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Mathematical Simulation of Damping Oscillation on Motorcycle Suspension

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Abstract

In this article, the resonance motion of a motorcycle suspension is investigated using a second-order differential equation approach. Load (F) variation is applied to the spring and absorber to determine the spring constant (k) and the damping constant (c), respectively. The second-order differential equation applied

$$m\frac{d^2x}{dt^2} + c\,\frac{dx}{dt} + kx = F(t)$$

the boundary conditions for the equation are $F(t)=0$, with displacement $x(t=0) = 0.1\ m$ and $dx/dt\ (t=0) = 0$. The suspension oscillations show under-damping behaviour. The suspension under damping phenomena, still gives comfortable under mass 640 kg for $\zeta = 0.25$ with the c = 700 Ns/m, and k = 3065 N/m.

Keywords: suspension, spring, absorber, differential equation

1. INTRODUCTION

This article describes the suspension movement using a second-order linear differential equation approach. The behaviour of an application in the fields of science and technology can be approached by The different equations[1]. Every motorcycle is equipped with a suspension whose main function is as a vibration damper [2]. If the suspension is rigid, it will feel uncomfortable when passing on an uneven road. Undamped vibration can affect the body and other engine components may become damaged quickly. Therefore, the suspension components need to be identified and cared for properly. An important system in the suspension of problems in dynamics concerns the free vibrations of systems [3]. The free vibration of a mass-spring system could be described as an oscillatory interchange between the kinetic and potential energy [4], and we could determine the natural frequency of oscillation by equating. To verify suspension system performances, a vibrating platform test bench has been used [5][6]. The discussion with the solution is a simple mass-spring system, recognizing is a model for more complex systems as well. The shock-absorber construction system can be drawn in general as a sketch (Figure 1).

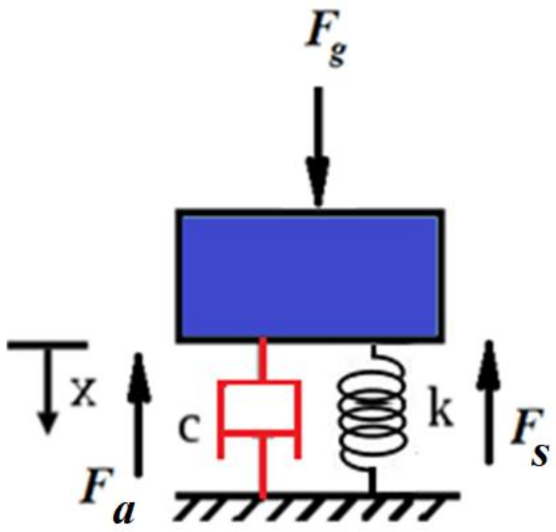

Figure 1. Viscously Damped Free Vibration

The system in Figure 1 is based on the law of equilibrium that the sum of the forces in a state of equilibrium. The equilibrium can be shown in Equation (1).

$$F_g + F_a + F_s = F(t) \quad (1)$$

F(t) is the influence of the external force acting on the spring and damping system. The simplest mechanical vibration equation occurs when $F(t) = 0$. The system is not the external force and it is named the undamped free vibration [7].

The mass (m) acting on the system in Figure 1 creates a gravitational force, which is expressed by equation (2)

$$F_g = mg \quad (2)$$

Let the g is the gravitational constant with $g = \frac{dv}{dt}$

$$g = \frac{dv}{dt} = \frac{d\left(dx/_{dt}\right)}{dt} = \frac{d^2x}{dt^2} \quad (3)$$

The spring is given a force then, the spring will give a force of spring back (F_s) of the same magnitude but in the opposite direction. A spring whose length is subjected to a force so that its length changes to x as displacement, then the magnitude of the recovered force can be written as follows according to Hooke's law in Equation (4).

$$F_s = -kx \tag{4}$$

k is the spring constant, which is a value that indicates the force required to change the length of the spring per meter [8].

Damping is experienced by most oscillatory systems such as harmonic oscillators. These systems, when displaced from their equilibrium position, experience a restoring force proportional to their displacement. If a frictional force proportional to the velocity is also applied, the system exhibits damped harmonic oscillation. The damping factor is determined by the ability of the system to respond to the applied load. The damping coefficient (c) with the damping force applied to a system moving in a fluid is given by Equation (5) [9] [10]

$$F_a = cv \tag{5}$$

v is the velocity, let the $v = \frac{dx}{dt}$. The damping force applied is expressed in Equation (6)

$$F_a = c\frac{dx}{dt} \tag{6}$$

Employing the model equation for the mass-spring system has been done, which is of second order differential equation. They analyze the displacement with various opposition forces applied on the suspension [11]. The Differential equation plays an important role in mathematics for the mechanical system in vibration [12] [13]. In general, the spring construction with damping with a certain mass without the influence of external forces (*F(t) = 0*) is formulated as an equation in the second-order homogeneous linear differential model [14] [14] [15]. The general behaviour of mass-spring systems can be extended to structures and elastic systems subjected to gravitational. The free vibrations of mass-spring in mathematical models as differential equations in Figure 1 are shown in Equation (7)

