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Local Stability Predator Prey Model with Harvesting and The Presence of Toxicity

Saiful Marom

Department of Mathematics Education,

Salatiga State Islamic University (UIN Salatiga), Salatiga, Indonesia

Abstract

The purpose of this study is to examine the dynamics of the prey-predator model with a harvesting function, a threshold, and the presence of poison in both prey and predator populations. The author uses theoretical methods and numerical simulations to investigate the limitations of the solution, the existence of the equilibrium point, and the local stability of the equilibrium point of a system that has been formed. Based on analytical and numerical studies, it can be concluded that when the system is given a predation policy in accordance with the threshold, it will maintain its stability.

Key Words: Predator Prey Models, Threshold Harvesting, Local Stability

1. Introduction

Population dynamics of one or more species in the prey-prey model is widely developed because of its many applications in various fields [1], [2], and [3]. Overexploitation of resources and extinction of a species by harvesting or poisoning are major concerns in ecology and industry [4] and [5]. In this research, we will study the harvesting component in a mathematical model and include a toxic component in an ecosystem to study the effects on one or more species [6] and [7]. This has attracted interest from the commercial harvesting industry and from many scientific communities, including the biological, ecological, and economic communities [8].

Most prey-prey models consider a constant harvest function [9] and [10]. Currently, the solution limitations of the prey-prey model in general and the stability of the model with rational harvesting of prey and quadratic harvesting of predators have been studied in [11]. Harvesting starting at $t = 0$ with a large population size is not very realistic. In this study, a threshold harvesting policy will be given by considering starting harvesting when the population x has reached a certain harvesting threshold T. Furthermore, the harvesting function with threshold will be defined as follows:

$$\mathrm{H}(x) = \begin{cases} 0 & \text{if } x \leq \mathrm{T}, \\ \frac{\mathrm{h}(x - \mathrm{T})}{\mathrm{h} + x - \mathrm{T}} & \text{if } x > \mathrm{T}. \end{cases} \quad (1)$$

Once the prey population reaches the size $x = \mathrm{T}$, the harvesting will start and increase until it reaches the harvesting threshold value h. The following numerical simulations will be given in relation to the harvesting function, namely with $h = 0{,}5$ and $T = 0{,}8$ so that obtained:

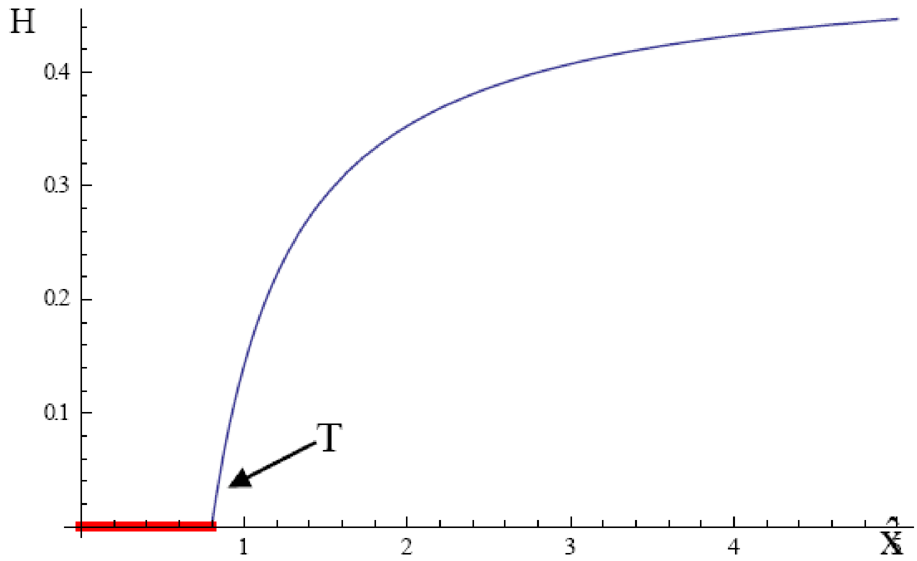

Figure 1: Harvesting function graphs

2. Methods

The focus of this research is to examine the local stability of the prey-prey model with Michaelis-Menten response function and harvesting function (1) in the presence of poison in the ecosystem.

