International Journal of Difference Equations (IJDE).
ISSN 0973-6069, Volume 18, Number 1, (2023) pp. 267-279

http://www.ripublication.com/ijde.htm

The Effects of Capital Policy on Banking Loan Dynamics: A Difference Equation Approach

Nurcahya Yulian Ashar[*a], **Moch. Fandi Ansori**[a], **and Hafidh Khoerul Fata** [a]

[a]*Department of Mathematics, Faculty of Science and Mathematics, Universitas Diponegoro, Jl. Prof. Jacub Rais, Semarang, 50275, Indonesia.*

Abstract

This research analyzes a central bank's capital adequacy ratio (CAR) policy using a dynamic loan model based on a gradient adjustment approach defined as a difference equation. The bank will distribute loans in accordance to the marginal loan profit in the next term. Bifurcation analysis explores how the CAR policy parameter affects loan dynamics. The analysis suggests CAR policy affects loan equilibrium stability. Transcritical bifurcation may instability the loan equilibrium if the CAR policy parameter is too high. The loan equilibrium may flip bifurcate and become chaotic if the CAR policy parameter is too low. Numerical simulations of the bifurcation diagram, Lyapunov exponent, chaotic trajectory, and cobweb diagram validate the analytical conclusions. We do a contour plot-based sensitivity analysis of the policy's interaction with other model parameters to assess their impact on loan stability.

Keywords: Capital adequacy ratio; Chaos; Flip bifurcation; Loan dynamics; Transcritical bifurcation

2000 MSC: 39A60, 39A33

1. INTRODUCTION

One of the goals of banking is to be able to sustain unforeseen losses, such as non-performing loans, investment losses, and operating losses. To achieve this goal, the capital adequacy ratio (CAR) regulation is essential. The regulation protects individual institutions from failing and lowers systemic risk, which helps maintain the stability of the financial market [8, 10, 7]. Because capital is viewed as a safety margin capable of absorbing possible losses, the regulator utilizes the CAR as a key indicator of the safety and soundness of depository institutions [25].

*Corresponding author's email ID: nurcahyayulian@lecturer.undip.ac.id (Nurcahya Yulian Ashar)

The scholarly literature on economics has examined how bank conduct and business cycles are related to capital restrictions [26, 14]. In order to determine if the CAR of subsidiaries and branches in developed and developing nations depends on the same criteria, authors in [23] tested the elements that influence international banks' CAR policies. The author in [20] examined the theoretical underpinnings of the CAR policy's maximum leverage ratio and compared it to the value-at-risk method. The study in [1], which was inspired by the large-scale bank failures that occurred during the financial crisis, investigated whether the CAR policy demanded by regulators is linked to bank failure by determining whether the association is impacted by the bank's proximity to the minimum needed capital ratios. Given that market share and leverage have a positive relationship, authors in [19] showed that the restrictions on CAR policy have indeed had an impact on firms' risky investment strategies. In [22], the authors looked at and contrasted how Islamic and conventional banks acted in relation to the CAR policy's ratio under various competitive conditions.

Since it has been confirmed that the CAR policy affects the banking industry, the policy will undoubtedly have an effect on the performance of individual banks. The proportion in the CAR policy parameter established by the regulator will result in the bank's optimal balance sheet portfolio. Because the policy is related to risk-weighted assets, which include loans, it is necessary to examine the policy's impact on bank loans. In this study, a dynamic model of a bank loan with a capital adequacy ratio policy is devised. Our model's gradient adjustment procedure determines the future quantity of money to be lent based on the sign of the loan's marginal profit. Several works [15, 16, 17, 11, 12] analyzed dynamic models based on the gradient adjustment process to evaluate various aspects of oligopoly markets in finance and economics, and they employed bifurcation theory to investigate the models' stability criteria. Using difference equation theory, specifically local stability and bifurcation theory, this study investigates the impact of CAR policy on the loan dynamics of a bank. Results indicate that the CAR policy can effect the stability of loan equilibrium. If the CAR policy parameter is set too high, the loan equilibrium may become unstable due to transcritical bifurcation and eventually reach zero. Until then, if the CAR policy parameter is adequately low, the loan equilibrium may lose stability due to flip bifurcation, resulting in chaotic behavior.

2. THE MODEL

In this paper, we consider a banking monopoly model. Suppose that the bank balance sheet consists of deposit and equity in the funding side, and reserve requirement (RR) and loan in the financing side, as shown in Table 1. The identity of balance sheet gives

$$L + R = D + E.$$

Table 1: Bank balance sheet's components.