$$m\frac{d^2x}{dt^2} + c\frac{\mathrm{dx}}{dt} + kx = 0 \tag{7}$$

Mathematical modelling of the working mechanism of the spring and shock absorber is used to that the greatest vibrations occur due to the un-sprung weight of the vehicle [10]. the suspension system is introduced in a vehicle to dampen the relative movement between the wheels and body of the vehicle and give better handling and comfort to travellers [3]. The machine's condition can be determined by the vibration amplitude and frequency, as both can reveal the severity and source of the machine's problem, respectively [16]. Mechanical vibration is the term used to describe the movement produced in mechanical parts due to the effect of external or internal forces on that parts. Each part can be considered composed of one or more spring-mass-damper systems subjected to an exciting force [17]. *F(t)= 0* is obtaining a special solution caused by homogeneous excitation which gives an understanding of the role of damping, with the equation being [11][12].

The solution of the equation with $m > 0$, $c > 0$ *and* $k > 0$, has characteristic equations with characteristic roots (λ_1 *and* λ_2)

$$\lambda_1 \; and \; \lambda_2 = -\frac{c}{2m} \pm \frac{1}{2m}\sqrt{c^2 - 4mk} \tag{8}$$

Let $\sqrt{-(c^2 + 4mk)}.\sqrt{-1} = \omega i$, then the character roots $\lambda_1, \lambda_2 = -\frac{c}{2m} \pm \omega i$ The general real solution is found by taking linear combinations of the two basic solutions. The solve equation is:

$$x(t) = e^{(-\frac{c}{2m}+\omega i)t} + e^{(-\frac{c}{2m}-\omega i)t} \tag{9}$$

$$x(t) = e^{-\frac{c}{2m}t}(A_1 \cos \omega t + A_2 \sin \omega t) \tag{10}$$

The form of the solution of the equation is called under-damped. Mathematical modelling on suspension systems is vastly used and the quarter-car model is the model most commonly used in the activities of research due to its simplicity[10] [18]. When the damping is enlarged so that it reaches a point where the system is no longer oscillating, we reach a critical damping point. When damping is added past this critical point the system is said to be in a super-damped state. The damping coefficient value needed to reach the critical damping point in the mass-spring-damper model is $c_c = 2\sqrt{mk}$. To characterize the amount of attenuation in the system, a ratio called the damping coefficient is used. This ratio is the ratio of the actual damping to the amount of damping required to reach the critical damping point. The formula for the damping ratio (ζ) (zeta) is shown in Equation (11) [5] [19]:

$$\zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{mk}} \tag{11}$$

For example, a metal structure will have a damping ratio of less than 0.05 while an automotive suspension will be in the range $\zeta = 0.2 - 0.3$. In this observation without external force on the mass of the spring system, the opposition forces may cause the displacements of the mass of the spring.

2. METHOD

The spring test equipment was carried out using a displacement with the variation of load [20]. The value of the spring constant is determined by the variation of the load. The load apply from 0 to 300 N to the spring then the displacement is recorded. The value of the spring constant (*k*) is the slope gradient of the curve of the load (*F*) to the displacement (*x*). The value of the spring constant (k) is determined according to Equation (4). The Absorber is tested by providing a compressive load from *F* = - 400 N and then, a rebound load, the speed in the rebound and compression stroke are recorded. The damping coefficient (*c*) in Equation (5) is the slope of the gradient of the rebound stroke curve the in load vs velocity chart. the boundary conditions for the differential equation are *F(t)=0*, with displacement *x(t=0)* = 0.1 m and *dx/dt (t*=0*)* = 0

3. RESULT

Figure 2 shows the chart of the force versus displacement. The gradient trendline of the curve shows a spring constant (*k*) or a measure of the stiffness of the spring. The spring constant from the gradient trendline of the curve in Figure 2 the spring constant *k* is 3065 N/m.

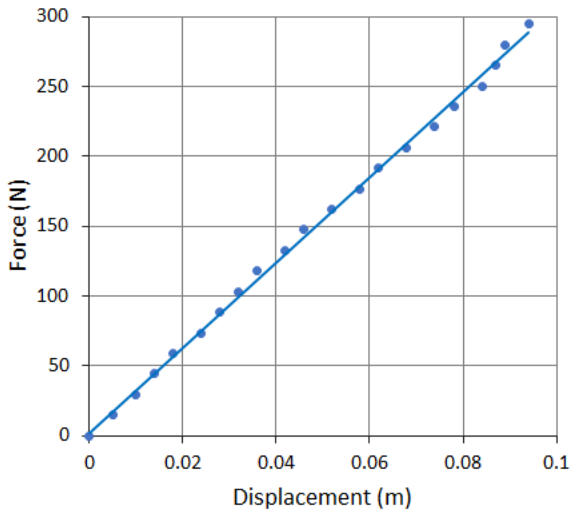

Figure 2. The effect of the load on the displacement in the spring test

Figure 3 shows the chart of the force versus velocity at condition rebound and compression stroke. A gradient of the curve of the force versus velocity at the rebound stroke shows a damping constant. The trendline of the curve in the force versus velocity at the rebound stroke determines the damping constant (*c)* is 700 Ns/m. The rebound stroke determines the damping constant because the rebound stroke determines the damping performance against the compressive load to return to its original position.