$$\frac{dx}{dt} = x(1 - x) - \frac{axy}{(1 + mx)} - jx^3 - H(x)$$

$$\frac{dy}{dt} = y\left(-d + \frac{bx}{(1+mx)}\right) - ly^2, \quad (2)$$

With x and y, the number of prey and predator populations Furthermore, the parameters a, b, d and m are positive constants, with a being the capture rate of prey, b being the conversion rate of prey, d being the natural mortality rate of predators, j being the mortality rate of prey because of toxins, l being the mortality rate of predators because of toxins, and $1 + mx$ being the Holling type II function. In this article, the equilibrium point of system (2), which is located in the first quadrant when the number of prey and predator populations is $x \geq 0$ dan $y \geq 0$.

3. Results and Discussion

Boundedness of solutions

The author begins to demonstrate the local stability of system (2) by showing the existence of solutions to system (2) starting and bounded at $R^2{}_+$ as in the theorem below:

Theorem 1.

Every solution of system (2) that starts in $R^2{}_+$ is uniformly bounded.

Proof:

Let $x \geq T$ and let $x = v + \frac{a}{b}$ then for all $c > 0$:

$$\frac{dv}{dt} + cv = -jx^3 - x^2 + (c+1)x - ly^2 + y\left(\left(\frac{a}{b}\right)(c - \tilde{d})\right) - \frac{h(x-T)}{h+x-T} \leq\leq \frac{(c+1)^2}{4} + y\left(\left(\frac{a}{b}\right)(c - \tilde{d})\right).$$

Let $c \leq d$. Then there exist $A = \frac{(c+1)^2}{4} > 0$ such that $\frac{\mathrm{dv}}{\mathrm{dt}} + \mathrm{cv} \leq \mathrm{A}$ or $\frac{dv}{dt} \leq A - cv$. Let $\frac{dr}{dt} = A - cr$, where $r(0) = v(0) = v_0$, the solution is:

$$r(t) = \frac{A}{c} + \left(v_0 - \frac{A}{c}\right)e^{-ct}$$

is bounded for $t \geq 0$. As a result, using a differential inequality (3), we get:

$$0 < v(t) \leq r(t) = \frac{A}{c} + \left(v_0 - \frac{A}{c}\right)e^{-ct}$$

As $t \to \infty$, such that:

$$0 < v(t) \leq r(t) = \frac{A}{c} + \left(v_0 - \frac{A}{c}\right)e^{-ct} \leq \frac{A}{c}$$

Therefore, every solutions of (2) where $x \geq T$, starting in $R^2{}_+$ is uniformly bounded.

Remarks 1: The proof for the case where $x < T$ follows directly from the above proof, using the same bound on c.

Equilibrium Point for $x < T$

When the prey population size is less than the harvesting threshold, the system (2) has two equilibrium points, namely:

$$P_1(x_1, y_1) = \left(\frac{-1 + \sqrt{1+4j}}{2j}, 0\right)$$

and

$$P_2(x_2, y_2) = \left(\frac{d + ly_2}{(\beta)}, \left(\frac{b}{a}\right)\left(\frac{b - dm - d - \alpha}{\beta^2}\right)\right)$$

With $\beta = b - dm - lmy_2$ and $\alpha = lmy_2 + ly_2 + j(d + ly_2)^2$.

Thus, for $(x,y) \in R^2{}_+$, then $b > (m+1)d + \alpha$, in other words, the prey conversion rate must be greater than the natural prey mortality rate and the toxic mortality rate of both prey and predator.

The general jacobian matrix of (2) for $x < T$ i around the equilibrium point is:

$$J(x,y) = \begin{bmatrix} 1 - 2x - \dfrac{ay}{(1+mx)^2} - 3jx^2 & -\dfrac{ax}{1+mx} \\ \dfrac{by}{(1+mx)^2} & -d + \dfrac{bx}{1+mx} - 2ly \end{bmatrix}$$

Thus, at $P_1(x_1,y_1)$ it becomes:

$$J(x_1,y_1) = \begin{bmatrix} \left(\dfrac{-(1+4j)+\sqrt{1+4j}}{2j}\right) & -\dfrac{\left(a\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)} \\ 0 & -d + \dfrac{\left(b\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)} \end{bmatrix}$$

