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The Hyperbolic Function Method for the Fractional (3+1)-dimensional Generalized Korteweg–de-Vries–Zakharov–Kuznetsov Equation in Plasma Physics

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Abstract

The fractional (3+1)-dimensional generalized Korteweg-de-Vries-Zakharov-Kuznetsov equation (gKdV-ZKe) is one of the nonlinear models to indicate the impact of magnetic fields on weak ion-acoustic wave in plasma comprised of cool and hot electrons. We have applied the hyperbolic function method to explore the diversity of wave structures. We extract the solitons in form of bright solitons, dark solitons, bright-dark combo solitons and other solitons Moreover, for the physical illustration, some of the obtained solutions are represented graphically
Keywords: Waves solution, The hyperbolic function method, The fractional (3+1)-dimensional gKdV-ZK equation, Soliton Solutions, β-Derivative

1. INTRODUCTION

Nonlinear partial differential equation (NLPDEs) perform a main role in different branches of physics and mathematics engineering, such as optical fibers, meteorology, fluid dynamics, chemical physics, theory of turbulence, solitary wave theory, among

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other fields [1]-[18]. Daring the previous decades, multiple impactful techniques are presented in order to get exact solutions for NLPDEs, such as the inverse scattering transform method [1], Kudryashov's method [2], (G'/G) - expansion method [3] , the sub - ODE [4] , an extended improved tanh - function method [5], auxiliary equation method [6], the solitary wave ansatz method [7], He's variational method [8], new modified extended direct algebraic [9] and lots of other [10]-[15].

In our work, we consider the (3+1)-dimensional generalized Korteweg-de-Vries-Zakharov-Kuznetsov eguation (gKdV-ZKe) [11]-[15]

$$u_t + \alpha u^2 u_x + \beta u_{xxx} + v(u_{yy} + u_{zz})_x = 0, \tag{1}$$

where u is wave profile defined in spatial x, y, z and temporal t variables in fluid ions. We are going to take these equation of the fractional (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in this work is categorized

$$D_t^{\gamma} u + \alpha u^2 D_x^{\gamma} u + \beta D_{xxx}^{3\gamma} u + v D_x^{\gamma}(D_{yy}^{2\gamma} u + D_{zz}^{2\gamma} u) = 0. \tag{2}$$

We divided the following work in to different sections.In section 3, we includes basic definitions of fraction derivatives. In section 3, the hyperbolic function method is described. In section 4, we consider the (3+1)-dimensional gKdV-ZKe and exact solutions is procured by the hyperbolic function method and the last section concludes the work.

2. BETA FRACTIONAL DERIVATIVE AND ITS PROPERTIES

The summary of Beta fractional derivative of order α which is used further in this paper is defined by the following expression [20]

$$ {}_0^A T_x^{\alpha}(H(x)) = \lim_{\epsilon \to 0} \frac{H\big(x + \epsilon(x + \frac{1}{\Gamma(\alpha)})\big) - H(x)}{\epsilon}, \qquad 0 < \alpha \leq 1. \tag{3}$$

Some important properties of the beta derivative famous formula can be listed as follows

$$\begin{aligned} {}_0^A T_x^{\alpha}(aH(x) \pm bF(x)) &= a{}_0^A T_x^{\alpha} H(x) + b{}_0^A T_x^{\alpha} F(x), \\ {}_0^A T_x^{\alpha}\Big(\frac{H(t)}{F(x)}\Big) &= t\frac{dH(t)}{dt}, \\ {}_0^A T_x^{\alpha}\big(H(x) * F(x)\big) &= H(x){}_0^A T_x^{\alpha} F(x) + F(x){}_0^A T_x^{\alpha} H(x), \\ {}_0^A T_x^{\alpha}\Big(\frac{H(x)}{F(x)}\Big) &= \frac{F(x){}_0^A T_x^{\alpha} H(x) - H(x){}_0^A T_x^{\alpha} F(x)}{F^2(x)}, \end{aligned} \tag{4}$$

taking $\varepsilon = \big(x + \frac{1}{\Gamma(\alpha)}\big)^{1-\alpha} h, h \longrightarrow 0$, when $\varepsilon \longrightarrow 0$, we have ${}_0^A T_x^{\alpha} H(x) = \big(x + \frac{1}{\Gamma(\alpha)}\big)^{1-\alpha} \frac{dH(x)}{dx}$ with $\xi = \frac{\mu}{\alpha}(x + \frac{1}{\Gamma(\alpha)})^{\alpha}$, where μ is a constant.

