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The Marichev–Saigo–Maeda Fractional Calculus Operator Associated with the Product of a General Class of Polynomial and Generalized K–Struve Function

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Abstract

In this paper, we introduce three theorems by using Marichev–Saigo–Maeda fractional calculus operator, applied on the product of Srivastava polynomial and K–Struve function with the help of some lemma. The results are presented in terms of Generalized K–Wright function. Also obtained some known and new results in special cases.

INRODUCTION

The wright function is widely used in the partial differential equation of fractional order which is amicable and broadly treated in papers by many authors including gorenflo–et–al [6].

For ζ_i, $\tau_j \in \mathbb{R} \setminus \{0\}$ and $a_i, b_j \in \mathbb{C}$, i=(1, p); j=(1, q) the generalized form of Wright function defined by Wright [11, 13–16] as following

$$p\Psi q\,(z) = p\Psi q\left[\begin{matrix}(a_i, \zeta_i)_{1,p}\\ ;\ z\\ \left((b_j, \tau_j)_{1,q}\right)\end{matrix}\right] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} \Gamma(a_i + n\,\zeta_i)\, z^n}{\prod_{j=1}^{q} \Gamma(b_j + n\tau_j)\, n!},\ z \in \mathbb{C} \tag{1.1}$$

where Γz is the Euler gamma function [4]. The condition for existence of (1.1) with its illustration in terms of Mellin–Barnes integral and the H–function obtained by Kilbas et al [9].

The generalized form of the above wright function (1.1) was given by Gehlot and Prajapati [5], as generalized K–Wright function define as

$$p\Psi^k q\,(z) = p\Psi^k q\left[\begin{matrix}(a_i,\zeta_i)_{1,p}\\ ;\ z\\ \left((b_j,\tau_j)_{1,q}\right)\end{matrix}\right] = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma_k(a_i+n\,\zeta_i)\,z^n}{\prod_{j=1}^{q}\Gamma_k(b_j+n\tau_j)\,n!},\ z\in\mathbb{C} \tag{1.2}$$

where $k \in \mathbb{R}^+$ and $(a_i + n\,\zeta_i)$, $\left(b_j + n\tau_j\right) \in \mathbb{C}\setminus k\mathbb{Z}^-$ for all $n \in \mathbb{N}_0$. The generalized k–gamma function [3] is defined as

$$\Gamma_k(z) = \int_0^{\infty} e^{-\frac{t^k}{k}}\, t^{z-1}\, dt;\ (\Re(z) > 0, k \in \mathbb{R}^+) \tag{1.3}$$

and

$$\Gamma_k(z) = \lim_{n\to\alpha}\frac{n!\,k^n(nk)^{\frac{z}{k}-1}}{(z)n_{,}k},\ k\in\mathbb{R}^+,\ z\in\mathbb{C}\setminus k\mathbb{Z}^- \tag{1.4}$$

Also

$$\Gamma_k(z) = (k)^{\frac{z}{k}-1}\,\Gamma\left(\frac{z}{k}\right) \tag{1.5}$$

where $(z)_{n,k}$ is the k–Pochammer symbol introduced by Daiz and Pariguan [3] defined for complex $z \in \mathbb{C}$ and $k \in \mathbb{R}$ as

$$(z)_{n,k} = \left\{\begin{matrix}1\ if\ (n=0)\\ z(z+k)(z+2k)..(z+(n-1)k)\ if\ (n\in N)\end{matrix}\right\} \tag{1.6}$$

On taking k=1, then generalized K–Wright function (1.2) diminishes to generalized wright function (1.1).

Saigo [17] defined the fractional integral operator with the gauss hypergeometric function as kernel, which are remarkable generalization of the Riemann–Liouville and Erdelyi–Kober fractional calculus operator [10].

