DTT Based Orthogonal Matrix for Image Compression

Tanveer Hasan and Muhammed Yusuf Khan

Faculty of Engineering & Technology,
Aligarh Muslim University, Aligarh

Abstract

This paper proposes an efficient orthogonal sparse $8 \times 8$ transform matrix for image compression. The transform matrix is derived from Signed Discrete Tchebichef Transform (SDCT). By using various natural test images, it is demonstrated that the Subjective Quality and Energy Compaction are comparable with Discrete Tchebichef Transform (DTT) with reduction in computation complexity.

Keywords: DTT, SDTT, Energy compaction.

I. INTRODUCTION:

Image Transforms using orthogonal kernel functions are used in image compression. Discrete Cosine Transform (DCT), used in JPEG compression standard [1] is one of them. The devices such as Personal digital assistants (PDAs), digital cameras and mobile phones need a lot of image transmission and processing capability. It becomes necessary to have image compression methods, which could be scalable and applicable to such portable devices. Discrete Tchebichef Transform (DTT), which is obtained from a discrete class of popular Tchebichef polynomials, is a orthonormal version of orthogonal transform. It has applications in image analysis and compression [2-8]. R. Mukundan [4] has proposed orthonormal version of Tchebichef moments and analysed some of their computational features. Mukundan and Hunt [6] have shown that, for natural images, DTT and DCT exhibit almost identical energy compactness. In order to compute Tchebichef moments, many fast algorithms have been developed [4], [8-9]. Ishwar et al. [8], have shown that DTT has lower complexity as it needs the evaluation of only algebraic expressions whereas; implementation of DCT requires integer approximation like Integer cosine transform (ICT) [10]. The features of DTT are as follows: i. A discrete domain of definition which matches exactly with image coordinates space. ii. Absence of numerical
approximation terms allows better representation of image features than others which is not possible using conventional transforms [11]. The paper is organized as follows section II gives details about Signed DTT & similar transforms used. Section III gives description about proposed algorithm and finally results are discussed in section IV.

II. SIGNED DISCRETE TCHEBICHEF TRANSFORM
For 8 x 8 input data matrix X, the two dimensional DTT, of order 8x8 is calculated as [8]:

\[ Y = T_{DTT} X T_{DTT}^T \]  

(1)

Where

\[
T_{DTT} = \begin{bmatrix}
0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 & 0.3536 \\
-0.5401 & -0.3858 & -0.2315 & -0.0772 & 0.0772 & 0.2315 & 0.3858 & 0.5401 \\
0.5401 & 0.0772 & -0.2315 & -0.3858 & -0.3858 & -0.2315 & 0.0772 & 0.5401 \\
-0.4308 & 0.3077 & 0.4308 & 0.1846 & -0.1846 & -0.4308 & -0.3077 & 0.4308 \\
0.2820 & -0.5238 & -0.1209 & 0.3626 & 0.3626 & -0.1209 & -0.5238 & 0.2820 \\
-0.1498 & 0.4922 & -0.3638 & -0.3210 & 0.3210 & 0.3638 & -0.4922 & 0.1498 \\
0.0615 & -0.3077 & 0.5539 & -0.3077 & 0.3077 & 0.5539 & -0.3077 & 0.0615 \\
-0.0171 & 0.1195 & -0.3585 & 0.5974 & -0.5974 & 0.3585 & -0.1195 & 0.0171
\end{bmatrix}
\]  

(4)

The SDTT is obtained by applying the sign function operator to the elements of DTT. Therefore, it is given as:

\[ T_{SDTT} = \frac{1}{\sqrt{N}} \text{sign}(T_{DTT}) \]  

(2)

where \( \text{sign}\{ \}\) is the signum function defined as:

\[
\text{sign}\{x\} = \begin{cases} 
+1, & \text{if } x > 0 \\
0, & \text{if } x = 0 \\
-1, & \text{if } x < 0 
\end{cases}
\]  

(3)