Financing	Funding
Reserve requirement R	Deposit D
Loan L	Equity E

RR is a portion of the deposit that must be maintained by the bank in the central bank as a part of RR policy. Thus,

$$R = \rho D, \quad 0 < \rho < 1.$$

The bank's equity will be covered by the CAR policy set by the central bank. CAR is the ratio of equity to risk weighted assets (RWA). Since the financing of the bank only consists of loan and RR, and RR is risk-free asset, then RWA is calculated by using loan only. We assume that loan has 100% risk profile. As a result, we can write,

$$E = \kappa L, \quad 0 < \kappa < 1,$$

where κ is set to the central bank-specified CAR minimum portion value. In this paper, we study the CAR policy's parameter κ.

Therefore, deposit variable becomes

$$D = \left(\frac{1-\kappa}{1-\rho}\right) L.$$

We follow the idea of [17] to use gradient adjustment process [9] to model the dynamics of banking loan where its distribution in the following period is based on the current marginal profit. The model is given as follows

$$L_{t+1} = L_t + \alpha_L L_t \frac{\partial \pi_t}{\partial L_t}, \tag{1}$$

where $\alpha_L > 0$ is called the adjustment speed parameter and π is bank's profit. This model is widely used in banking modelling to analyze the role of banking operating cost, reserve requirement policy, and macroprudential policy [5, 4, 3, 6].

The bank's profit is calculated as follows

$$\pi_t = r_L L_t - r_D D_t - r_E E_t - C_t,$$

where r_L and r_D are loan and deposit interest rates, respectively. We follow from the Monti-Klein model [21, 24] that they are assumed as $r_L = a_L - b_L L$ and $r_D = a_D + b_D D$ with $a_L, b_L, a_D, b_D > 0$; r_E is cost of equity with $0 < r_E < 1$;

and $C = c_D D - cDL + c_L L$ is bank operating cost with $0 < c, c_D, c_L < 1$. Here we add $-cDL$ as depiction of the economies of scope in banking [13].

Therefore, we have the calculation of bank's profit and its partial derivative as

$$\pi_t = \left(a_L - \left[r_E\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) L_t - \left[b_L - c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t^2.$$

and

$$\frac{\partial \pi_t}{\partial L_t} = a_L - \left[r_E\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right] - 2\left[b_L - c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t, \tag{2}$$

The final form of the banking loan model is given by

$$L_{t+1} = L_t + \alpha_L L_t \left(a_L - \left[r_E\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right] - 2\left[b_L - c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t\right). \tag{3}$$

3. MODEL ANALYSIS

By putting $L_{t+1} = L_t$ and solving for L, we have two equilibriums as follows

$$L^*_{(0)} = 0 \quad \text{and} \quad L^*_{(1)} = \frac{a_L - \left[r_E\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]}{2\left[b_L - c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right]}.$$

The loan equilibrium $L^*_{(1)}$ must be positive for it to have economic significance, in other words

$$a_L > r_E\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right). \tag{4}$$

Therefore, if the loan interest rate is relatively high, there exists a positive loan equilibrium.

Consider rewriting the difference equation (3) as $L_{t+1} = f(L_t)$. [2] asserts that the one-dimensional map (3) is stable if $|f'(L^*)| < 1$. Based on this, the stability of the obtained loan equilibrium values is provided by the subsequent theorem.

Theorem 1. *The loan equilibrium $L^*_{(0)}$ is unstable. In the meantime, the loan equilibrium $L^*_{(1)}$ is stable if $\kappa > \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\alpha_L[r_E(1-\rho)-(a_D+c_D)]}$.*

Proof. The first derivative of the right-hand side of (3) is given by

$$f'(L_t) = 1+\alpha_L\left(a_L - \left[r_E\kappa + c_L + (a_D+c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right)-4\alpha_L\left[b_L + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right]L_t.$$

For the equilibrium $L^*_{(0)}$, based on the positivity condition (4), we have

$$f'(L^*_{(0)}) = 1+\alpha_L\left(a_L - \left[r_E\kappa + c_L + (a_D+c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) > 1.$$

Consequently, $L^*_{(0)}$ is unstable.
For the equilibrium $L^*_{(1)}$, we get

$$f'(L^*_{(1)}) = 1-\alpha_L\left(a_L - \left[r_E\kappa + c_L + (a_D+c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right). \tag{5}$$

It is evident from the positivity condition (4) that $f'(L^*_{(1)}) < 1$. However, $f'(L^*_{(1)}) > -1$ is satisfied if

$$\kappa > \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\alpha_L\left[r_E(1-\rho)-(a_D+c_D)\right]}.$$

So, $L^*_{(1)}$ is stable if $\kappa > \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\alpha_L[r_E(1-\rho)-(a_D+c_D)]}$. □

We follow the Jury stability requirement for a one-dimensional map as stated in [18]. There are two critera: (i) the condition $f'(L^*_{(1)}) = 1$ implies that the loan equilibrium will become unstable due to transcritical bifurcation, (ii) whereas the condition $f'(L^*_{(1)}) = -1$ implies that the loan equilibrium will lose stability owing to flip or period-doubling bifurcation. The purpose of this article is to investigate the CAR policy parameter represented by κ. Therefore, κ will function as the bifurcation parameter. The following theorem is arrived at through a sequence of simple calculations.