For $\xi, \tau, \beta \in \mathbb{C}$ and $x \in \mathbb{R}^+$ with $\Re(\xi) > 0$, the left–hand and the right–hand sided generalized fractional integral operator connected with Gauss hypergeometric function are define as below:

$$(I_{0^+}^{\xi,\tau,\beta}\, f)(x) = \frac{x^{-\xi-\tau}}{\Gamma(\xi)}\int_0^x (x-t)^{\xi-1}\, {}_2F_1(\xi+\tau, -\beta; \xi; 1-\frac{t}{x})\, f(t)dt \tag{1.7}$$

and

$$(I_{-}^{\xi,\tau,\beta}\, f)(x) = \frac{1}{\Gamma(\xi)}\int_x^{\infty}\frac{(t-x)^{\xi-1}}{t^{\xi+\tau}}\, {}_2F_1(\xi+\tau, -\beta; \xi; 1-\frac{x}{t})\, f(t)dt \tag{1.8}$$

respectively. Here ${}_2F_1(\xi, \tau;\ \beta;\ z)$ is the Gauss hypergeometric function [10] defined for $z \in \mathbb{C}$, $|z| < 1$ and $\xi, \tau \in \mathbb{C}$, $\beta \in \mathbb{C}\setminus \mathbb{Z}_0^-$ by

$$ {}_2F_1(\xi,\tau;\ \beta;\ z) = \sum_{n=0}^{\infty} \frac{(\xi)_n\,(\tau)_n\, z^n}{(\beta)_n\, n!} $$

where

$$ (z)_n = (z)_{n,1} $$

By substituting $\tau = -\xi$ and τ=0 in equation (1.7), we get corresponding R–L and Erdelyi–Kober fractional operator respectively.
Marichev (11) was introduced and studied fractional calculus operators which are the generalization of the Saigo operator, later generalized by Saigo and Maeda (18). For $\xi, \xi', \tau, \tau', \beta \in \mathbb{C}$ and $x \in \mathbb{R}^+$ with $\Re(\beta) > 0$, the left-hand and the right-hand sided MSM fractional integral operator associated with third Appell function F_3 are defined as

$$ (I_{0^+}^{\xi,\xi',\tau,\tau',\beta} f)(x) = \frac{x^{-\xi}}{\Gamma(\beta)} \int_0^x \frac{(x-t)^{\beta-1}}{t^{\xi'}} F_3(\xi,\ \xi',\tau,\tau',\beta, 1-\frac{t}{x}, 1-\frac{x}{t}) f(t)dt \tag{1.9} $$

And

$$ (I_{-}^{\xi,\xi',\tau,\tau',\beta} f)(x) = \frac{x^{-\xi'}}{\Gamma(\beta)} \int_x^{\infty} \frac{(t-x)^{\beta-1}}{t^{\xi}} F_3(\xi,\ \xi',\tau,\tau',\beta, 1-\frac{x}{t}, 1-\frac{t}{x},) f(t)dt \tag{1.10} $$

The third Appell function(16) is defined by

$$ F_3(\xi,\ \xi',\tau,\tau',\beta,x,y) = \sum_{m,n=0}^{\infty} \frac{(\xi)_m\,(\xi')_n\,(\tau)_m (\tau')_n\, x^m\, y^n}{(\beta)_{m+n}\, n!\, m!}, \max\{|x|,|y|\} < 1 \tag{1.11} $$

The Srivastava polynomial defined by Srivastava [21] (pp. 1, eq.1), [12] (pp. 11, eq. 7) in the following manner

$$ S_w^u[x] = \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^s \quad w=0,1,2, \tag{1.12} $$

where w is an arbitrary positive intenger and the coefficient $A_{w,s}$ $(w,s) > 0$ are the arbitrary constant real or complex. This polynomial provide a large number spectrum of well-known polynomial as one of its particular cases on appropriately specializing the coefficient $A_{w,s}$ particularly by setting $u = 1, A_{w,s} = \frac{s!}{(-W)u,s}$ for s=k and $A_{w,s} = 0$ for $s \neq k$ the above polynomial leads to a power function.

