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Connected total perfect dominating set in fuzzy graph

S.Revathi[1], C.V.R. Harinarayanan[2] and R.Muthuraj[3]

[1]Assistant Professor, Department of Mathematics, Saranathan College of Engineering Trichy – 620 001, Tamilnadu, India. E- mail: revathi.soundar@gmail.com

[2]Assistant Professor, Department of Mathematics, Government Arts College, Pudukkottai – 637 001, Tamilnadu, India. E-mail:CVRHNS@yahoo.com

[3]Assistant Professor, PG & Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India. E-mail: rmr1973@yahoo.co.in

Abstract

In this paper, the new kind of parameter total perfect domination number in a fuzzy graph is defined and established the parametric conditions. Another new kind of parameter connected total perfect domination number is defined and established the parametric conditions. The properties of total perfect domination number and connected total perfect domination numbers are discussed.

Keywords: Dominating set, perfect dominating set, total perfect dominating set, connected total perfect dominating set, connected total perfect domination number.

Mathematical Classification: 03E72,05C07, 05C69, 05C72, 05C76.

I INTRODUCTION

Graph theory is one of the most various branches of mathematics with applications to wide variety of subjects. In many real world problem we get only partial information about the problem, the vagueness in the description and uncertainty has led to the growth of fuzzy graph theory. A mathematical frame work to describe uncertainty in real life situation was first suggested by L. A. Zadeh [17]. Rosenfeld [14] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. One such interesting graph theoretic concept is domination in graphs. The study of dominating set in graphs was begun by Ore and Berge[11,1]. Sampathkumar and Walikar [15] introduced the concept of connected domination in graphs. The concept of Perfect domination, Total domination was introduced by Cockayne et al[3]. A remarkable beginning in fuzzy graphs for the concept domination was made in A.Somasundaram and S.Somasundaram [16] . They obtained several bounds for domination number using effective edges in fuzzy graph. A. Nagoor Gani and V.T Chandrasekaran [10] discussed domination in fuzzy graph using strong arcs. S.

Revathi et al.[12] studied about the concept of perfect domination in fuzzy graphs. They defined the perfect domination number of a fuzzy graph and obtained the relation between perfect domination number and independent domination number of a fuzzy graph. In this paper we discuss the connected total perfect domination number of a Fuzzy graph and introduce a new graph theoretic parameter known as connected total perfect domination in fuzzy graphs.

II Basic definitions:

2.1 Definition[1]

A **fuzzy graph** $G = (\sigma, \mu)$ is a pair of membership functions or fuzzy sets

$\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

2.2 Definition[1]

Let $G = (\sigma, \mu)$ be a fuzzy graph. The fuzzy graph $H = (\tau, \rho)$ is said to be a **fuzzy sub graph** of G, if τ is a fuzzy sub set of σ and ρ is a fuzzy sub set of μ.

2.3 Definition[1]

A fuzzy sub graph $H = (\tau, \rho)$ of $G = (\sigma, \mu)$ is said to be a **spanning fuzzy sub graph** of G if $\sigma(u) = \tau(u)$ for all vertices u in H and $\tau(u, v) \leq \mu(u, v)$ for all arcs (u, v) in H.

2.4 Definition[7]

The **order** p and **size** q of a fuzzy graph $G = (\sigma, \mu)$ are defined by $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{(u,v) \in E} \mu(u, v)$.

2.5 Definition[1]

An arc (u, v) is said to be a **strong arc or strong edge**, if $\mu(u, v) \geq \mu^{\infty}(u, v)$ and the vertex u is said to be a **strong neighbor to v or v is a strong neighbor to u**, otherwise it is called weak arc or weak edge.

A node u is said to be **isolated** in G if $\mu(u, v) = 0$ for all $v \neq u$, $v \in V$.

2.6 Definition[1]

The **strong arc neighbourhood degree** of a vertex v is defined by sum of the membership values of the strong adjacent vertices of v and is denoted by $d_N(v)$. That is, $d_N(v) = \sum_{u \in N_s(v)} \sigma(u)$.