Theorem 2. *When $\kappa = \kappa^T$, where*

$$\kappa^T = \frac{(a_L-c_L)(1-\rho)-(a_D+c_D)}{r_E(1-\rho)-(a_D+c_D)},$$

the equilibrium $L^*_{(1)}$ may become unstable due to transcritical bifurcation. In contrast, when $\kappa = \kappa^F$, where

$$\kappa^F = \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\alpha_L\left[r_E(1-\rho)-(a_D+c_D)\right]},$$

the equilibrium $L^*_{(1)}$ may become unstable which resulting flip bifurcation.

Proof. The following equation can be solved directly to determine the transcritical bifurcation value,

$$f'(L^*_{(1)}) = 1 - \alpha_L \left(a_L - \left[r_E\kappa + c_L + (a_D + c_D) \left(\frac{1-\kappa}{1-\rho} \right) \right] \right) = 1$$

for parameter κ, which resulting $\kappa = \kappa^T$. Similarly, by solving

$$f'(L^*_{(1)}) = 1 - \alpha_L \left(a_L - \left[r_E\kappa + c_L + (a_D + c_D) \left(\frac{1-\kappa}{1-\rho} \right) \right] \right) = -1$$

for parameter κ, the value of the flip bifurcation is determined as $\kappa = \kappa^F$. □

From Theorem 2 we get

$$\begin{aligned} \kappa^F &= \frac{\alpha_L(a_L - c_L)(1-\rho) - \alpha_L(a_D + c_D) - 2(1-\rho)}{\alpha_L \left[r_E(1-\rho) - (a_D + c_D) \right]} \\ &= \frac{(a_L - c_L)(1-\rho) - (a_D + c_D)}{r_E(1-\rho) - (a_D + c_D)} - \frac{2(1-\rho)}{\alpha_L \left[r_E(1-\rho) - (a_D + c_D) \right]} \\ &= \kappa^T - \frac{2(1-\rho)}{\alpha_L \left[r_E(1-\rho) - (a_D + c_D) \right]}. \end{aligned}$$

If $\Delta\kappa := \frac{2(1-\rho)}{\alpha_L[r_E(1-\rho)-(a_D+c_D)]}$ is positive (or negative), then $\kappa^F < \kappa^T$ (or $\kappa^F > \kappa^T$). The positivity of $\Delta\kappa$ is determined by the positivity of $P := r_E(1-\rho) - (a_D + c_D)$. Figure 1 depicts the transcritical and flip bifurcations curves versus the parameter r_E, ρ, a_D, and c_D.

As a result, we have upper and lower bounds for the parameter κ, that is $\kappa^F < \kappa < \kappa^T$ if $r_E(1-\rho) > a_D + c_D$, or $\kappa^F < \kappa < \kappa^T$ if $r_E(1-\rho) < a_D + c_D$, to ensure the stability of the loan equilibrium.

Given that $0 < \kappa < 1$, both transcritical and flip bifurcations must have values between 0 and 1 in order to have economic meanings.

$$\begin{aligned} 0 < \kappa^T < 1 \quad &\Leftrightarrow \quad 0 < \frac{(a_L - c_L)(1-\rho) - (a_D + c_D)}{r_E(1-\rho) - (a_D + c_D)} < 1 \\ &\Leftrightarrow \quad a_L < r_E + c_L < \frac{(a_D + c_D)}{(1-\rho)} + c_L. \end{aligned}$$

In the meantime, we have the following for the case of flip bifurcation value,

$$\begin{aligned} 0 < \kappa^F < 1 \quad &\Leftrightarrow \quad 0 < \frac{\alpha_L(a_L - c_L)(1-\rho) - \alpha_L(a_D + c_D) - 2(1-\rho)}{\alpha_L \left[r_E(1-\rho) - (a_D + c_D) \right]} < 1 \\ &\Leftrightarrow \quad \frac{2}{a_L - \left(c_L + \frac{(a_D + c_D)}{1-\rho} \right)} < \alpha_L < \frac{2}{a_L - (r_E + c_L)}. \end{aligned}$$

As a result, we have the next corollary.