Next, we will find the eigenvalues of the matrix $J(x_1,y_1)$, so that:

$$det\left(\lambda I - J(x_1,y_1)\right) = 0$$

$$\Leftrightarrow \begin{vmatrix} \lambda - \left(\dfrac{-(1+4j)+\sqrt{1+4j}}{2j}\right) & \left(\dfrac{\left(a\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)}\right) \\ 0 & \lambda + d - \left(\dfrac{\left(b\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)}\right) \end{vmatrix} = 0$$

So the eigenvalue obtained is:

$$\lambda_1 = \frac{-(1+4j)+\sqrt{1+4j}}{2j}$$

and

$$\lambda_2 = -d + \frac{\left(b\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)}$$

Thus, based on the eigenvalues obtained, the author concludes that the $P_1(x_1,y_1)$ equilibrium point of (2) has the following properties:

If j > 0 then $1+4j > 0$ results in $\sqrt{1+4j} > 0$ thus obtained:

$$\lambda_1 = \frac{-(1+4j)+\sqrt{1+4j}}{2j} < 0$$

and

$$\lambda_2 = -d + \frac{\left(b\left(-1+\sqrt{1+4j}\right)\right)}{\left(2j+m\left(-1+\sqrt{1+4j}\right)\right)} < 0$$

then the equilibrium point $P_1(x_1, 0) = \left(\frac{-1+\sqrt{1+4j}}{2j}, 0\right)$ is Local Asymptotic Stable.

Thus, at $P_2(x_2, y_2) = \left(\frac{d+ly_2}{(\beta)}, \left(\frac{b}{a}\right)\left(\frac{b-dm-d-\alpha}{\beta^2}\right)\right)$ it becomes:

$$J(x_2, y_2) = \begin{bmatrix} 1-2x_2-\dfrac{ay_2}{(1+mx_2)^2}-3jx^2 & -\dfrac{ax_2}{1+mx_2} \\ \dfrac{by}{(1+mx_2)^2} & -d+\dfrac{bx_2}{1+mx_2}-2ly_2 \end{bmatrix}$$

Next, we will find the eigenvalue of the matrix $J(x_2, y_2)$, so that we get:

$$det\left(\lambda I - J(x_2, y_2)\right) = 0$$

$$\Leftrightarrow \begin{vmatrix} \lambda - \dfrac{\beta^2\eta-\mu(2b\beta+3bj\mu)}{b\beta^2} & \dfrac{a\mu}{b} \\ -\dfrac{\left(b-\tilde{d}m-\tilde{d}-\alpha\right)}{a} & \lambda+2ly_2 \end{vmatrix} = 0$$

So the eigenvalue is obtained:

$$\lambda_{1,2} = \frac{A \pm \sqrt{A^2-4B}}{2}$$

with $A = -2ly_2 + \frac{\beta^2\eta-\mu(2b\beta+3bj\mu)}{b\beta^2}$ and

$$B = -\left(\frac{\beta^2\eta-\mu(2b\beta+3bj\mu)}{b\beta^2}\right)(2ly_2) + \frac{\mu}{b}(b-dm-d-\alpha)$$

Suppose $\Delta = A^2 - 4B$, so that it is obtained:

$$\Delta = 4l^2y_2^2 + \left(\frac{\beta^2\eta-\mu(2b\beta+3bj\mu)}{b\beta^2}\right)^2 + 4\left(\frac{\beta^2\eta-\mu(2b\beta+3bj\mu)}{b\beta^2}\right) - \frac{4\mu}{b}\left(b-\tilde{d}m-\tilde{d}-\alpha\right)$$

Thus, based on the eigenvalues obtained, the author concludes that the $P_2(x_2, y_2)$ equilibrium point of (2) has the following properties:

1. If $\Delta \geq 0$ or $\Delta < 0$ and $b > d(m+1) + \alpha + \left(\frac{b\left(\beta^2\eta-\mu(2b\beta+3bj\mu)\right)}{\mu\beta^2}\right)(ly_2)$, $b > \eta\left(\frac{\beta^2}{\beta^2 2ly_2+\mu(2\beta+3j\mu)}\right)$ then the equilibrium point $P_2(x_2, y_2)$ is local asymptotically stable.

