Modeling and Algorithm Development for Cattle Feed Mix Formulation

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Abstract
The objective of this paper is to develop the algorithms for optimal feed mix of dairy cows at different stages of livestock. These algorithms have been proposed to formulate feed mix at minimum cost and maximum shelf life for dairy cattle. Different forms of mathematical programming have been used to develop these algorithms such as stochastic and weighted goal programming. These algorithms incorporate nutrient variability of different nutrients present in the feed ingredients which has not been done by the available softwares. These algorithms also minimize the deviations of the objectives of cost minimization and shelf life maximization.

Keywords: Cattles; feed mix; algorithms; cost minimization; shelf-life maximization.

INTRODUCTION
Feed mix is a mixture of different feed ingredients used for animal feeding. A balanced feed mix provides required nutrition to the cattle at different stages of livestock therefore, in dairy industry, formulation and computation of balanced feed mix is of utmost importance. It provides better yields, productivity and nutrient utilization. For achieving these objectives, optimization should be done in such a
manner that optimal feed mix can fulfil the nutritional requirements of the animal at different stages of production and livestock. Optimization models are in use for more than a half century at commercial level and for livestock management. A linear programming technique has been developed for defining feed formulation problem [1]. Various animal feed formulation techniques have been reviewed in an article [2]. Another review article has been presented for formulating animal feed mix on the basis of various programming techniques [3]. An LP model has been presented to find the least-cost ration for drought maintenance of dry adult sheep. Results showed a considerable reduction in the feeding cost compared to currently recommended standards [4]. A linear programming model has been developed to find the least cost ration for broilers of age 6 to 10 weeks for the utilization of locally available and non-conventional feed stuff-Duckweed (Lemna paucicostata) [5]. A model has been developed for optimal beef production systems in Ireland in which the objective is to maximize farm gross margin under a constraint set of animal nutritional requirements [6]. A linear model has been developed for the Nigerian Poultry Industry and it has been found that cost was reduced by 9% compared to existing practice of the farm [7]. A nonlinear programming model has been developed for weight gain in sheep [8]. A paper has been presented in which LP, LP with a margin of safety (LPMS) and SP models have been used to formulate poultry rations at least cost, with a given probability level to meet nutrient requirements, set by the NRC in 1984. LP has formulated least cost feed mix but has not the ability to consider nutrient variability. The LPMS and SP models have met poultry nutritional requirements at different confidence levels varying from P 0.5 to 0.90. The SP model has produced lower cost feed mix than LPMS [9]. Linear and stochastic programming techniques have been used for incorporating nutrient variability in animal feed formulation [10]. To consider nutrient variability in least-cost feed formulation model for African catfish SP technique has been used [11]. Goal Programming has been presented as a tool for formulating feed mixes using one hundred and fifty food raw materials. Results by GP showed improvement over those of LP [12]. A handy spreadsheet tool has been developed for the formulation of a daily cow feed mix supported by linear programming and weighted goal programming techniques [13]. A multi criteria programming model has been developed using goal programming technique to find optimized feed blend[14]. A model has been developed using a combination of LP and WGP. In this model, multiple goals have been incorporated for optimization. The method has been tested and concluded to more accurate and useful results in practice by WGP as compared to LP [15].A paper has been presented to formulate ruminant ration under bi-objective criteria [16]. The nutritional requirements of dairy cattle are different at different weights as per NRC recommendations [17].

In this paper, objectives are to develop the algorithms:

- to achieve the nutrient variability included feed mix at lowest possible cost which satisfy the nutritional requirement of the cattles at different weight stages.
- to achieve the nutrient variability included feed mix with maximum shelf life which satisfy the nutritional requirement of the cattles at different weight stages.
- to minimize the deviations of the achieved objectives.
This paper presents the technological intervention to the field of animal nutrition by developing algorithms to achieve the above objectives. Previously, cost minimization and shelf life maximization models have been developed [18, 19] but these models were solved by available software. This technology proposes development of algorithms to achieve these two objectives with inclusion of nutrient variability. The density of nutrient contents of different feed ingredients may change considerably. While formulating the optimal feed mix using algorithm based on linear programming model, this variation cannot be considered and cause over-formulation or under-formulation which results in higher cost, over or under achievement of nutrient requirements, adverse effect on the growth rate of the animals etc. Therefore, to reduce the risk of over and under achievement of nutrients, it is essential to consider this variation while developing the algorithms for animal feed mix formulation. This technology is an attempt to deal with this nutrient variability. These algorithms have been developed to incorporate nutrient variability. In addition to this, one of the algorithm is developed to minimize the deviations of the objectives which is done innovatively first time in the area of feed mix formulation.