$$ S_w^u[x] = x^k \ (k \in Z^+ \ with\ k \leq w) \tag{1.13} $$

The generalized k-Struve function was defined by Nisar K S [13] as

$$\mathrm{S}_{\mathrm{v,c}}^{\mathrm{k}}(t) = \sum_{\mathrm{n}=0}^{\infty} \frac{(-\mathrm{c})^{\mathrm{n}}}{\Gamma_{\mathrm{k}}\left(\mathrm{nk}+\mathrm{v}+\frac{3\mathrm{k}}{2}\right)\Gamma\left(\mathrm{n}+\frac{3}{2}\right)n!}\left(\frac{\mathrm{t}}{2}\right)^{2\mathrm{n}+\frac{\mathrm{v}}{\mathrm{k}}+1} \quad (\mathrm{k} \in \mathbb{R}^+, \mathrm{c} \in \mathbb{R}; \mathrm{v} > -1) \tag{1.14}$$

By putting k=1 and c=1 in (1.11), it will be reduced to Struve function of order v is defined by [1] as:

$$\mathrm{H}_{\mathrm{v}}(\mathrm{t}) = \sum_{\mathrm{n}=0}^{\infty} \frac{(-1)^{\mathrm{n}}}{\Gamma\left(\mathrm{n}+\mathrm{v}+\frac{3}{2}\right)\Gamma\left(\mathrm{n}+\frac{3}{2}\right)n!}\left(\frac{\mathrm{t}}{2}\right)^{2\mathrm{n}+\mathrm{v}+1} \tag{1.15}$$

To study more about Struve function, their generalization and properties the revered reader is call to consider references [2, 7, 8, 14, 15, 20, 22, 23].

The following MSM integral operator are required here [18, p.394] to obtain the MSM fractional integration of generalized k–Struve function.

Lemma 1: Let $\xi, \xi', \tau, \tau', \beta, \eta \in \mathbb{C}$ such that $\Re(\xi) > 0$

(i) If $\Re(\eta) > 0$ max $\{0, \Re(\xi' - \tau'), \Re(\xi + \xi' + \tau - \beta)\}$

$$\left(\mathrm{I}_{0^+}^{\xi,\xi',\tau,\tau',\beta}\, t^{\eta-1}\right)(x) = \frac{\Gamma(\eta)\Gamma(-\xi'+\tau'+\eta)\,\Gamma(-\xi-\xi'-\tau+\beta+\eta)}{\Gamma(\tau'+\eta)\,\Gamma(-\xi-\xi'+\beta+\eta)\,\Gamma(-\xi'-\tau+\beta+\eta)}\, x^{-\xi-\xi'+\beta+\eta-1} \tag{1.16}$$

(ii) If $\Re(\eta) > 0$ max $\{\Re(\tau), \Re(-\xi - \xi' - \beta), \Re(-\xi - \tau' + \beta)\}$, then

$$\left(\mathrm{I}_{-}^{\xi,\xi',\tau,\tau',\beta}\, t^{-\eta}\right)(x) = \frac{\Gamma(-\tau+\eta)\Gamma(\xi+\xi'-\beta+\eta)\,\Gamma(\xi+\tau'-\beta+\eta)}{\Gamma(\eta)\,\Gamma(\xi-\tau+\eta)\,\Gamma(\xi+\xi'+\tau'-\beta+\eta)}\, x^{-\xi-\xi'+\beta-\eta} \tag{1.17}$$

MAIN RESULTS

Theorem 1. Let $\xi, \xi', \tau, \tau', \beta, \eta \in \mathbb{C}$ and $\mathrm{k} \in \mathbb{R}^+$, be such that $\Re(\beta) > 0$, $\Re(\frac{\sigma}{\mathrm{k}}) >$ max $\{0, \Re(\xi' - \tau'), \Re(\xi + \xi' + \tau - \beta)\}$.Also let $\mathrm{c} \in \mathbb{R}$; v >-1, then for t > 0