2. If $\Delta \geq 0$ or $\Delta < 0$ and $b > d(m+1) + \alpha + \left(\frac{b\left(\beta^2\eta - \mu(2b\beta + 3bj\mu)\right)}{\mu\beta^2}\right)(ly_2)$, as well as $b < \eta\left(\frac{\beta^2}{\beta^2 2ly_2 + \mu(2\beta + 3j\mu)}\right)$ then the equilibrium point $P_2(x_2, y_2)$ is unstable.

Equilibrium Point for $x \geq T$

When the prey population size is less than the harvesting threshold, the system (2) has two equilibrium points, namely:

$P_3(x_3, y_3) = (x^*, 0)$

With x^* is one of the positive real solutions of equation:

$jx^4 - (j - jh - 1)x^3 + (h - T - 1)x^2 + Tx - Th = 0$

and

$$P_4(x_4, y_4) = \left(\frac{d + ly_4}{b - dm - mly_4}, \frac{(1 + mx_4)\left(x_4(1 - x_4 - jx_4^2)(h + x_4 - T) - h(x_4 - T)\right)}{a(h + x_4 - T)x_4}\right)$$

With $\beta = b - dm - lmy_2$ and $\alpha = lmy_2 + ly_2 + j(d + ly_2)^2$.

Thus for $(x, y) \in R^2{}_+$ then $b > (m + l)d + \alpha$, in other words, the prey conversion rate must be greater than the natural prey mortality rate and the toxic mortality rate of both prey and predator.

The general jacobian matrix of (2) for $x \geq T$ around the equilibrium point is:

$$J(x, y) = \begin{bmatrix} 1 - 2x - \frac{ay}{(1 + mx)^2} - 3jx^2 & -\frac{ax}{1 + mx} \\ \frac{by}{(1 + mx)^2} & -d + \frac{bx}{1 + mx} - 2ly \end{bmatrix}$$

Thus, at $P_3(x_3, y_3)$ it becomes:

$$J(x_3, y_3) = \begin{bmatrix} 1 - 2x_3 - 3jx_3^2 - \frac{h^2}{(h + x_3 - T)^2} & -\frac{ax_3}{1 + mx_3} \\ 0 & -\tilde{d} + \frac{bx_3}{1 + mx_3} \end{bmatrix}$$

Next, we will find the eigenvalue of the matrix $J(x_3, y_3)$, so that it is obtained:

$det\left(\lambda I - J(x_3, y_3)\right) = 0$

$$\Leftrightarrow \begin{vmatrix} \lambda - 1 + 2x_3 + 3x_3^2 + \frac{h^2}{(h + x_3 - T)^2} & \frac{ax_3}{1 + mx_3} \\ 0 & \lambda + \tilde{d} - \frac{bx_3}{1 + mx_3} \end{vmatrix} = 0$$

So the eigenvalue is obtained:

$$\lambda_1 = 1 - 2x_3 - 3x_3^2 - \frac{h^2}{(h + x_3 - T)^2}$$

and

$$\lambda_2 = \frac{bx_3}{1 + mx_3} - d$$

Thus, based on the eigenvalues obtained, the author concludes that the $P_3(x_3, y_3)$ equilibrium point of (2) has the following properties:

If $\frac{h^2}{(h+x_3-T)^2} \geq 0$ then $1 - 2x_3 - 3x_3^2 - \frac{h^2}{(h+x_3-T)^2} < 1 - 2x_3 - 3x_3^2$

Suppose $x_3 = T$ then obtained $1 - 2x_3 - 3x_3^2 - \frac{h^2}{(h+x_3-T)^2} < 1 - 2T - 3T^2$ so that $\lambda_1 < 0$ is only satisfied when $T > \frac{1}{3}$ and if $\lambda_2 < 0$ then obtained $x_3 < \frac{d}{(b-dm)}$. Based on this, we can conclude that $P_3(x_3, y_3)$ is locally asymptotically stable when $T > \frac{1}{3}$ and d< $\frac{d}{(b-dm)}$.