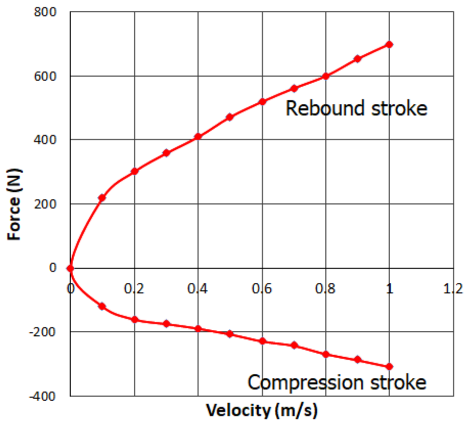

Figure 3. Effect of load on the velocity at damping in absorber test at condition rebound and compression stroke

The solution of the equation with $m > 0$, c = 700 Ns/m, and k = 3065 N/m the change of form over damping to under damping reaches for the damping ratio ($\zeta\, c_c = c$), $c_c = 2\sqrt{mk}$, for ζ =1 then $c_c = c$, let $c = 2\sqrt{mk}$ give solution for m = 40 kg. Figure 4 shows the oscillatory motion curve of the test results with mass variations m = 30 kg and 50 kg. The damping behaviour shows the change the over damping to the under damping behaviour.

For m = 30 kg, then λ_1 and $\lambda_2 = -5.84$ and -17.49

The general solution of the equation, let $x(t) = A\,(e^{-5.84t} + e^{-17.49t})$. It is an equation shows over damping phenomena.

For m = 50 kg, then λ_1 and $\lambda_2 = -7 \pm 3.51i$

The general solution of the equation, let $x\,(t) = e^{-7t}\,(A_1 \cos 3.5t + A_2 \sin 3.5t)$. It is an equation shows under damping phenomena.

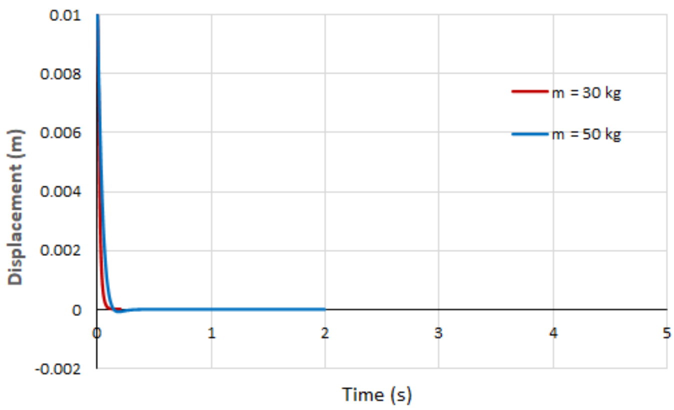

Figure 4. The characteristics of the curve on changes from over damping to under damping are influenced by mass

An automotive suspension will be in the range of damping ratio ζ = 0.2 - 0.3., if ζ = 0.25 the characteristic of the automotive suspension shows the motion of the displacement in the time shown in Figure 5. The characteristic equations with characteristic roots (λ_1 *and* λ_2) in the the damping ratio ζ = 0.25 give solution for m = 640 kg. $x\ (t) = \ e^{-0.55t}\ (A_1 \cos 2.12t + \ A_2 \sin 2.12t)$. The general solution for the mass is under damping phenomena, it is still give comfortable when passing on an uneven road.

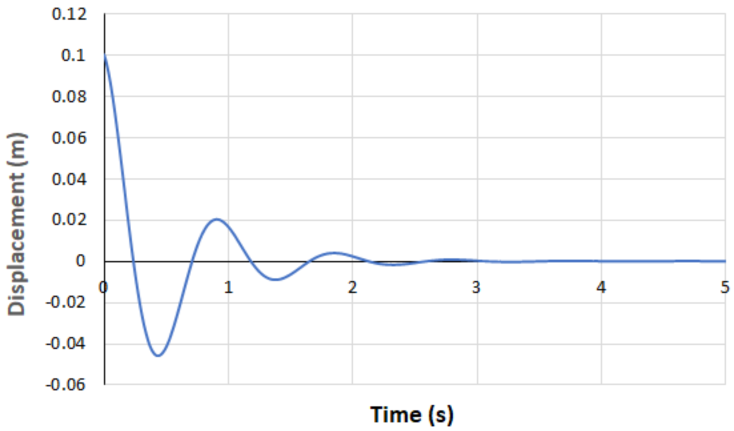

Figure 5. The behaviour suspension moving with the damped oscillation for ζ = 0.25

4. CONCLUSION

The simulation of the oscillating motion of the suspension shows damped free vibration behaviour. The damped oscillatory behaviour of the suspension provides comfort under vibration effects. The suspension under damping phenomena, still gives comfortable under mass 640 kg for ζ = 0.25.

Conflict of interest.

The authors declare no conflicts of interest.

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