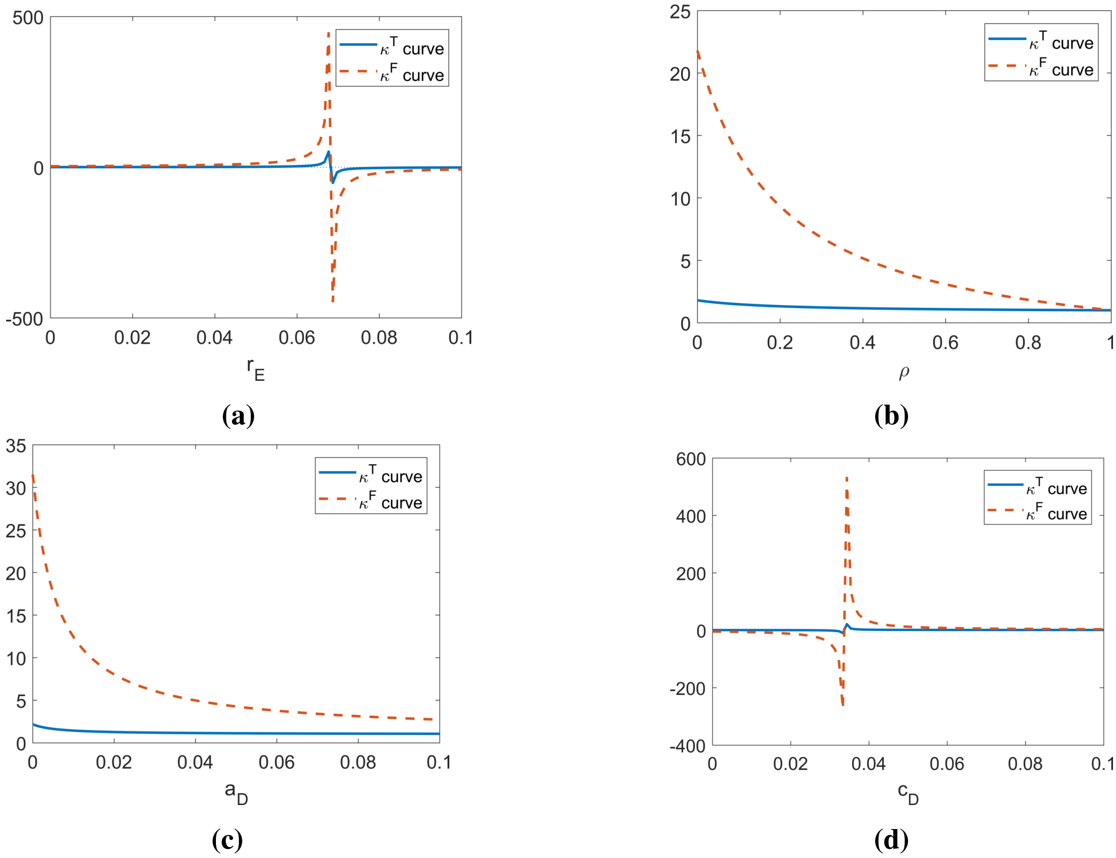


Figure 1: Transcritical bifurcation curve κ^T and flip bifurcation curve κ^F versus parameter (a) r_E, (b) ρ, (c) a_D, and c_D.

Corollary 1. *There will be a transcritical bifurcation value κ^T if*

$$a_L < r_E + c_L < \frac{(a_D + c_D)}{(1-\rho)} + c_L.$$

Meanwhile, the flip bifurcation κ^F will exist if

$$\frac{2}{a_L - \left(c_L + \frac{(a_D+c_D)}{1-\rho}\right)} < \alpha_L < \frac{2}{a_L - (r_E + c_L)}.$$