Thus, at $P_4(x_4, y_4) = \left(\frac{d+ly_4}{b-dm-mly_4}, \frac{(1+mx_4)\left(x_4(1-x_4-jx_4^2)(h+x_4-T)-h(x_4-T)\right)}{a(h+x_4-T)x_4}\right)$ it becomes:

$$J(x_4, y_4) = \begin{bmatrix} 1 - 2x_4 - \frac{ay_4}{(1 + mx_4)^2} - 3jx_4^2 - \frac{h^2}{(h + x_4 - T)^2} & -\frac{ax_4}{1 + mx_4} \\ \frac{by_4}{(1 + mx_4)^2} & -d + \frac{bx_4}{1 + mx_4} - 2ly_4 \end{bmatrix}$$

Next, we will find the eigenvalue of the matrix $J(x_4, y_4)$, so that it is obtained:

$$det\big(\lambda I - J(x_4, y_4)\big) = 0$$

$$\Leftrightarrow \begin{vmatrix} \lambda - \left(1 - 2x_4 - 3jx_4^2 - \left(\frac{\left(x_4(1 - x_4 - jx_4^2)(h + x_4 - T) - h(x_4 - T)\right)}{(1 + mx_4)(h + x_4 - T)x_4}\right)\right) - \frac{h^2}{(h + x_4 - T)^2} & \frac{a(\tilde{d} + ly_4)}{b} \\ -\frac{b\left(x_4(1 - x_4 - jx_4^2)(h + x_4 - T) - h(x_4 - T)\right)}{a(1 + mx_4)(h + x_4 - T)x_4} & \lambda + ly_4 \end{vmatrix} = 0$$

with $A = \left(x_4(1 - x_4 - jx_4^2)(h + x_4 - T) - h(x_4 - T)\right)$ and $B = (1 + mx_4)(h + x_4 - T)x_4$ so that it is obtained $E = \frac{A}{B} + \frac{h^2}{(h+x-T)^2}$.

Suppose:

$$\Delta = (ly_4 + 2x_4 + 3jx_4^2 + E - 1)^2 - 4\left(\frac{A(d + ly_4)}{B}\right)$$

So the eigenvalue is:

$$\lambda_{1,2} = \frac{-(ly_4 + 2x_4 + 3jx_4^2 + E - 1) \pm \sqrt{(ly_4 + 2x_4 + 3jx_4^2 + E - 1)^2 - 4\left(\frac{A(d + ly_4)}{B}\right)}}{2}$$

Thus, based on the eigenvalues obtained, the author concludes that the $P_4(x_4, y_4)$ equilibrium point of (2) has the following properties:

1. If $\Delta > 0$ and $K = b - m(d + ly_4)$, $L = d + ly_4, B = (1 + mx_4)(h + x_4 - T)x_4$, $A = \left(x_4(1 - x_4 - jx_4^2)(h + x_4 - T) - h(x_4 - T)\right)$, $K\left(K\left(1 - (E + ly_4)\right) - 2L\right) > 3j$ then the equilibrium point $P_4(x_4, y_4)$ is unstable.
2. If $\left(\frac{\left(K\left(K\left(1-(E+ly_4)\right)-2L\right)\right)-3jL}{K^2}\right)^2 > 4\left(\frac{AL}{B}\right)$ or $\left(\frac{\left(K\left(K\left(1-(E+ly_4)\right)-2L\right)\right)-3jL}{K^2}\right)^2 < 4\left(\frac{AL}{B}\right)$ and $K\left(K\left(1 - (E + ly_4)\right) - 2L\right) < 3j$ then the equilibrium point $P_4(x_4, y_4)$ is local asymptotically stable.
3. If $\left(\frac{\left(K\left(K\left(1-(E+ly_4)\right)-2L\right)\right)-3jL}{K^2}\right)^2 < 4\left(\frac{AL}{B}\right)$ and $K\left(K\left(1 - (E + ly_4)\right) - 2L\right) < 3j$ then the equilibrium point $P_4(x_4, y_4)$ is unstable.

It is interesting to see how the stability properties of some equilibrium points of model (2) may change for cases where the prey population is greater than or equal to the threshold and the prey population is less than the threshold population. For example, how does harvesting affect the stability of the ecosystem equilibrium of $P_2(x_2, y_2)$, and what effect does it have on possible periodic solutions.