2.7 Definition[11]

Let G be a fuzzy graph and the **minimum strong arc neighbourhood degree** of the fuzzy graph G is defined by $\delta_N(G) = \min\{d_N(u) : u \in V(G)\}$ and the **maximum strong arc neighbourhood degree** of G is $\Delta_N(G) = \max\{d_N(u) : u \in V(G)\}$.

2.8Definition[11]

A fuzzy graph $G = (\sigma, \mu)$ is said to be a **complete fuzzy graph** if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$ and is denoted by K_σ.

2.9 Definition[11]

A fuzzy graph $G = (\sigma, \mu)$ with bipartition (V_1, V_2) is said to be a **complete bipartite fuzzy graph** if every vertex in V_1 has a strong neighbor in V_2 and vice versa. If V_1 and V_2 have m, n vertices respectively, then the complete bipartite fuzzy graph is denoted by $K_{m,n}$.

2.10 Definition[9]

Let $G = (\sigma, \mu)$be a fuzzy graph. Let $u, v \in V$. The vertex u dominates the vertex v in G if (u, v) is a strong arc or strong edge. A subset P of V is called a **perfect dominating set** of G if each vertex v not in P is dominated by exactly one vertex of P.

2.11 Definition[9]

A perfect dominating set P of a fuzzy graph G is said to be a **minimal perfect dominating set,** if for $v \in P$, $P-\{v\}$ is not a perfect dominating set of G.

2.12 Definition[9]

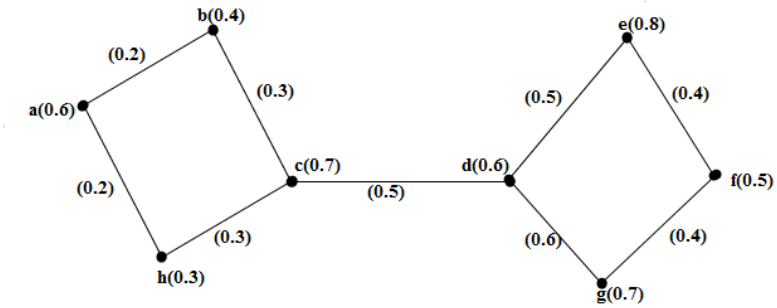
The minimum fuzzy cardinality of a minimal perfect dominating set of G is called the **perfect domination number** of a fuzzy graph G. It is denoted by $\gamma_{Pf}(G)$.

2.13 Definition[9]

The maximum fuzzy cardinality of a minimal perfect dominating set of G is called the **upperperfect domination number** of a fuzzy graph G. It is denoted by $\Gamma_{Pf}(G)$.

2.14 Example

Consider the fuzzy graph G.



Here, perfect dominating sets are {a, d, g} , {f, c, h}, {a, d, e} and {f, c, b}

Minimal perfect dominating set P = {f, c, h}

Perfect domination number $\gamma_{Pf}(G) = 1.5$

Upper perfect domination number $\Gamma_{Pf}(G) = 2$.

2.15 Definition[10]

Let $G = (\sigma, \mu)$ be a fuzzy graph. Let u, v$\in$V. The vertex u dominates v in G if (u, v) is a strong arc or strong edge. A perfect dominating set P_c of a fuzzy graph is said to be a **connected perfect dominating set**(CPDS) if the induced sub graph $<P_c>$ is connected.

2.16 Definition[10]

A perfect dominating set P_c of a fuzzy graph G is said to be a **minimal connected perfect dominating set** if for each vertex v in P_c, P_c-{v} is not a connected perfect dominating set of G.

2.17 Definition[10]

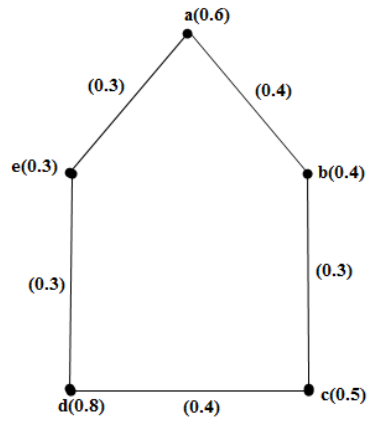
The minimum fuzzy cardinality of a connected perfect dominating set of a fuzzy graph G is called the **connected perfect domination number** of G and is denoted by $\gamma_{cPf}(G)$.