$$\left\{\mathrm{I}_{0^+}^{\xi,\xi',\tau,\tau',\beta}\left(t^{\frac{\sigma}{\mathrm{k}}-1}\mathrm{S}_{\mathrm{v,c}}^{\mathrm{k}}(t)\mathrm{S}_{\mathrm{w}}^{\mathrm{u}}[t^{\mu}]\right\}(x) = \frac{\mathrm{k}^{\beta+\frac{1}{2}}x^{-\xi-\xi'+\beta+\frac{\sigma}{\mathrm{k}}+\frac{v}{\mathrm{k}}}}{2^{\frac{v}{\mathrm{k}}+1}}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}\,x^{s\mu}$$

$$\times\, {}_3\Psi^{\mathrm{k}}{}_5\left[\begin{matrix}(\sigma+v+\mu sk+k,2k),(-k\xi'+k\tau'+\sigma+v+\mu sk+k,2k),(-k\xi-k\xi'-k\tau+\mathrm{k}\beta+\sigma+v+\mu sk+k,2k)\\(k\tau'+\sigma+v+\mu sk+k,2k),(-k\xi-k\xi'+\mathrm{k}\beta+\sigma+v+\mu sk+k,2k),(-k\xi'-k\tau+\mathrm{k}\beta+\sigma+v+\mu sk+k,2k),\left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right)\end{matrix}\,\middle|\,-\frac{cx^2k}{4}\right] \tag{2.1}$$

Proof:-By using the definition of (1.12), (1.14) and taking the left-hand sided MSM fractional integral operator inside the summation the left hand side of (2.1) becomes

$$\sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s} \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+ v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right) n!\; 2^{2n+\frac{v}{k}+1}}$$

$$(I_{0^+}^{\xi,\xi',\tau,\tau',\beta}\{t^{\frac{\sigma}{k}+2n+\frac{v}{k}+\mu s+1-1}\})(x)$$

Making use of lemma (1.16), we obtain

$$\frac{x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\; x^{\mu s} \sum_{n=0}^{\infty} \frac{\Gamma\left(\frac{\sigma}{k}+\frac{v}{k}+2n+\mu s+1\right)}{\Gamma_k\left(nk+ v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right) n!}$$

$$\times \frac{\Gamma\left(-\xi-\xi'-\tau+\beta+\frac{\sigma}{k}+\frac{v}{k}+\mu s+2n+1\right)\Gamma\left(-\xi'-\tau'+\frac{\sigma}{k}+\frac{v}{k}+\mu s+2n+1\right)}{\Gamma\left(\tau'+\frac{\sigma}{k}+\frac{v}{k}+\mu s+2n+1\right)\Gamma\left(-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}+\mu s+2n+1\right)\Gamma\left(-\xi'-\tau+\beta+\frac{\sigma}{k}+\frac{v}{k}+\mu s+2n+1\right)}\left(\frac{-cx^2k}{4}\right)^n$$

Now using (1.5) on above term, then we get

$$\frac{x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1} k^{-\beta-\frac{1}{2}}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\; x^{\mu s}$$

$$\sum_{n=0}^{\infty} \frac{\Gamma_k(\sigma+v+\mu sk+k+2nk)}{\Gamma_k(k\tau'+\sigma+v+\mu sk+k+2nk)\,\Gamma_k(-k\xi-k\xi'+k\beta+\sigma+v+\mu sk+k+2nk)}$$

$$\times \frac{\Gamma_k(-k\xi'+k\tau'+\sigma+v+\mu sk+k+2nk)\,\Gamma_k(-k\xi-k\xi'-k\tau+k\beta+\sigma+v+\mu sk+k+2nk)}{\Gamma_k(-k\xi'-k\tau+k\beta+\sigma+v+\mu sk+k+2k)\,\Gamma_k\left(nk+ v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right) n!}\left(\frac{-cx^2k}{4}\right)^n$$

Using the definition of (1.2) in the above term we arrive at the result (2.1).