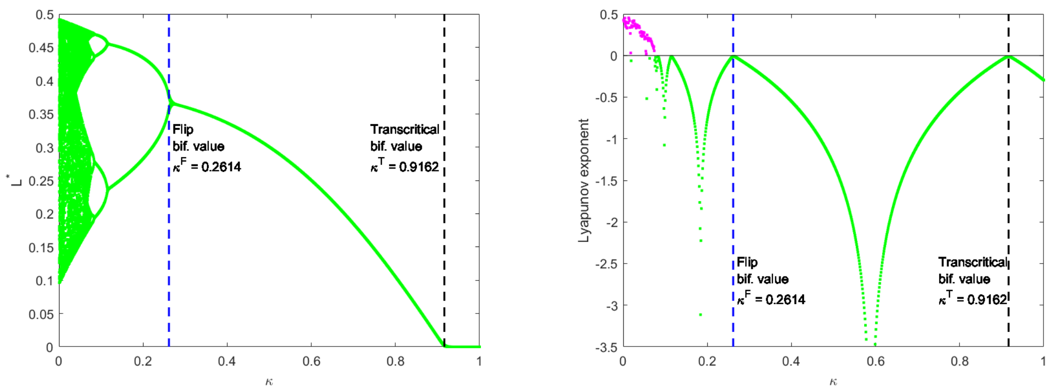
4. SIMULATION AND SENSITIVITY ANALYSIS

In this section, the results of the previous section are illustrated and validated using several numerical simulations using parameter values listed in Table 2. Although the values of these parameters were chosen primarily for simulation purposes, they nonetheless satisfy the criteria in Corollary 1 and the positivity condition in (4). Based on the values from Table 2, we have $\kappa^F = 0.2614$ and $\kappa^T = 0.9162$ as the bifurcation values of the CAR policy parameter.

Table 2: The value of parameters for simulation.

Parameter	Description	Value
ρ	RR policy parameter	0.12
a_L	Parameter of loan interest rate	0.092
b_L	Parameter of loan interest rate	0.05
a_D	Parameter of deposit interest rate	0.01
b_D	Parameter of deposit interest rate	0.05
r_E	Cost of equity	0.05
κ	CAR policy parameter	Vary
c	Economies of scope	0.05
c_D	Marginal cost of deposit	0.05
c_L	Marginal cost of loan	0.05
α_L	Speed of adjustment	32

Using bifurcation diagrams, we examine the loan map to provide comprehensive insights on the map's dynamics, including when it is stable or unstable and when it can behave chaotically. A bifurcation diagram is provided for the parameter κ in Figure 2a. The graph shows that the loan equilibrium is zero when the CAR parameter is greater than the transcritical bifurcation value, $\kappa > \kappa^T$. We reach a positive and stable loan equilibrium when the CAR parameter falls between the transcritical and flip bifurcation levels, $\kappa^F < \kappa < \kappa^T$. Lowering the CAR parameter starting from $\kappa < \kappa^F$ will make the loan equilibrium raising within this range. The loan equilibrium behaves periodically 2-period, 4-period, 8-period, and so on even resulting chaos.



(a) **(b)**

Figure 2: Bifurcation diagram of the CAR policy parameter κ and (b) the corresponding Lyapunov exponent.

In Figure 2b, we plot the graph of the Lyapunov exponent, which is related to Figure 2a to depict the proof the existence of chaos as shown by the magenta dots which is a mark for that the Lyapunov exponent is greater than zero. The calculation of Lyapunov exponent is given below

$$h(L) = \frac{1}{N_2} \sum_{t=N_1+1}^{N_1+N_2} \ln |f'(L_t)|,$$

for some big natural numbers N_1 and N_2. As stated in [2], the positive Lyapunov exponent corresponds to chaos.

We investigate how changes in the CAR policy parameter κ affecting the loan trajectory through time. Figure 3 shows a graph that shows convergent loan trajectory with quite big κ. A larger loan equilibrium is obtained by using a smaller value for κ. When the value of κ is significantly bigger, the trajectory of the loan behaves periodically with bigger amplitude.

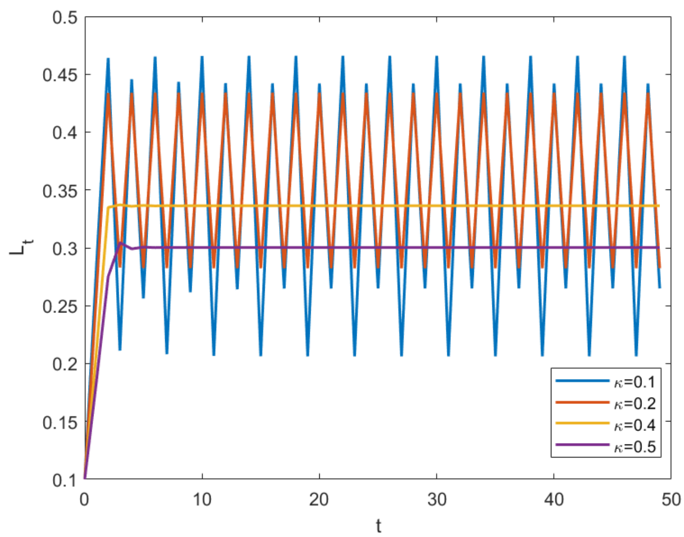


Figure 3: Graphs of loan L_t versus time t for various values of the CAR policy parameter κ.