2.18 Definition[10]

The maximum fuzzy cardinality of a minimal connected perfect dominating set of G is called the **upperconnected perfect domination number** of a fuzzy graph G. It is denoted by $\Gamma_{cPf}(G)$.

2.19 Example

Consider the fuzzy graph G.



Connected perfect dominating set P_c = {a, b, c}

Connected perfect domination number $\gamma_{cPf}(G) = 1.5$

Upper connected perfect domination number = 1.7.

MAIN RESULTS:

III Total perfect dominating set in fuzzy graph

In this section, the new concept of total perfect dominating set in fuzzy graph are introduced.

3.1 Definition

A perfect dominating set P_T in a fuzzy graph G is said to be total perfect dominating set if every vertex of G is dominates to at least one vertex of P_T.

3.2 Definition

The minimum fuzzy cardinality of a total perfect dominating set is the total perfect domination number and it is denoted by $\gamma_{Tpf}(G)$.

3.3 Definition

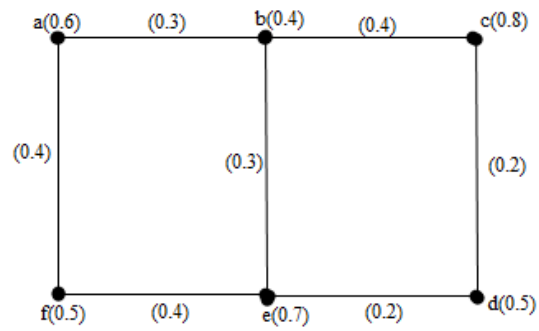
A total perfect dominating set P_T of a fuzzy graph G is said to be a minimal total perfect dominating set if for each vertex v in P_T, P_T-{v} is not a dominating set of G.

3.4 Definition

The upper total perfect domination number of a fuzzy graph G, denoted by $\Gamma_{Tpf}(G)$ is the maximum fuzzy cardinality of a minimal total perfect dominating set.

3.5 Example

Consider the fuzzy graph G.



Perfect dominating sets are {a, d}, {c, f}, {b, e}

Total perfect dominating set = {b, e}

Total perfect domination number = 1.1.

3.6 Theorem

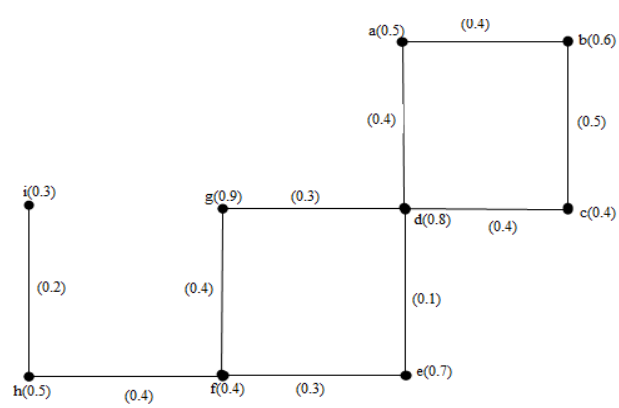
Every total perfect dominating set need not be a connected perfect dominating set.

Proof

Let G be a fuzzy graph without isolated vertices. A Total perfect dominating set P_T , by the definition of total perfect dominating set, a dominating set P_T if every vertex v in V- P_T is dominated by exactly one vertex in P_T and every vertex of G is dominates to at least one vertex of P_T.Here < P_T> satisfies either connected perfect dominating set or not. Therefore P_T need not be a connected perfect dominating set of a fuzzy graph G.

3.7 Example

Consider the fuzzy graph G.



Total perfect dominating set of G = {b, c, f, h} is not a connected perfect dominating set of G.

Total perfect domination number = 1.9.