Algorithm based on linear programming model is used to achieve cattle feed mix at lowest cost for different stages of livestock. It is also used to maximize the shelf life of cattle feed mix. Shelf life can be increased by reducing water content for different stages of livestock. Algorithm based on stochastic programming model is used to incorporate nutrient variability so that the risk of not meeting the nutrient requirement can be minimized. Practically, feed mix formulation is a complex process. It cannot be confined to the achievement of one objective only. Real life problems need a solution which satisfies multiple conflicting objectives on a priority basis. To overcome this drawback Multi-criteria Goal programming model can be used. GP is used with weights and priorities for prioritizing multiple goals of a single farm holder and also to assign weightage to the goals of same priority level. Priorities have been introduced to achieve multiple target values simultaneously with different preferences. Weights have been introduced to give weightage to different goals in the same priority level.

METHODS

Three algorithms have been developed to achieve the said objectives and a combination of mathematical programming has been used for this purpose. In developing algorithms the objective functions are taken as cost minimization and water content minimization for maximum shelf life. Constraints have been formulated on the basis of nutritional requirements of the animals at different stages of livestock. These algorithms will provide the optimal feed mix with lowest cost and minimum water content to reach different weight class of dairy cattle. Firstly algorithm 1 has been developed to achieve the optimal feed mix with rigid nutrient constraints. Then, algorithm 2 has been developed to incorporate nutritional variability of different nutrients. It minimizes the risk of not meeting the nutrient requirement. Finally, algorithm 3 for developing animal feed mix provides the optimal solution with the
satisfaction of the constraints depending on the priorities and weights of goals. These algorithms are capable of considering different priorities and weights associated with the goals and therefore providing more practical results.

**Notations used for developing mathematical models:**

- $z$ objective function, $a_{ij}$ amount of $i^{th}$ nutrient available in the $j^{th}$ feed ingredient, $x_j$ quantity of $j^{th}$ feed ingredient in the feed mix, $b_i$ minimum requirement of $i^{th}$ nutrient, $c_j$ per unit cost of feed ingredient $j$, $i$ index identifying feed nutrient components with $i = 1, 2, \ldots, m$, $j$ index identifying feed ingredients with $j = 1, 2, \ldots, n$.

Algorithm 1 has been developed for formulating and computing feed mix with lowest possible cost and minimum water content on the basis of linear programming model.

**Linear Programming Model for computing feed mix:**

$$
\begin{align*}
\min z &= \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}x_j \\
\text{s.t.} \sum_{j=1}^{n} a_{ij}x_j &\geq b_i \\
x_j &\geq 0, b_i \geq 0
\end{align*}
$$

(1)

**Algorithm for computation of feed mix at minimum cost and water content (Algorithm 1):**

- **Step 1:** Define nature of objective function (max. or min.).
- **Step 2:** Input number of decision variables (i.e. $j$).
- **Step 3:** Input cost coefficients (per unit cost of each feed ingredient ($x_j$)) $c_j$ for $j=1$ to 16 to formulate objective function.
- **Step 4:** Input technological coefficients $a_{ij}$ and requirement variables(nutrient requirements) $b_i$ for $i=1$ to 5 and $j=1$ to 16 to formulate the constraints.
- **Step 5:** Formulate the linear mathematical model.
- **Step 6:** Introduce artificial variables to get basis matrix as we are not getting identity matrix as basis matrix.
- **Step 7:** Construct auxiliary LPP $\min f_a = \sum a_k$ for $k = 1$ to 5.
Step 8: Construct simplex table of phase I with 5 basic variables.

Step 9: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \leq 0 \forall j$ for minimization

(ii) $z_j - c_j \geq 0 \forall j$ for maximization.

Step 10: (i) If optimality condition is satisfied then

(a) stop

(b) write the optimal solution of phase I, go to step 12.