Theorem 2. Let $\xi, \xi', \tau, \tau', \beta, \eta \in \mathbb{C}$ and $k \in \mathbb{R}^+$, be such that $\Re(\beta) > 0$, $\Re(\frac{\sigma}{k}) > \max\{\Re(\tau), \Re(-\xi-\xi'-\tau'), \Re(-\xi-\tau'+\beta)\}$.Also let $c \in \mathbb{R}$; $v > -1$, then for $t > 0$

$$\{I_{-}^{\xi,\xi',\tau,\tau',\beta}(t^{\frac{\sigma}{k}-1}S_{v,c}^{k}(t)S_{w}^{u}[t^{\mu}]\}(x) = \frac{k^{\beta+\frac{1}{2}}x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s}$$

$$\times\, {}_3\Psi^{k}{}_5\left[\begin{matrix}(-k\tau-\sigma-v-\mu sk,-2k),(k\xi+k\tau'-k\beta-\sigma-v-\mu sk,-2k),(k\xi+k\xi'-k\beta-\sigma-v-\mu sk,-2k)\\(-\sigma-v-\mu sk,-2k),(k\xi+k\xi'+k\tau'-k\beta-\sigma-v-\mu sk,-2k),(k\xi-k\tau-\sigma-v-\mu sk,-2k),\left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right)\end{matrix}\;\middle|\; -\frac{cx^2k}{4}\right] \quad (2.2)$$

Proof: By using the definition of (1.14), (1.12) and taking the right-hand sided MSM fractional integral operator inside the summation the left hand side of (2.2) becomes

$$\sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s} \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+ v+\frac{3k}{2}\right) \Gamma\left(n+\frac{3}{2}\right) n!\, 2^{2n+\frac{v}{k}+1}}$$

$$\left(I_{-}^{\xi,\xi',\tau,\tau',\beta} \{t^{-(-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n)}\}\right)(x)$$

On applying lemma (1.17), we get

$$\frac{x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s} \sum_{n=0}^{\infty} \frac{\Gamma\left(-\tau-\frac{\sigma}{k}-\frac{v}{k}-2n-\mu s\right)}{\Gamma_k\left(nk+ v+\frac{3k}{2}\right) \Gamma\left(n+\frac{3}{2}\right) n!}$$

$$\times \frac{\Gamma\left(\xi+\xi'-\beta-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n\right) \Gamma\left(\xi+\tau'-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n\right)}{\Gamma\left(-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n\right)\Gamma\left(\xi-\tau-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n\right) \Gamma\left(\xi+\xi'+\tau'-\beta-\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n\right)} \left(\frac{-cx^2k}{4}\right)^n$$

Now making use of k–gamma function (1.5), we get

$$\frac{x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1} k^{-\beta-\frac{1}{2}}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s} \sum_{n=0}^{\infty} \frac{\Gamma_k(-k\tau-\sigma-v-\mu sk-2nk)}{\Gamma_k(-\sigma-v-\mu sk-2nk)\, \Gamma_k(k\xi-k\tau-\sigma-v-\mu sk-2nk)}$$

$$\times \frac{\Gamma_k(k\xi+k\xi'-k\beta-\sigma-v-\mu sk-2nk)\, \Gamma_k(k\xi+k\tau'-k\beta-\sigma-v-\mu sk-2nk)}{\Gamma_k(k\xi+k\xi'+k\tau'-k\beta-\sigma-v-\mu sk-2nk)\, \Gamma_k\left(nk+ v+\frac{3k}{2}\right) \Gamma_k\left(nk+\frac{3k}{2}\right) n!} \left(\frac{-cx^2k}{4}\right)^n$$

Using the definition of (1.2) in the above term we arrive at the result (2.2).