Several advantages of SDTT are apparent. These are (i) All the elements are 0,1,-1. No multiplication operation or transcendental expressions are required. Unlike WHT (Beauchamp, 1984) [12], RSWT (Pendar and Cover, 1992)[13] and SDFT (Haweel and Alhasan,1995)[14], the transform order need not be an integer power of 2. SDTT maintains the periodicity and spectral structure of its originating DTT and maintains good de-correlation and energy compaction characteristics. As an example, the 8 x 8 SDTT transform matrix is given by:

\[
T_{SDTT} = \left( \frac{1}{\sqrt{8}} \right) \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
-1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & -1 & -1
\end{bmatrix}
\]  

(4)
III. PROPOSED 8×8 TRANSFORM MATRIX

The proposed 8 × 8 transform matrix can be obtained by appropriately inserting some 0s and 0.5s into the SDT matrix in equation (4). The proposed matrix contains 24 zeros. The matrix is obtained by analyzing work of Hawel [15], Senapati et al [16] & Bougueaee [17]. Multiplication of input pixel with 0.5 is just a shift operation. So multipliers are not required during transform stage. This makes the transformation faster. The matrix is shown in equation (5).

\[ T_1 = \begin{pmatrix} 
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0.5 & -0.5 & -1 & -1 & -0.5 & 0.5 & 1 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
-1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\
0.5 & 0 & 0 & -0.5 & -0.5 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 
\end{pmatrix} \]  \hspace{1cm} (5)

where \( D \) is a diagonal matrix. It is expressed as: \( D = \text{diag}(1, \sqrt{2}, 2\sqrt{2}/5, 2, 1, \sqrt{2}, 2\sqrt{2}/5, 2) \). Take,

\[ D_2 = \begin{pmatrix} 
D 
\end{pmatrix} \]  \hspace{1cm} (6)

The transform matrix of equation (5) can be represented as:

\[ T_1 = D_2 T_2 \]  \hspace{1cm} (7)

\[ T_2 = \begin{pmatrix} 
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0.5 & -0.5 & -1 & -1 & -0.5 & 0.5 & 1 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
-1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\
0.5 & 0 & 0 & -0.5 & -0.5 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 
\end{pmatrix} \]

It can be observed that the matrix \( T_1 \) satisfies the property of orthogonality, i.e., \( T_1^T = T_1^{-1} \) where \( T \) denotes the transpose operation. So, we can use the same matrix for image encoding and image decoding. Let \( X \) be an 8 × 8 block of image data and let \( Y \) be its corresponding matrix, in transformed domain. Then, the forward transform operation will be:

\[ Y = T_1 X T_1^T \]  \hspace{1cm} (8)

Since \( T_1 \) is orthogonal, we can reconstruct the image by using the reverse transform given as:

\[ X = T_1^T Y T_1 \]  \hspace{1cm} (9)

The proposed matrix in equation (5) can be used for image compression.
IV. RESULTS
The parameter Compaction Ratio is helpful in evaluating the performance of the system.

**Compaction Ratio**: The energy distribution after transformation is such that major portion of energy is in low frequency coefficients and energy goes on decreasing for high frequency coefficients. The energy distribution follows zig zag pattern, as shown in following Figure 1. The energy is proportional to square of amplitude. The compaction ratio is equal to the ratio of sum of squares of first four coefficients of transformed matrix to sum of squares of all sixty four coefficients. This parameter indicates the fraction of energy in first four low frequency coefficients to total energy of 8x8 block.

![Figure 1. Zig Zag scanning of coefficients](image)

**Table 1**: Results for Various Images

<table>
<thead>
<tr>
<th>S.No</th>
<th>Test Image</th>
<th>Compaction Ratio DTT</th>
<th>Compaction Ratio Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Boat</td>
<td>98.9552</td>
<td>98.7556</td>
</tr>
<tr>
<td>2</td>
<td>Cameraman</td>
<td>97.8264</td>
<td>97.5351</td>
</tr>
<tr>
<td>3</td>
<td>Car</td>
<td>94.4845</td>
<td>94.2332</td>
</tr>
<tr>
<td>4</td>
<td>Circuit Board</td>
<td>99.9552</td>
<td>98.9766</td>
</tr>
</tbody>
</table>

The results are obtained for various test images by calculating Energy Compaction using DTT matrix & proposed matrix. The results are summarized in Table 1. The energy compaction and subjective quality of reconstructed image is comparable in both cases (using DTT & Proposed Transform) with lesser computational complexity for proposed transform in comparison to DTT.
REFERENCES