3.8 Theorem

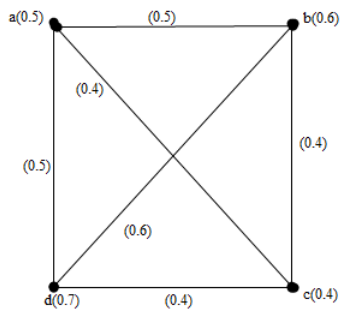
Let G be a complete fuzzy graph then total perfect dominating set P_T does not exists.

Proof

Since G is a complete fuzzy graph, all arcs are strong and each vertex is dominate to all other vertices. Then perfect dominating set exists which is minimum of $\sigma(u)$. It has only one vertex and it is not satisfy the definition of total perfect dominating set. That is, it is not satisfy the every vertex of G dominates to at least one vertex of P_T.

3.9 Example

Consider the complete fuzzy graph G.



Here, Total perfect dominating set does not exists for any complete fuzzy graph.

IVConnected total perfect dominating set in fuzzy graph

In this section, the new concept of connected total perfect dominating set in fuzzy graph is introduced and also define the notation of various kinds of minimum connected total perfect dominating set and connected total perfect domination number of a fuzzy graph are introduced.

4.1 Definition

Let $G=(\sigma,\mu)$be a fuzzy graph. Let u, v$\in$V. The vertex u dominates v in G if (u, v) is a strong arc or strong edge. A total perfect fuzzy dominating set P_{ct} is said to be a **connected total perfect dominating set** if the induced sub graph $<P_{ct}>$ is connected.

4.2 Definition

A perfect dominating set P_{ct} of a fuzzy graph G is said to be a **minimal connected total perfect dominating set** if for each vertex v in P_{ct}, P_{ct}-{v} is not a connected total perfect dominating set of G.

4.3 Definition

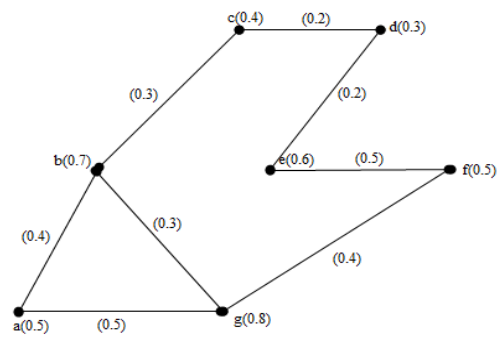
The minimum fuzzy cardinality of a connected total perfect dominating set of a fuzzy graph G is called the **connected total perfect domination number** of G and is denoted by $\gamma_{ctp}(G)$.

4.4 Definition

The maximum fuzzy cardinality of a minimal connected total perfect dominating set of G is called the **upperconnected total perfect domination number** of a fuzzy graph G. It is denoted by $\Gamma_{ctp}(G)$.

4.5 Example

Consider the fuzzy graph G.



Connected total perfect dominating sets are {a, b, c, d, e}, {b, c, d, e, f}, {c, d, e, f, g}, {d, e, f, g, a}, {e, f, g, a, b}, {f, g, a, b, c}, {g, a, b, c, d}

Minimal connected total perfect dominating set = {a, b, c, d, e} or {b, c, d, e, f}

Connected total perfect domination number = 2.5

Upper connected total perfect domination number = 3.1.

4.6 Theorem

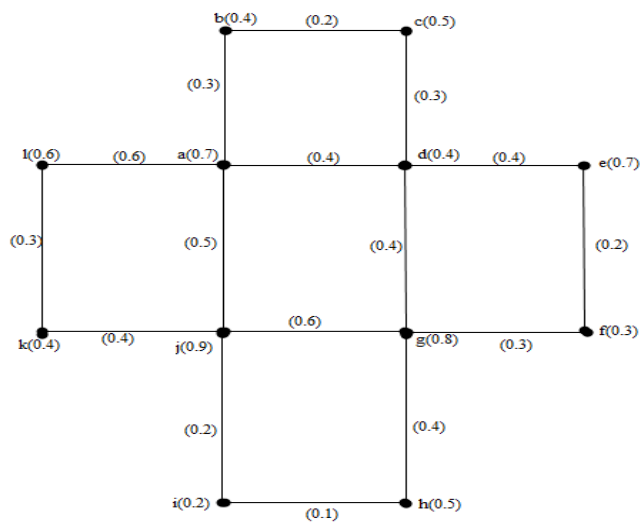
Every connected total perfect dominating set is a total perfect dominating set of a fuzzy graph G.