3. HYPERBOLIC FUNCTION METHOD

Following are the major steps of hyperbolic function method. [16]-[19] Consider a nonlinear partial differential equation (NLPDE) as,

$$G(u, u_x, u_y, u_z, u_t, u_{tt}, u_{xx}, u_{xt}, u_{yy}, u_{zz}...) = 0, \tag{5}$$

where $u = u(x, y, z, t)$ is an unknown function.

Step 1. Suppose a wave transformation,

$$u(x, y, z, t) = U(\xi), \xi = x + y + z - ct. \tag{6}$$

Utilizing Eq.(6) reduces the Eq.(5) into an ordinary differential equation (OED), given as follows,

$$H(U, U', U'', ...) = 0. \tag{7}$$

Step 2. Let the solution of Eq.(7) can be written as

$$U(\xi) = \sum_{j=1}^{N} \sinh^{j-1}(w)[B_j \sinh(w) + A_j \cosh(w)] + A_0, \tag{8}$$

where w is some special functions. By computing the value of N via the homogeneous balance technique, inserting Eq.(8) in Eq.(7), and equating coefficients, we will acquire a nonlinear algebraic system whose solution finally results in exact solutions of Eq.(5).

$CaseI$: It should be mentioned that applying the separation of variables method on $\frac{dw}{d\xi} = \sinh(w)$ yields, $\sinh(w) = \pm\text{csch}(\xi), \cosh(w) = -\coth(\xi)$ and $\sinh(w) = \pm i\text{sech}(\xi), \cosh(w) = -\tanh(\xi)$. Consequently, Eq.(8) can be rewritten as

$$U(\xi) = \sum_{j=1}^{N} \pm\text{csch}^{j-1}(\xi)[\pm B_j\text{csch}(\xi) - A_j \coth(\xi)] + A_0. \tag{9}$$

and

$$U(\xi) = \sum_{j=1}^{N} \pm i\text{sech}^{j-1}(\xi)[\pm iB_j sech(\xi) - A_j \tan(\xi)] + A_0. \tag{10}$$

$CaseII$: It is clear that from $\frac{dw}{d\xi}$= cosh(w), we have $\sinh(w) = -\cot(\xi), \cosh(w) = \pm\csc(\xi)$ and $\sinh(w) = \tan(\xi), \cosh(w) = \pm\sec(\xi)$. Accordingly, Eq.(8) can be rewritten as

$$U(\xi) = \sum_{j=1}^{N} -\cot^{j-1}(\xi)[-B_j \cot(\xi) \pm A_j \csc(\xi)] + A_0 \tag{11}$$

and

$$U(\xi) = \sum_{j=1}^{N} \tan^{j-1}(\xi)[B_j \tan(\xi) \pm A_j \sec(\xi)] + A_0. \tag{12}$$

4. APPLYING THE HYPERBOLIC FUNCTION METHOD

Let the (3+1)-dimensional gKdV-ZKe can be converted in to a ODE by using the given transformation

$$\begin{aligned} u(x,y,z,t) &= U(\xi), \\ \xi &= \frac{1}{\gamma}(x+\frac{1}{\Gamma(\gamma)})^{\gamma} + \frac{1}{\gamma}(y+\frac{1}{\Gamma(\gamma)})^{\gamma} + \frac{1}{\gamma}(z+\frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{c}{\gamma}(t+\frac{1}{\Gamma(\gamma)})^{\gamma}, \end{aligned} \tag{13}$$

here, c is speed of the wave. Now by substituting Eq.(14) into Eq.(2), it becomes

$$-cU' + \alpha U^2 U' + \beta U''' + 2vU'' = 0. \tag{14}$$

Integrating Eq.(14) with respect to ξ and letting integration constant to be zero, we gain

$$-cU + \alpha \frac{U^3}{3} + (\beta + 2v)U'' = 0. \tag{15}$$

Applying homogeneous balance technique on U^3 and U" in Eq.(15), then we have $N = 1$.Put $N = 1$ in Eq.(8).
Case: $\frac{dw}{d\xi} = \sinh(w)$.