It is always fascinating to see how a map behaves when its dynamics turn unstable, like periodic or even chaotic. A map can be represented most simply by its trajectory. There, we may directly observe the dynamics of the route from the start till equilibrium. Another way to look into qualitative behavior is to display the cobweb diagram of the map. We show the loan trajectory for $\kappa = 0.15$ in Figure 4a. It is clear that the trajectory continually swings across a two-period cycle. The cobweb diagram for this trajectory is shown in Figure 4b. The green trajectory line only hits the blue dashed curve twice when the duration t is large, indicating that the map is about to enter a two-period cycle. The loan trajectory for $\kappa = 0.015$ is shown in Figure 4c. In this case, the chaotic behavior or the unpredictability of the trajectory can be immediately observed. The cobweb diagram that goes along with this chaotic track is shown in Figure 4d. The blue dashed curve is repeatedly crossed by the green trajectory line when t is large.

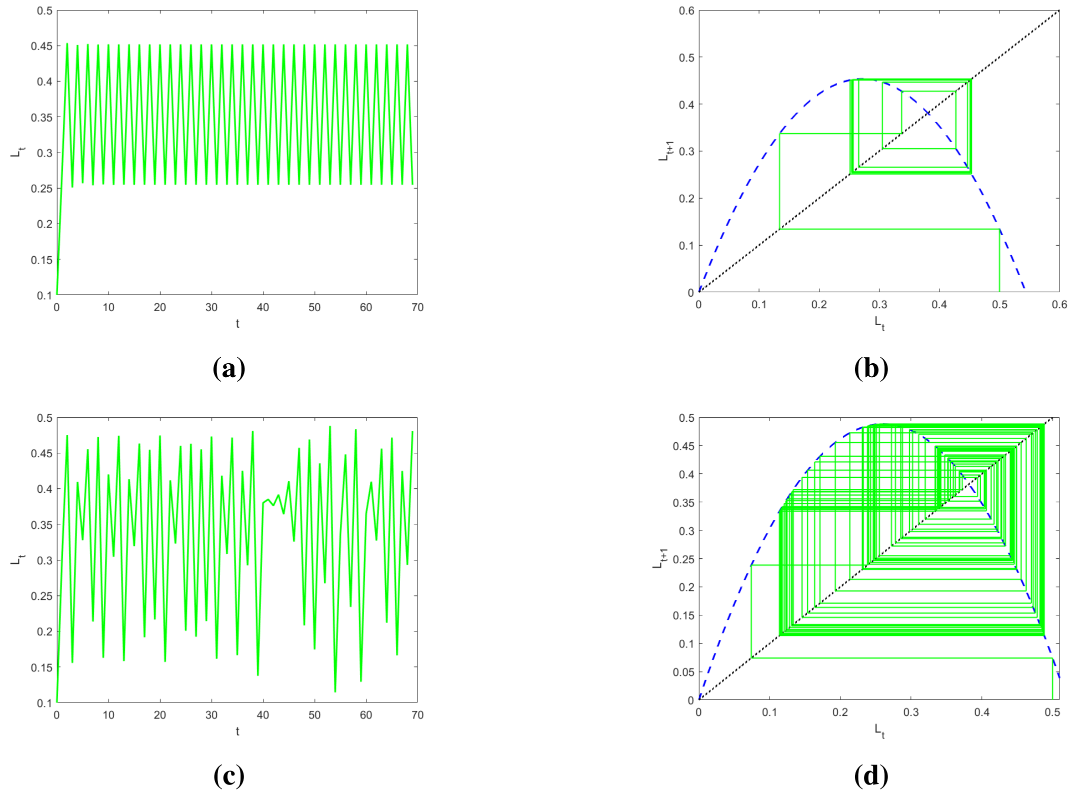


Figure 4: Panel (a) shows the loan trajectory with a 2-period equilibrium when the CAR policy parameter $\kappa = 0.15$ and panel (b) is the respective cobweb diagram of panel (a). Panel (c) shows the loan trajectory with chaotic behavior when the equity to loan ratio parameter $\kappa = 0.015$ and panel (d) is the respective cobweb diagram of panel (c).

It has been demonstrated in Theorem 1 that the stability of the loan equilibrium $L^*_{(1)}$ requires $\kappa > \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\alpha_L[r_E(1-\rho)-(a_D+c_D)]}$. Now, by defining $S := \frac{\alpha_L(a_L-c_L)(1-\rho)-\alpha_L(a_D+c_D)-2(1-\rho)}{\kappa\alpha_L[r_E(1-\rho)-(a_D+c_D)]}$, Theorem 1 can be rewritten as: $L^*_{(1)}$ is stable if $S < 1$. It is interesting to examine how the parameter κ interacts with other parameters from $Q = \{a_L, r_E, \rho, c_L, a_D, c_D\}$ in order to preserve loan stability, given that the goal of this paper is to analyze the CAR policy, which is represented by the parameter κ. In Figure 5, we show the contour plot of S in the $\kappa - q$ plane, where $q \in Q$. By looking at the colorbar with a value below one, we can see where the loan equilibrium is stable in that location.