(ii) else

(a) find leaving variable,

(b) entering variable and

(c) the pivot element.

(d) Construct the simplex table.

Step 11: Repeat step 9-10.

Step 12: Construct the simplex table of Phase II.

Step 13: Check optimality condition using $z_j - c_j$

(i) $z_j - c_j \leq 0 \forall j$ for minimization

(ii) $z_j - c_j \geq 0 \forall j$ for maximization.

Step 14: (i) If optimality condition is satisfied then,

(a) stop

(b) write the optimal solution(feed mix) of the problem.

(ii) else

(a) find leaving variable

(b) entering variable

(c) the pivot element.

(d) construct the simplex table.

Step 15: Repeat step 13-14.
Flow chart for algorithm 1 is shown in fig. 1.

![Flow Chart for algorithm 1](image-url)

**Figure 1:** Flow Chart for algorithm 1
Algorithm 1 gives the optimum feed mix with minimum cost and water content but it does not take into account the nutritional variability.

Algorithm 2 has been developed for the same objectives as above but with the incorporation of nutritional variability of different nutrients as it is an important factor in computing feed mix. This variation, if not considered, can affect the growth rate of animal negatively. Therefore, with this algorithm the effect of nutritional variability of different nutrients on the feed mix can be controlled. In the presence of variability, it is possible to determine the probability that the nutrient concentration in the feed mix meets or exceeds the specified requirements in the feed blend. Therefore stochastic programming models are introduced to consider the variability of nutrients present in different feed ingredients. To introduce the variability of nutrient components, nonlinear variance of each nutrient ingredient is added at a desired probability level in the mathematical model. $\sigma_{ij}^2$ represents variance of nutrient $i$ in ingredient $j$ and it is included with a certain probability level, $z$ represents level of probability and rest of the variables are defined as above.

### Stochastic Programming Model for computing feed mix:

$$\text{Min} \quad z = \sum c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} - z \left( \frac{\sum_{j=1}^{n} \sigma_{ij}^2}{\sigma_{ij}^2} \right) x_j \geq b_i$$

$$x_j \geq 0, b_i \geq 0$$

(2)

### Algorithm for computation of feed mix at minimum cost and water content including nutrient variability (Algorithm 2):

Step 1: Define nature of objective function (max. or min.).

Step 2: Input number of decision variables (i.e. $j$).

Step 3: Input cost coefficients $c_j$ for $j=1$ to $16$ to formulate objective function.

Step 4: Input technological coefficients $a_{ij}$ and requirement variables $b_i$ for $i=1$ to $5$ and $j=1$ to $16$ to formulate the constraints.

Step 5: Input probability level and standard deviation $\sigma_{ij}$ for $i=1$ to $5$ and $j=1$ to $16$.

Step 6: Calculate $z$ for the given probability level.

Step 7: Formulate the Stochastic mathematical model.

Step 8: Introduce artificial variables to get basis matrix as we are not getting identity matrix as basis matrix.

Step 9: Construct auxiliary LPP $\min f_a = \sum a_k$ for $k=1$ to $5$.
Step 10: Construct simplex table of phase I with 5 basic variables.

Step 11: Check optimality condition using \( z_j - c_j \)

(i) \( z_j - c_j \leq 0 \forall j \) for minimization

(ii) \( z_j - c_j \geq 0 \forall j \) for maximization.

Step 12: (i) If optimality condition is satisfied then

(a) stop

(b) write the optimal solution of phase I, go to step 14.

(ii) else

(a) find leaving variable,

(b) entering variable and

(c) the pivot element.

(d) Construct the simplex table.

Step 13: Repeat step 11-12.

Step 14: construct the simplex table of Phase II.

Step 15: Check optimality condition using \( z_j - c_j \)

(i) \( z_j - c_j \leq 0 \forall j \) for minimization

(ii) \( z_j - c_j \geq 0 \forall j \) for maximization.

Step 16: (i) If optimality condition is satisfied then,

(a) stop

(b) write the optimal solution (feed mix) of the problem.

(ii) else

(a) find leaving variable

(b) entering variable

(c) the pivot element.

(d) construct the simplex table.

Step 17: Repeat step 15-16.