$$U(\xi) = B_1 \sinh(w) + A_1 \cosh(w) + A_0, \tag{16}$$

which A_1 and B_1 cannot be zero together. By setting the Eq.(16) into Eq.(15), and comparing coefficients, we will gain a system of nonlinear algebraic equations as

$$-\alpha {B_1}^2 A_0 - cA_0 + \frac{1}{3}\alpha {A_0}^3 = 0, \tag{17}$$

$${A_0}^2 B_1 \alpha + {A_1}^2 B_1 \alpha + B_1\beta - B_1 c + 2B_1 v = 0, \tag{18}$$

$$2\beta B_1 + 4vB_1 + \alpha B_1 {A_1}^2 + \frac{1}{3}\alpha {B_1}^3 = 0, \tag{19}$$

$${A_0}^2 A_1 \alpha - A_1 {B_1}^2 \alpha - 2A_1\beta - A_1 c - 4A_1 v = 0, \tag{20}$$

$$2A_0 A_1 B_1 \alpha = 0, \tag{21}$$

$$A_0 {A_1}^2 \alpha + A_0 {B_1}^2 \alpha = 0, \tag{22}$$

$$\alpha {B_1}^2 A_1 + 4vA_1 + 2\beta A_1 + \frac{1}{3}\alpha {A_1}^3 = 0. \tag{23}$$

On solving the produced algebraic system of Eq.(17) - Eq.(23) via Maple, the following set of coefficient for the exact solution of Eq.(2) can be obtained, which is given as which its solution provides the following sets:
Set 1

$$A_0 = 0, A_1 = \pm\sqrt{-\frac{6\beta + 12v}{\alpha}}, B_1 = 0, c = -2\beta - 4v.$$

Therefore, the following dark and bright soliton in the fluid ions propagated via a $(3+1)$ -dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be gained

$$u_{1,2}(\xi) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \coth(\xi), \tag{24}$$

and

$$u_{3,4}(\xi) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \tanh(\xi), \tag{25}$$

where $\xi = \frac{2\beta+4v}{\gamma}(t + \frac{1}{\Gamma(\gamma)})^\gamma + \frac{1}{\gamma}(x + \frac{1}{\Gamma(\gamma)})^\gamma + \frac{1}{\gamma}(y + \frac{1}{\Gamma(\gamma)})^\gamma + \frac{1}{\gamma}(z + \frac{1}{\Gamma(\gamma)})^\gamma$.
Set 2

$$A_0 = 0, A_1 = 0, B_1 = \pm\sqrt{-\frac{6\beta + 12v}{\alpha}}, c = \beta + 2v.$$

Consequently, the following dark and bright soliton in the fluid ions propagated via a$(3 + 1)$-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be acquired

$$u_{5,6}(\xi) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \operatorname{csch}(\xi), \tag{26}$$

and

$$u_{7,8}(x, y, z, t) = \pm i\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \operatorname{sech}(\xi). \tag{27}$$

where $\xi = \frac{\beta+2v}{\gamma}(t + \frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(x + \frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(y + \frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(z + \frac{1}{\Gamma(\gamma)})^\gamma$.
Set 3

$$A_0 = 0, A_1 = -\sqrt{-\frac{1}{2}\frac{3\beta + 6v}{\alpha}}, B_1 = \pm\sqrt{-\frac{1}{2}\frac{3\beta + 6v}{\alpha}}, c = -\frac{1}{2}\beta - v.$$

Subsequently, the following bright-dark combo solitons in the fluid ions propagated via a (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be obtained

$$u_{9,10}(\xi) = \pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \operatorname{csch}(\xi) - \frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}} \coth(\xi), \tag{28}$$

and

$$u_{11,12}\left(x,y,z,t\right)=\pm\frac{1}{2}i\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\operatorname{sech}(\xi)-\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\tanh(\xi),\qquad(29)$$

where $\xi=\frac{\beta+2v}{2\gamma}(t+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(x+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(y+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(z+\frac{1}{\Gamma(\gamma)})^{\gamma}$.
Set 4

$$A_0=0,A_1=-\sqrt{-\frac{1}{2}\frac{3\beta+6v}{\alpha}},B_1=\pm\sqrt{-\frac{1}{2}\frac{3\beta+6v}{\alpha}},c=-\frac{1}{2}\beta-v.$$