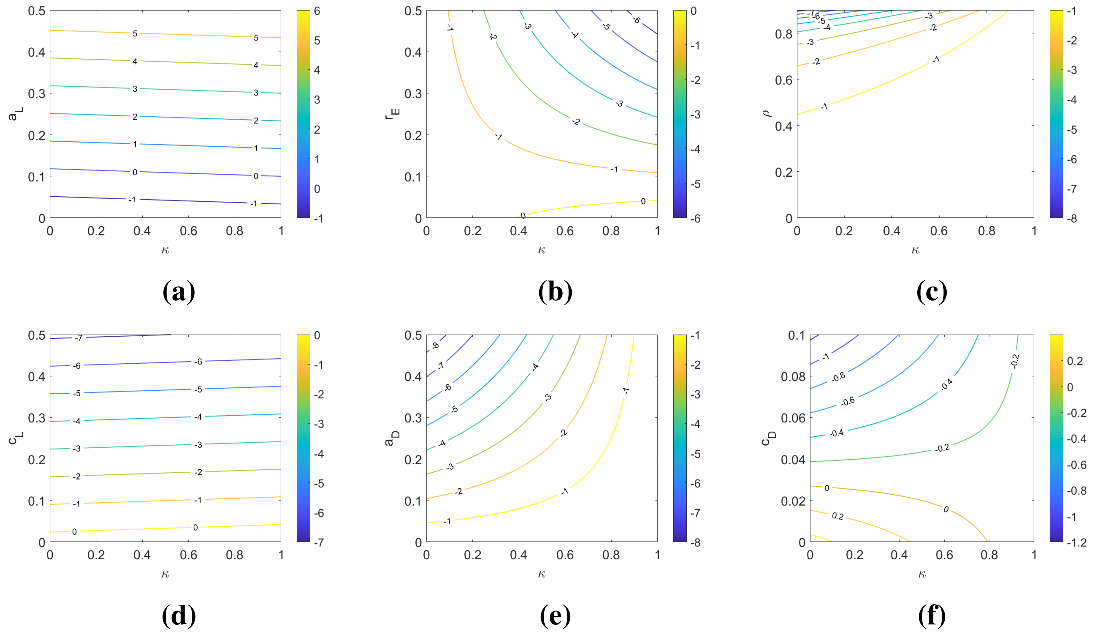


Figure 5: Contour plot of S in a plane of combination of parameter κ with (a) a_L, (b) r_E, (c) ρ, (d) c_L, (e) a_D, and (f) c_D.

The contour plots in Figure 5 can be seen as a sensitivity analysis of the CAR policy parameter (κ) in combination with other parameters that have been introduced in the banking industry, loan interest rate parameter (a_L), cost of equity (r_E), RR policy parameter (ρ), marginal cost of loan (c_L), deposit interest rate parameter (a_D), and marginal cost of deposit (c_D), in contributing to the stability of loans. It is clear from Figures 5b, 5c, 5d, 5e, and 5f that the combinations of κ with r_E, ρ, c_L, a_D, and c_D do not cause the loan to become unstable. On the other hand, for a particular combination of κ with a_L, in Figure 5a, we can see that the loan equilibrium is unstable for low κ and high a_L.

5. CONCLUSION

The CAR policy parameter of the central bank is always a time-varying issue that needs to be examined in light of how economic cycles behave. In order to ascertain how the policy might impact the loan dynamics of a bank, this study investigates the issue. An increase in the loan equilibrium is produced by lowering the CAR policy parameter. The results of this study suggest that the CAR policy parameter should neither be overly high or low. If the CAR policy parameter becomes too high in the future, the loan will be revoked. On the other hand, a loan could become unstable and destabilize the economy in the interim if the CAR policy parameter is too low.

ACKNOWLEDGMENT

This study is financed using Dana Selain APBN FSM UNDIP T.A. 2023 with grant number 22.D/UN7.F8/PP/II/2023.

REFERENCES

[1] Heba Abou-El-Sood. Are regulatory capital adequacy ratios good indicators of bank failure? evidence from us banks. *International Review of Financial Analysis*, 48(December):292–302, 2016.

[2] K. Alligood, T. Sauer, , and J. Yorke. *Chaos: An Introduction to Dynamical Systems*. Springer-Verlag, New York, 1996.