Example 1:

Given the parameters of $r = 10, n = 22.5, k = 1, p = 5, f = 2, s = 25$ and the time $1,2, \ldots, 200$ in system (2) so that it is obtained:

$$a = 2.5, b = \frac{nK}{r} = 2.25, d = \frac{f}{r} = 0.2, m = pK = 5$$

and then we get $P_2(x_2, y_2) = (0.16, 0.61)$ According to the figure below, System (2) is unstable:

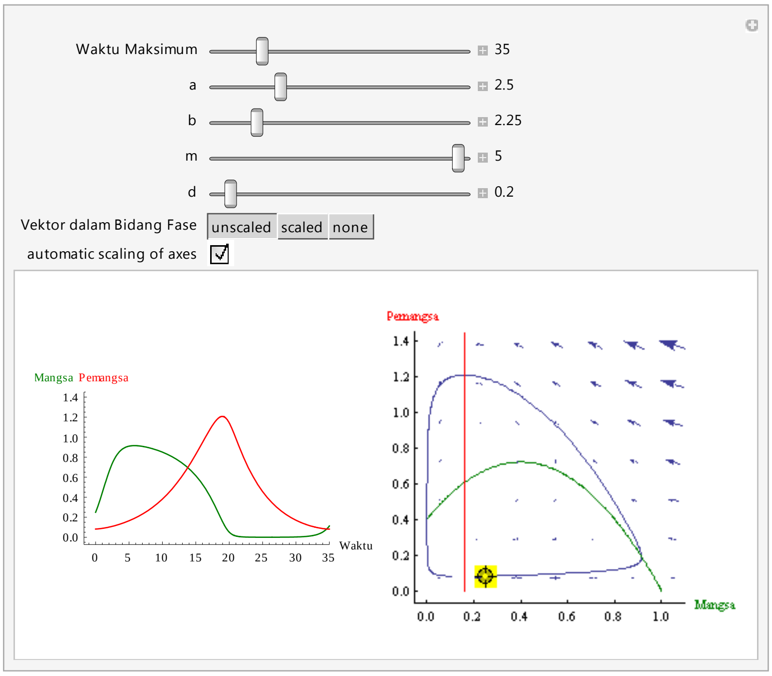

Figure 2: Numerical Simulation of $P_2(x_2, y_2)$without harvesting

From Figure 2, it can be seen that if the growth rate of the predator population due to eating prey is greater than $d(1+m)$, then both populations, namely the prey and predator populations, exist. In this case, there is coexistence between the prey population and the predator population. If the growth rate of the prey population due to eating prey is greater than $d\frac{m(1+m)}{(m-1)}$, then the prey population density increases and the predator population density decreases, so that over time the population density of prey and predators decreases, resulting in the extinction of the prey and predator populations. So the population density of prey and predators does not approach the equilibrium point $P_2(x_2, y_2) = (0.16,0.61)$ in other words, the equilibrium point in system (2) is unstable.

Example 1:

Given the parameters $r = 10, n = 26, k = 1, p = 1.5, f = 10, s = 10, g = 0.001, i = 0.001$ and time 1,2,...,200 in system (2) so as to obtain:

$a = 1, b = 2.6, d = 1, m = 1.5, j = 0.0001, i = 0.0001$

and then obtained $P_2(x_2, y_2) = (0.9, 0.21)$ in system (2) is locally asymptotically stable according to the figure below:

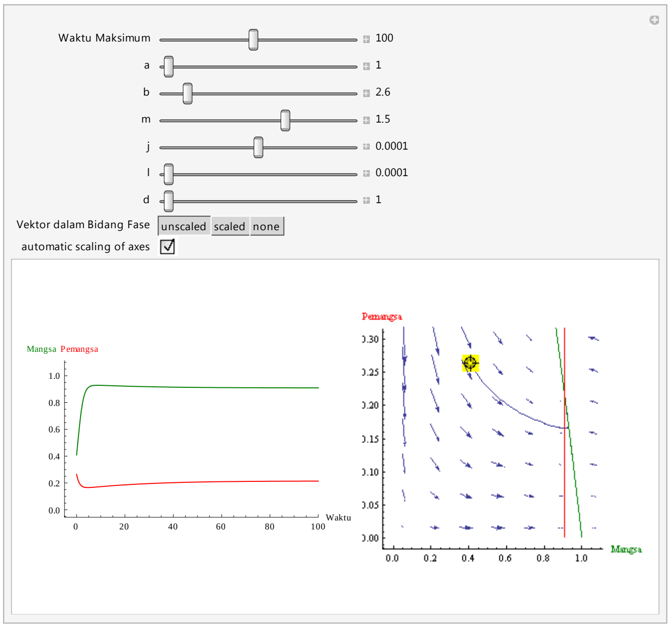

Figure 3: Numerical Simulation of $P_2(x_2, y_2)$ with harvesting

From Figure 3, it can be seen that if the growth rate of the predator population due to eating prey is more than $b > d(m+1) + \alpha + \left(\frac{b\left(\beta^2\eta - \mu(2b\beta + 3bj\mu)\right)}{\mu\beta^2}\right)(ly_2)$ then both populations, namely the prey population and the predator population, exist. In this case, there is coexistence between the prey population and the predator population. If the growth rate of the prey population due to eating prey is more than $\eta\left(\frac{\beta^2}{\beta^2 2ly_2 + \mu(2\beta + 3j\mu)}\right)$ then the density of the prey population and the predator population approaches the equilibrium point$P_2(x_2, y_2)$

4. Conclusion

The author has studied the prey-predator system with threshold harvesting and added the presence of toxins in both prey and predator populations. By considering the harvesting threshold rule, it is more realistic than the constant harvesting function. Although there are toxins in both populations, the harvesting rule guarantees the existence of the prey population. Theoretical, analytical, and numerical simulations have revealed many dynamics, and the author has also described the existence of solutions and equilibrium stability for system (2).

5. References

[1] W. Hogarth, “Stability of a predator-prey model with harvesting,” *Ecol. Modell.*, vol. 62, no. 1, pp. 83–106, 1992, doi: 10.1016/0304-3800(92)90083-Q.

[2] J. Huang, "Bifurcation analysis in a predator-prey model with constant-yield predator harvesting," *Discret. Contin. Dyn. Syst. - Ser. B*, vol. 18, no. 8, pp. 2101–2121, 2013, doi: 10.3934/dcdsb.2013.18.2101.

[3] Q. J. A. Khan, "Predator–prey harvesting model with fatal disease in prey," *Math. Methods Appl. Sci.*, vol. 39, no. 10, pp. 2647–2658, 2016, doi: 10.1002/mma.3718.

[4] T. K. Ang and H. M. Safuan, "Harvesting in a toxicated intraguild predator–prey fishery model with variable carrying capacity," *Chaos, Solitons & Fractals*, 2019, [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0960077919302103.

[5] Y. Louartassi, A. Alla, K. Hattaf, and A. Nabil, "Dynamics of a predator–prey model with harvesting and reserve area for prey in the presence of competition and toxicity," *J. Appl. Math. …*, 2019, doi: 10.1007/s12190-018-1181-0.

[6] A. A. Majeed and M. H. Ismaeeb, "The bifurcation analysis of prey-predator model in the presence of stage structured with harvesting and toxicity," *J. Phys. Conf. Ser.*, 2019, doi: 10.1088/1742-6596/1362/1/012155.

[7] M. H. I. A. A. Majeed, "The Dynamical behavior of stage structured prey-predator model in the harvesting and toxin presence," *J. Southwest Jiaotong Univ.*, 2019, [Online]. Available: http://jsju.org/index.php/journal/article/view/407.

[8] X. Yu, S. Yuan, and T. Zhang, "About the optimal harvesting of a fuzzy predator–prey system: a bioeconomic model incorporating prey refuge and predator mutual interference," *Nonlinear Dyn.*, 2018, doi: 10.1007/s11071-018-4480-y.

[9] S. Al-Momen and R. K. Naji, "The dynamics of modified Leslie-Gower predator-prey model under the influence of nonlinear harvesting and fear effect," *Iraqi J. Sci*. iasj.net, 2022, [Online]. Available: https://www.iasj.net/iasj/download/2b1ca015f0604b81.

[10] L. Ji, "Qualitative analysis of a predator-prey model with constant-rate prey harvesting incorporating a constant prey refuge," *Nonlinear Anal. Real World Appl.*, vol. 11, no. 4, pp. 2285–2295, 2010, doi: 10.1016/j.nonrwa.2009.07.003.

[11] T. Asfaw, "Dynamics of generalized time dependent predator prey model with nonlinear harvesting," *Int. J. Math. Anal.*, vol. 3, no. 29, pp. 1473–1485, 2009, [Online]. Available: https://api.elsevier.com/content/abstract/scopus_id/78449262041.