Finally, the following bright-dark combo solitons in the fluid ions propagated via a (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetso equation in plasma physics can be obtained

$$u_{13,14}\left(x,y,z,t\right)=\pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\operatorname{csch}(\xi)+\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\coth(\xi),\qquad(30)$$

and

$$u_{15,16}\left(x,y,z,t\right)=\pm\frac{1}{2}i\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\operatorname{sech}(\xi)+\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha}-\frac{12v}{\alpha}}\tanh(\xi),\qquad(31)$$

where $\xi=\frac{\beta+2v}{2\gamma}(t+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(x+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(y+\frac{1}{\Gamma(\gamma)})^{\gamma}+\frac{1}{\gamma}(z+\frac{1}{\Gamma(\gamma)})^{\gamma}$.
Case: $\frac{dw}{d\xi}=\cosh(w)$. In a manner similar to that accomplished above, we will procure a system of nonlinear algebraic equations as

$$\alpha {A_1}^2A_0-cA_0+\frac{1}{3}\alpha {A_0}^3=0,\qquad(32)$$
$${A_0}^2B_1\alpha+{A_1}^2B_1\alpha+2B_1\beta-B_1c+4B_1v=0,\qquad(33)$$
$$A_0{A_1}^2\alpha+A_0{B_1}^2\alpha=0,\qquad(34)$$
$${A_1}^2B_1\alpha+4B_1v+2\beta B_1+\frac{1}{3}\alpha {B_1}^3=0,\qquad(35)$$
$${A_0}^2A_1\alpha-A_1{B_1}^2\alpha-A_1\beta-A_1c-2A_1v=0,\qquad(36)$$
$$2A_0A_1B_1\alpha=0,\qquad(37)$$
$$2\beta A_1+4vA_1+\alpha {B_1}^2A_1+\frac{1}{3}\alpha {A_1}^3=0.\qquad(38)$$

On solving the produced system via Maple, it yields:
Set 1

$$A_0=0,A_1=0,B_1=\sqrt{-\frac{6\beta+12v}{\alpha}},c=2\beta+4v.$$

Therefore, the following dark and bright soliton in the fluid ions propagated via a (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics be secured

$$u_{17,18}(x,y,z,t) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\cot(\xi), \tag{39}$$

and

$$u_{19,20}(x,y,z,t) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\tan((\xi), \tag{40}$$

where $\xi = \frac{2\beta+4v}{\gamma}(t + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(x + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(y + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(z + \frac{1}{\Gamma(\gamma)})^{\gamma}$.
Set 2

$$A_0 = 0, A_1 = \sqrt{-\frac{6\beta + 12v}{\alpha}}, B_1 = 0, c = 2\beta + 4v.$$

Subsequently, the following dark and bright soliton in the fluid ions propagated via a (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be gained

$$u_{21,22}(x,y,z,t) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\csc(\xi), \tag{41}$$

and

$$u_{23,24}(x,y,z,t) = \pm\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\sec(\xi). \tag{42}$$

where $\xi = \frac{2\beta+4v}{\gamma}(t + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(x + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(y + \frac{1}{\Gamma(\gamma)})^{\gamma} - \frac{1}{\gamma}(z + \frac{1}{\Gamma(\gamma)})^{\gamma}$.
Set 3

$$A_0 = 0, A_1 = \sqrt{-\frac{1}{2}\frac{3\beta + 6v}{\alpha}}, B_1 = \pm\sqrt{-\frac{1}{2}\frac{3\beta + 6v}{\alpha}}, c = \frac{1}{2}\beta + v.$$

Consequently, the following bright-dark combo solitons in the fluid ions propagated via a (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be derived

$$u_{25,26}(x,y,z,t) = \pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\cot(\xi) - \frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\csc(\xi), \tag{43}$$

and

$$u_{27,28}(x,y,z,t) = \pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\tan(\xi) + \frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\sec(\xi). \tag{44}$$

where $\xi = \frac{\beta+2v}{2\gamma}(t+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(x+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(y+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(z+\frac{1}{\Gamma(\gamma)})^\gamma$.
Set 4

$$A_0 = 0, A_1 = -\sqrt{-\frac{1}{2}\frac{3\beta+6v}{\alpha}}, B_1 = \pm\sqrt{-\frac{1}{2}\frac{3\beta+6v}{\alpha}}, c = \frac{1}{2}\beta + v.$$

At the end, the following bright-dark combo solitons in the fluid ions propagated via a (3+1) -dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation in plasma physics can be acquired

$$u_{29,30}(x,y,z,t) = \pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\cot(\xi) + \frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\csc(\xi), \tag{45}$$

and

$$u_{31,32}(x,y,z,t) = \pm\frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\tan(\xi) - \frac{1}{2}\sqrt{-\frac{6\beta}{\alpha} - \frac{12v}{\alpha}}\sec(\xi). \tag{46}$$

where $\xi = \frac{\beta+2v}{2\gamma}(t+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(x+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(y+\frac{1}{\Gamma(\gamma)})^\gamma - \frac{1}{\gamma}(z+\frac{1}{\Gamma(\gamma)})^\gamma$.

