
<td>ANN</td>
<td>14.30364872</td>
<td>20.77185505</td>
<td>52.92930282</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AM</td>
<td>14.17030464</td>
<td>20.76171916</td>
<td>52.97489368</td>
</tr>
</tbody>
</table>
One can observe from Table 3 that the switching angles estimated by trained ANN are almost same as actual values estimated using AM. Simulation model of 7 level multilevel inverter with reference voltage generation based on neural network switching control is shown in Figure 5. The output voltage waveform and its bar chart of the FFT of the seven level inverter based on neural network for modulation index of 0.6 are shown in Figure 6 (a) & (b) and both voltage waveform and its FFT bar chart for modulation index of 0.8 are shown in Figure 7 respectively.

**Figure 5.** Simulink model of neural network based 7-level Inverter
Figure 6. Output voltage waveform and its Bar chat of FFT for 0.6 modulation index
In FFT bar chart, fundamental magnitude of output voltage is 100%, which is at 50 Hz and magnitude other harmonics components is show in percentage of fundamental voltage, which are higher frequency. It is noticed that 5th and 7th order harmonics are completely eliminated from the output voltage of the seven level inverter. In addition, magnitude of output voltage is increased by the ratio of modulation index and THD is improved from 16.97% at 0.6 modulation index to 11.16% at 0.8 modulation index.

5. CONCLUSION
In this paper a neural network based switching angle control of seven level multilevel inverter is implemented on SIMULINK platform. The main feature of the presented work is that the neural network designed is trained to calculate switching angles for given modulation index. For this, training data is collected from solutions of SHE equation obtained using algebraic method. Algebraic method is used to since it given solutions in terms of switching angles which corresponds to minimum THD. This has been proved using comparative study between different SHE equation solving techniques that algebraic method gives minimum THD.

In this paper two major advantages achieved

(i) Implementation of neural network based switching control, which can calculate switching angles for given modulation index in minimum time. This makes the proposed system applicable for closed loop control in real time.

(ii) Since the neural network is training with data collected from algebraic method hence the switching angle solution produced will ensure minimum THD.

At the end it is expected that the proposed methodology can be extended to designed a closed loop control for a particular type of load.
REFERENCES