Flow chart for algorithm 2 is shown in fig. 2.
Figure 2: Flow Chart for algorithm 2
Now algorithm has been developed with an emphasis on minimizing the deviations of the above formulated models. Mathematical model has been formulated with the help of weighted goal programming. Two goals are formed for minimizing the deviations for cost and water content minimization. For each goal, the objective functions of stochastic programming model are reconstructed as constraints with deviation variables. Rest of the constraints are same as in stochastic programming model. The objective function of the weighted goal programming model has been defined with the help of priorities, weights and deviation variables, corresponding to the constraints of cost and water content minimization, which is to be minimized as both of the objectives may not be fully satisfied simultaneously.

**Weighted Goal Programming Model for determination of feed blend:**

\[
\min z = P_1 \sum_{k=1}^{2} w_{1k} \frac{d_{ik}^+}{z_k} + P_2 \sum_{i=1}^{5} w_{2i} \frac{d_{2i}^+ + d_{2i}^-}{b_i}
\]

\[
s.t. \sum_{j=1}^{16} c_{ij} x_j - d_{ik}^+ = z_k \quad (3a)
\]

\[
\sum_{j=1}^{5} a_{ij} x_j + d_{2i}^- - d_{2i}^+ = b_i \quad (3b)
\]

\[
x_j \geq 0, b_i \geq 0
\]

k=1, 2 (corresponding to objectives as goals)
i=1 to 5 (corresponding to constraints as goals)

In the above models, the equations denoted by (3a) are the goals corresponding to the objectives of cost minimization and water content minimization with only over-achievement \( d_{ik}^+ \). These goals have been given priority \( P_1 \) with weights \( w_{11} \) and \( w_{12} \).
The equations denoted by (3b) are the goals corresponding to the nutritional requirement constraints with over and under achievement \( d_{2i}^+ \) and \( d_{2i}^- \). These goals have been given priority \( P_2 \) with weights \( w_{21}, w_{22}, w_{23}, w_{24}, \) and \( w_{25} \).

In developing weighted goal programming model normalization technique has been used to overcome the issue of different units of goals used for developing objective function.

**Algorithm for weighted goal programming (Algorithm 3):**

Step 1: Set the goals.

Step 2: Set the priorities and weights of the goals.
Step 3: Input goals as constraints.

Step 4: Input hard constraints.

Step 5: Add deviation variables to the goal constraints.

Step 6: Identify the variables to be minimized in the objective function.

Step 7: Write the weighted goal programming model.

Step 8: Identify goals with highest priority.

Step 9: Write the simplex table corresponding to this goal.

Step 10: Apply simplex algorithm.

Step 11: Find the optimal solution of this problem.

Step 12: (i) If alternative optimal solution exists then

a. Find the slack or surplus variables with negative value of $z_j - c_j$

b. Drop those slack or surplus variables from the table.

c. Delete objective function row.

d. Add constraints of next highest priority level.

e. Make solution feasible.

f. Add objective function corresponding to priority.

g. Repeat step 9-12.

(ii) else

a. Stop

b. Write the optimal solution of the problem.

Flow chart for algorithm 3 is shown in figure 3.
Figure 3: Flow Chart of algorithm 3
RESULTS AND DISCUSSION
Algorithms are developed for computation of optimal feed mix for dairy cattle. Models have been developed by using linear, stochastic and goal programming models and proposes a useful procedure for determination of the optimal livestock feed mix. This paper represents an innovative approach towards introduction of technology and leads to software development in the area of animal feed mix formulation. The paper represents algorithmic approach to bi-criteria model and can be extended to multi-criteria models. Algorithm 1 is providing the optimal feed mix. Algorithm 2 is incorporating nutritional variability of different nutrients to minimize the risk of not meeting the nutrient requirement. Algorithm 3 is providing the optimal solution with the satisfaction of the constraints depending on the priorities and weights of goals. Therefore it can be used to obtain more balanced feed mix which can optimize multiple parameters at the same time. The algorithms have been applied and verified on the data of dairy cattle. Data for composition of feed ingredients with cost, water content and nutrients are given by table 1.