Theorem 3: Let $\xi, \xi', \tau, \tau', \beta, \eta \in \mathbb{C}$ and $k \in \mathbb{R}^+$, be such that $\Re(\beta) > 0$, $\Re(\frac{\sigma}{k}) > \max\{\Re(\tau), \Re(-\xi-\xi'+\beta), \Re(-\xi-\tau'+\beta)\}$.Also let $c \in \mathbb{R}$; $v > -1$, then for $t > 0$

$$I_{-}^{\xi,\xi',\tau,\tau',\beta}\left(t^{-\frac{\sigma}{k}} S_{v,c}^{k}(t) S_w^u[t^\mu]\right\}(x) = \frac{k^{\beta+\frac{1}{2}} x^{-\xi-\xi'+\beta-\frac{\sigma}{k}+\frac{v}{k}+1}}{2^{\frac{v}{k}+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s}$$

$$\times {}_3\Psi^k{}_5\left[\begin{matrix}(-k\tau+\sigma-v-\mu sk-k,-2k),(k\xi+k\xi'-k\beta+\sigma-v-\mu sk-k,-2k),(k\xi+k\tau'-k\beta+\sigma-v-\mu sk-k,-2k)\\(\sigma-v-\mu sk-k,-2k),(k\xi+k\xi'+k\tau'-k\beta+\sigma-v-\mu sk-k,-2k),(k\xi-k\tau+\sigma-v-\mu sk-k,-2k),\left(v+\frac{3k}{2},k\right),\left(\frac{3k}{2},k\right)\end{matrix} \;\middle|\; -\frac{cx^2k}{4}\right] \tag{2.3}$$

Proof:-By using the definition of (1.14), (1.12) and taking the right-hand sided MSM fractional integral operator inside the summation the left hand side of (2.3) becomes

$$\sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s} \sum_{n=0}^{\infty} \frac{(-c)^n}{\Gamma_k\left(nk+\ v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right) n!\ 2^{2n+\frac{v}{k}+1}}$$

$$(I_{-}^{\xi,\xi',\tau,\tau',\beta}\{t^{-(-\frac{v}{k}+\frac{\sigma}{k}-\mu s-2n-1)}\})(x)$$

On applying lemma (1.17), we get

$$\frac{x^{-\xi-\xi'+\beta+\frac{\sigma}{k}+\frac{v}{k}}}{2^{\frac{v}{k}+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\ x^{\mu s} \sum_{n=0}^{\infty} \frac{\Gamma\left(-\tau+\frac{\sigma}{k}-\frac{v}{k}-2n-\mu s-1\right)}{\Gamma_k\left(nk+\ v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right) n!}$$

$$\times \frac{\Gamma\left(\xi+\xi'-\beta+\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n-1\right)\Gamma\left(\xi+\tau'-\beta+\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n-1\right)}{\Gamma\left(\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n-1\right)\Gamma\left(\xi-\tau+\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n-1\right)\Gamma\left(\xi+\xi'+\tau'-\beta+\frac{\sigma}{k}-\frac{v}{k}-\mu s-2n-1\right)}\left(\frac{-cx^2k}{4}\right)^n$$

Now making use of k–gamma function (1.5) in the above term, we get

$$\frac{x^{-\xi-\xi'+\beta-\frac{\sigma}{k}+\frac{v}{k}+1}}{2^{\frac{v}{k}+1}k^{-\beta-\frac{1}{2}}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\ x^{\mu s} \sum_{n=0}^{\infty} \frac{\Gamma_k(-k\tau+\sigma-v-\mu sk-k-2nk)}{\Gamma_k(\sigma-v-\mu sk-k-2nk)\,\Gamma_k(k\xi-k\tau+\sigma-v-\mu sk-k-2nk)}$$

$$\times \frac{\Gamma_k(k\xi+k\xi'-k\beta+\sigma-v-\mu sk-k-2nk)\,\Gamma_k(k\xi+k\tau'-k\beta+\sigma-v-\mu sk-k-2nk)}{\Gamma_k(k\xi+k\xi'+k\tau'-k\beta+\sigma-v-\mu sk-k-2nk)\,\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma_k\left(nk+\frac{3k}{2}\right)n!}\left(\frac{-cx^2k}{4}\right)^n$$

Using the definition of (1.2) in the above term we arrive at the result (2.3).