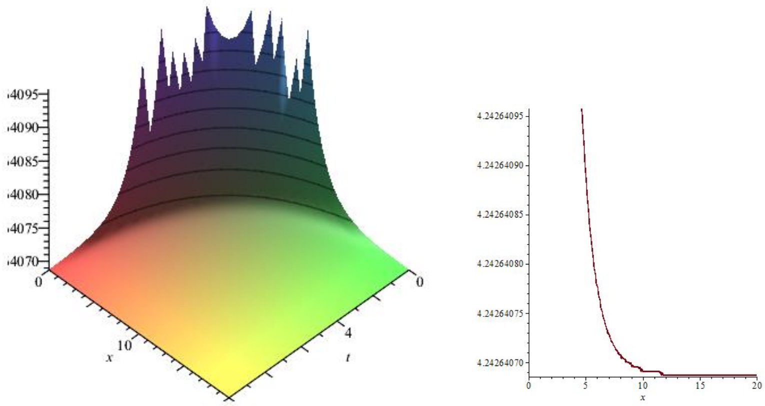

Figure 1: (Left) 3D-plot of $u_1(x,y,zt)$ for the parameter $y = z = \alpha = -1, \beta = v = 1$, and for beta with $\gamma = 0.5$. (Right) 2D-plot of $u_1(x,t)$ with $t = 1$.

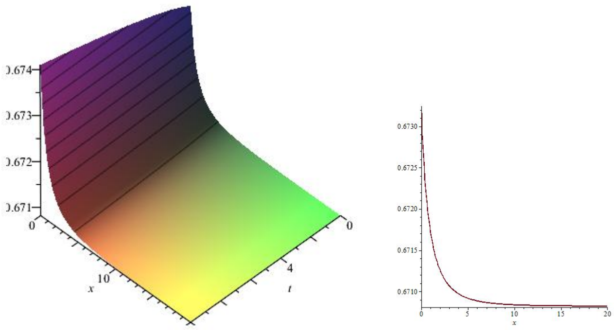

Figure 2: (Left) 3D-plot of $u_9(x, y, zt)$ for the parameter $y = z = \alpha = 1, \beta = v = -0.1$, and for beta with $\gamma = 0.5$. (Right) 2D-plot of $u_9(x, t)$ with $t = 1$.

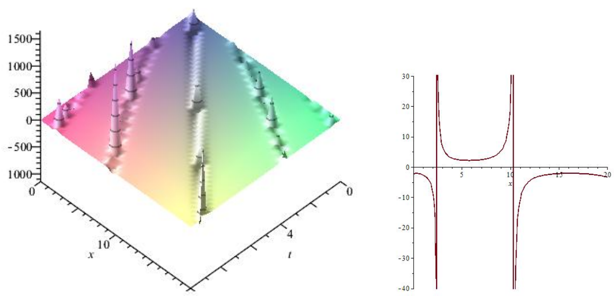

Figure 3: (Left) 3D-plot of $u_{23}(x, y, zt)$ for the parameter $y = z = 1, \alpha = 4, \beta = v = -1$, and for beta with $\gamma = 0.5$. (Right) 2D-plot of $u_9(x, t)$ with $t = 1$.

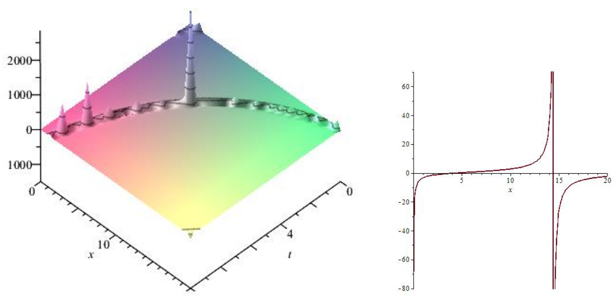

Figure 4: (Left) 3D-plot of $u_{27}(x, y, zt)$ for the parameter $y = z = \alpha = 2, \beta = -1, v = 2$, and for beta with $\gamma = 0.5$. (Right) 2D-plot of $u_{27}(x, t)$ with $t = 1$.

5. CONCLUSION

In this work, bright - solitons, dark solitons, bright - dark combo solitons and other solitons of the (3+1)-dimensional generalized Korteweg–de-Vries–Zakharov–Kuznetsov equation are derived by using the hyperbolic function method. These results will certainly be help full to explain physics phenomena in nonlinear complex systems.

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