Table 1: Composition of feed ingredients with cost, water content and nutritional composition

<table>
<thead>
<tr>
<th>Notation</th>
<th>Feed ingredients</th>
<th>Price (Rs./Kg.)</th>
<th>Water content (on the basis of DM)</th>
<th>Metabolizable energy(ME) (mj/kg of feed)</th>
<th>Crude Protein (CP) (g/kg)</th>
<th>NDF (g/kg)</th>
<th>DM (g/kg)</th>
<th>Ca (g/kg)</th>
<th>P (g/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>Alfalfa hay</td>
<td>14</td>
<td>.11</td>
<td>7.51</td>
<td>163</td>
<td>400</td>
<td>894</td>
<td>15</td>
<td>2.3</td>
</tr>
<tr>
<td>x2</td>
<td>Barley grain</td>
<td>10</td>
<td>.13</td>
<td>10.80</td>
<td>103</td>
<td>189</td>
<td>871</td>
<td>0.7</td>
<td>3.4</td>
</tr>
<tr>
<td>x3</td>
<td>Sugarbeet pulp</td>
<td>15</td>
<td>.11</td>
<td>9.99</td>
<td>83</td>
<td>429</td>
<td>892</td>
<td>13.83</td>
<td>0.89</td>
</tr>
<tr>
<td>x4</td>
<td>Cottonseed meal (high fibre, low oil)</td>
<td>18</td>
<td>.10</td>
<td>9.2</td>
<td>360</td>
<td>330</td>
<td>902</td>
<td>2.62</td>
<td>11</td>
</tr>
<tr>
<td>x5</td>
<td>Soyabean meal(high protein-dehulled)</td>
<td>28</td>
<td>.12</td>
<td>11.98</td>
<td>471</td>
<td>97</td>
<td>881</td>
<td>3.17</td>
<td>6.70</td>
</tr>
<tr>
<td>x6</td>
<td>Sunflower meal(solvent-extracted, dehulled or non-dehulled)</td>
<td>16</td>
<td>.11</td>
<td>8.10</td>
<td>288</td>
<td>400</td>
<td>890</td>
<td>3.92</td>
<td>10.32</td>
</tr>
<tr>
<td>x7</td>
<td>Wheat bran</td>
<td>19</td>
<td>.13</td>
<td>9.57</td>
<td>151</td>
<td>394</td>
<td>870</td>
<td>1.22</td>
<td>9.66</td>
</tr>
<tr>
<td>x8</td>
<td>Maize grain high moisture</td>
<td>23</td>
<td>.35</td>
<td>8.84</td>
<td>62</td>
<td>89</td>
<td>650</td>
<td>0.32</td>
<td>2.01</td>
</tr>
<tr>
<td>x9</td>
<td>Sorghum grain</td>
<td>17</td>
<td>.13</td>
<td>11.80</td>
<td>94</td>
<td>96</td>
<td>874</td>
<td>0.26</td>
<td>2.88</td>
</tr>
<tr>
<td>x10</td>
<td>Groundnut meal(solvent-extracted)</td>
<td>25</td>
<td>.11</td>
<td>11.16</td>
<td>489</td>
<td>217</td>
<td>893</td>
<td>1.52</td>
<td>5.54</td>
</tr>
<tr>
<td>x11</td>
<td>Rice bran(fibre 11-20%)</td>
<td>10</td>
<td>.10</td>
<td>9.11</td>
<td>115</td>
<td>310</td>
<td>902</td>
<td>0.63</td>
<td>12.45</td>
</tr>
</tbody>
</table>
Table 2: Minimum Nutritional Requirement of different nutrients at different weights of dairy cattle to reach at 600 kg weight

<table>
<thead>
<tr>
<th>Wt. class/Nutrient</th>
<th>ME(MJ)</th>
<th>CP(g)</th>
<th>DM(g)</th>
<th>Ca(g)</th>
<th>P(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 kg</td>
<td>43.71</td>
<td>533</td>
<td>5000</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>300 kg</td>
<td>57.11</td>
<td>671</td>
<td>6670</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>450 kg</td>
<td>67.37</td>
<td>749</td>
<td>7870</td>
<td>23</td>
<td>18</td>
</tr>
<tr>
<td>600 kg</td>
<td>82.06</td>
<td>879</td>
<td>9580</td>
<td>25</td>
<td>18</td>
</tr>
</tbody>
</table>

Then algorithm 1 is used to solve the model defined by equation (1) for cost minimization and shelf life maximization with rigid nutrient constraints. Optimal value of objective function and feed ingredients according to algorithm 1 has been shown in table III. Graphical results for feed ingredients corresponding to algorithm 1 have been shown in figure 4.