SPECIAL CASES

1. On taking $k \to 1$ and $c = 1$ in (2.1), the generalized k–Struve functions yields to Struve function of order v, so we get the following result.

$$\{I_{0^+}^{\xi,\xi',\tau,\tau',\beta}(t^{\sigma-1}H_v(t)S_w^u[t^{\mu}]\}(x) = \frac{x^{-\xi-\xi'+\beta+\sigma+v}}{2^{v+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{s\mu} \times$$

$$_3\Psi_5\left[\begin{matrix}(\sigma+v+\mu s+1,2),(-\xi'+\tau'+\sigma+v+\mu s+1,2),(-\xi-\xi'-\tau+\beta+\sigma+v+\mu s+1,2)\\(\tau'+\sigma+v+\mu s+1,2),(-\xi-\xi'+\beta+\sigma+v+\mu s+1,2),(-\xi'-\tau+\beta+\sigma+v+\mu s+1,2),\left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right)\end{matrix}\middle| -\frac{cx^2}{4}\right]$$

2. On taking $k \to 1$ and $c = 1$ in (2.2), the generalized k–Struve functions yields to Struve function of order v, so we get the following result.

$$I_{-}^{\xi,\xi',\tau,\tau',\beta}\,(t^{\sigma-1}H_v(t)S_w^u[t^{\mu}]\,\}(x) = \frac{x^{-\xi-\xi'+\beta+\sigma+v}}{2^{v+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s} \times$$
$$_3\Psi_5\left[\begin{matrix}(-\tau-\sigma-v-\mu s,-2),(\xi+\tau'-\beta-\sigma-v-\mu s,-2),(\xi+\xi'-\beta-\sigma-v-\mu s,-2)\\(-\sigma-v-\mu s,-2),(\xi+\xi'+\tau'-\beta-\sigma-v-\mu s,-2),(\xi-\tau-\sigma-v-\mu s,-2),\left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right)\end{matrix}\,\middle|\, -\frac{cx^2}{4}\right]$$

3. On taking k → 1 and c = 1 in (2.3), the generalized k–Struve functions yields to Struve function of order v, so we get the following result.

$$I_{-}^{\xi,\xi',\tau,\tau',\beta}\,(t^{-\sigma}H_v(t)S_w^u[t^{\mu}]\,\}(x) = \frac{x^{-\xi-\xi'+\beta-\sigma+v+1}}{2^{v+1}} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}\, x^{\mu s}$$
$$\times\, _3\Psi_5\left[\begin{matrix}(-\tau+\sigma-v-\mu s-1,-2),(\xi+\tau'-\beta+\sigma-v-\mu s-1,-2),(\xi+\xi'-\beta+\sigma-v-\mu s-1,-2)\\(\sigma-v-\mu s-1,-2),(\xi+\xi'+\tau'-\beta+\sigma-v-\mu s-1,-2),(\xi-\tau+\sigma-v-\mu s-1,-2),\left(v+\frac{3}{2},1\right),\left(\frac{3}{2},1\right)\end{matrix}\,\middle|\, -\frac{cx^2}{4}\right]$$

4. On setting w=0, $A_{0,0} = 1$, then S_0^u [x] → 1 in (2.1), we arrive at the known result given by Seema Kabra [19, pp. 596, eq. 2.1]
5. On setting w=0, $A_{0,0} = 1$, then S_0^u [x] → 1 in (2.2), we arrive at the known result given by Seema Kabra [19, pp. 597, eq. 2.2]
6. On setting w=0, $A_{0,0} = 1$, then S_0^u [x] → 1 in (2.3), we arrive at the known result given by Seema Kabra [19, pp. 598, eq. 2.3]

CONCLUSION

Due to the generalization of Riemann–Liouville, Weyl, Erdelyi–Kobder and Saigo's fractional calculus operator. MSM fractional calculus operator have a compelling advantage, that was the reason many authors are referred to as general operator. Now we are close out of this paper by highlighting that our results (Theorem 1–3) can be deduced as the special case involving familiar fractional calculus operator as above said. The generalized k–Struve function defined in (1.14) has the property that a number of special function appear to be the special cases. Various special cases involving integral relating to the k–Struve function have been exposed in the earlier research worked by various authors with different arguments.

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