[3] M. F. Ansori and S. Hariyanto. Analysis of banking deposit cost in the dynamics of loan: Bifurcation and chaos perspectives. *BAREKENG: Journal of Mathematics and Its Application*, 16(4):1283–1292, 2022.

[4] M. F. Ansori and S. Khabibah. The role of cost of loan in banking loan dynamics: Bifurcation and chaos analysis. *BAREKENG: Journal of Mathematics and Its Application*, 16(3):1031–1038, 2022.

[5] M. F. Ansori, N. Sumarti, and K. A. Sidarto. Analyzing a macroprudential instrument during the covid-19 pandemic using border collision bifurcation. *Rect@*, 22(2):113–125, 2022.

[6] M. F. Ansori, G. Theorista, and Winson. Difference equation-based banking loan dynamics with reserve requirement policy. *International Journal of Difference Equations*, 18(1):35–48, 2023.

[7] Kenneth Baldwin, Maryam Alhalboni, and Mohamad Husam Helmi. A structural model of "alpha" for the capital adequacy ratios of islamic banks. *J. Int. Financ. Markets, Inst. Money*, 60(May):267–283, 2019.

[8] A. N. Berger, R. J. Herring, and G. P. Szegö. The role of capital in financial institutions. *J. Bank. Finan.*, 19(3-4):393–430, 1995.

[9] G. I. Bischi, C. Chiarella, M. Kopel, and F. Szidarovszky. *Nonlinear Oligopolies: Stability and Bifurcations*. Springer-Verlag, Berlin, 2010.

[10] M. Bitar, K. Pukthuanthong, and T. Walker. The effect of capital ratios on the risk, efficiency and profitability of banks: evidence from oecd countries. *J. Int. Financ. Markets, Inst. Money*, 53:227–262, 2018.

[11] S. Brianzoni and G. Campisi. Dynamical analysis of a banking duopoly model with capital regulation and asymmetric costs. *Discrete Contin. Dyn. Syst. - B*, 26:5807–5825, 2021.

[12] S. Brianzoni, G. Campisi, and A. Colasante. Nonlinear banking duopoly model

with capital regulation: The case of italy. *Chaos Solitons Fractals*, 160:112209, 2022.

[13] Eleni Dalla and Erotokritos Varelas. Regulation & oligopoly in banking: The role of banking cost structure. *Journal of Economics and Business*, 104(July-August):105836, 2019.

[14] I. Drumond. Bank capital requirements, business cycle fluctuations and the basel accords: a synthesis. *Journal of Economic Surveys*, 23(5):798–830, 2009.

[15] A. A. Elsadany. Dynamics of a delayed duopoly game with bounded rationality. *Mathematical and Computer Modelling*, 52:1479–1489, 2010.

[16] A. A. Elsadany. Dynamics of a cournot duopoly game with bounded rationality based on relative profit maximization. *Appl. Math. Comput.*, 294:253–263, 2017.

[17] L. Fanti. The dynamics of a banking duopoly with capital regulations. *Econ. Model.*, 37:340–349, 2014.

[18] G. Gandolfo. *Economic Dynamics: Methods and Models*. Elsevier Science Publisher BV, Amsterdam, The Netherlands, 2 edition, 1986.

[19] Shirley J. Ho and Su-Chu Hsu. Leverage, performance and capital adequacy ratio in taiwan's banking industry. *Japan and the World Economy*, 22(4):264–272, 2010.

[20] Robert Jarrow. A leverage ratio rule for capital adequacy. *Journal of Banking & Finance*, 37(3):973–976, mar 2013.

[21] M. A. Klein. A theory of the banking firm. *Journal of Money, Credit, and Banking*, 3:205–218, 1971.

[22] Salma Louati, Ilhem Gargouri Abida, and Younes Boujelbene. Capital adequacy implications on islamic and non-islamic bank's behavior: Does market power matter? *Borsa Istanbul Review*, 15(3):192–204, 2015.

[23] Mehdi Mili, Jean Michel Sahut, Hatem Trimeche, and Frédéric Teulon. Determinants of the capital adequacy ratio of foreign banks' subsidiaries: The role of interbank market and regulation. *Research in International Business and Finance*, 42:442–453, dec 2017.

[24] M. Monti. Deposit, credit and interest rates determination under alternative objective functions. In G. P. Szego and K. Shell, editors, *Mathematical methods in investment and finance*. Amsterdam, 1972.

[25] J. F. Sinkey. *Commercial Bank Financial Management in the Financial Services Industry*. MacMillan Publishing Company, 3rd edition, 1989.

[26] D. VanHoose. Theories of bank behavior under capital regulations. *Journal of Banking and Finance*, 31:3680–3697, 2007.