Table 3: Results from algorithm 1

<table>
<thead>
<tr>
<th>Feed Ingredients</th>
<th>Min cost</th>
<th>Min Water content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>x_1 Alfalfa hay</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>x_2 Barley grain</td>
<td>1.56</td>
<td>.59</td>
</tr>
<tr>
<td>x_3 Sugarbeetpulp</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x_4 Cottonseed meal (high fibre, low oil)</td>
<td>.66</td>
<td>.5</td>
</tr>
<tr>
<td>x_11 Rice bran(fibre 11-20%)</td>
<td>0</td>
<td>2.46</td>
</tr>
<tr>
<td>x_12 Oats grain</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x_13 Wheat straw</td>
<td>3.30</td>
<td>3.83</td>
</tr>
<tr>
<td>x_15 Canola meal(solvent-extracted)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Objective Function</td>
<td>51.29</td>
<td>66.35</td>
</tr>
</tbody>
</table>
Algorithm 2 is used to solve the model defined by equation 2 for cost minimization and shelf life maximization with variable nutrient concentration. Optimal value of objective function and feed ingredients according to algorithm 2 has been shown in table IV. Graphical results for feed ingredients corresponding to algorithm 2 have been shown in figure 5.

Table 4: Results from algorithm 2

<table>
<thead>
<tr>
<th>Feed Ingredients/ weight class</th>
<th>Min cost</th>
<th>Min water content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>x1   Alfalfa hay</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>x2   Barley grain</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x3   Sugarbeetpulp</td>
<td>2.49</td>
<td>3.14</td>
</tr>
<tr>
<td>x4   Cottonseed meal (high fibre, low oil)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x11  Rice bran (fibre 11-20%)</td>
<td>2.3</td>
<td>3.02</td>
</tr>
<tr>
<td>x12  Oats grain</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x13  Wheat straw</td>
<td>0.28</td>
<td>0.83</td>
</tr>
<tr>
<td>x15  Canola meal (solvent-extracted)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Objective Function</td>
<td>69.87</td>
<td>89.89</td>
</tr>
</tbody>
</table>
Tabular data and figures are depicting that though minimum values of objective functions have been achieved by using algorithm 1, more feed ingredients have been included in the feed mix achieved by algorithm 2. These results are showing that in terms of nutritional variability inclusion algorithm 2 is giving better results. Results from algorithm 3 are controlling over-achievement and under-achievement of values of different nutrients. Priorities are introduced in the model as $P_1$ and $P_2$. $P_1$ is the priority for both of the objectives and $P_2$ is for all the other constraints.

Results for the numerical data of dairy cattle according to algorithm 3 have been represented in table V. This table provides optimal values of objective function, feed ingredients and deviation variables.

Graphical view of values of deviations and of feed ingredients from algorithm 3 is shown in figure 6 and figure 7.

**Table 5: Results obtained from algorithm 3**

<table>
<thead>
<tr>
<th>Variables/weight clas</th>
<th>200</th>
<th>300</th>
<th>450</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{21}'$</td>
<td>0.201</td>
<td>0.261</td>
<td>0</td>
<td>1.665</td>
</tr>
<tr>
<td>$d_{23}'$</td>
<td>279.154</td>
<td>388.922</td>
<td>580.528</td>
<td>774.521</td>
</tr>
<tr>
<td>$d_{24}'$</td>
<td>0.386</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_3$</td>
<td>2.987</td>
<td>3.667</td>
<td>4.325</td>
<td>4.901</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.187</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$x_9$</td>
<td>0</td>
<td>0.015</td>
<td>0.288</td>
<td>0.568</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>0</td>
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<td>0.026</td>
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<td>2.908</td>
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<td>0.122</td>
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<td>$x_{15}$</td>
<td>0.030</td>
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<td>Objective value</td>
<td>2.663</td>
<td>2.652</td>
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CONCLUSION

In this paper, algorithms have been developed for computation of dairy cattle feed mix with lowest cost and maximum shelf life. Models of linear, stochastic and weighted goal programming with priority functions have been used for developing algorithms. It has shown a step by step approach to refine the results and make them more effective for better results.
REFERENCES