Proof

By using the definition of (7.3.1) connected total perfect dominating set , if the induced sub graph <P_{ct}> is connected in the connected total perfect dominating set of a fuzzy graph G. It is clear that, if every vertex in G is dominates to at least one vertex in P_{ct} which is the definition of total perfect dominating set of a fuzzy graph G. Therefore, Every connected total perfect dominating set is a total perfect dominating set of a fuzzy graph G.

4.7 Example

Consider the fuzzy graph G.



Connected total perfect dominating set = {a, d, g, j} is also a total perfect dominating set.

Connected total perfect domination number = 2.8 .

4.8 Theorem

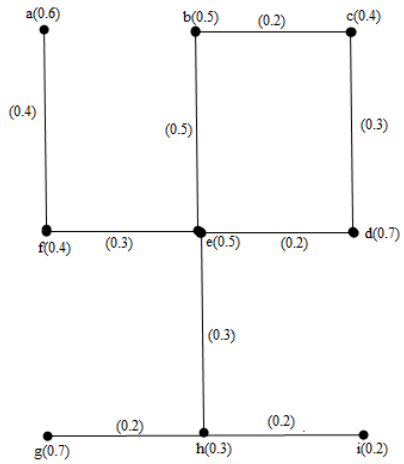
For any connected fuzzy graph G, then $\gamma_p(G) \leq \gamma_{tp}(G) \leq \gamma_{ctp}(G)$.

Proof

Since any nontrivial connected total perfect dominating set is also a total perfect dominating set and every total perfect dominating set is a perfect dominating set, therefore $\gamma_p(G) \leq \gamma_{tp}(G) \leq \gamma_{ctp}(G)$ for any connected fuzzy graph G.

4.9 Example

Consider the fuzzy graph G.



Connected total perfect dominating set = {b, e, f, h}

Connected total perfect domination number = 1.6

Total perfect dominating set = {b, e, h, f}

Total perfect domination number = 1.6

Perfect dominating set = {a, h, c}

Perfect domination number = 1.3 .Therefore, $\gamma_{pf}(G) \leq \gamma_{tp}(G) \leq \gamma_{ctp}(G)$.

4.10 Theorem

For any connected fuzzy graph of G with maximum degree Δ, $\frac{p}{2(\Delta+1)} \leq \gamma_{ctp}(G) \leq 2q - p+1$.

Proof

First we consider the lower bound. Each membership values of connected fuzzy graph can dominate atmost maximum degree Δ membership values and itself. Hence

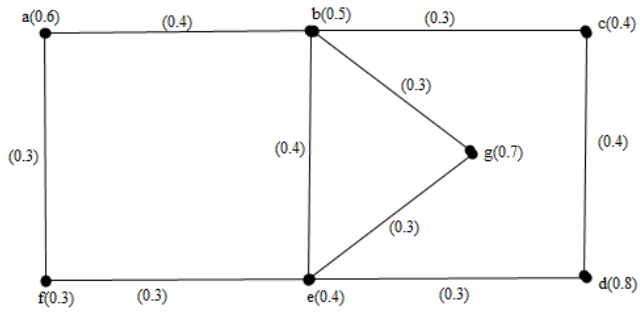
$\frac{p}{2(\Delta+1)} \leq \gamma_{ctp}(G)$ _____ (1)

For the upper bound, from the definition of connected total perfect dominating set of a fuzzy graph, we have $\gamma_{ctp}(G) \leq p-1$ then $\gamma_{ctp}(G) \leq 2(p-1)-p-1 \leq 2q-p+1$ _____ (2)

From (1) and (2), $\frac{p}{2(\Delta+1)} \leq \gamma_{ctp}(G) \leq 2q - p+1$.

4.11 Example

Consider the fuzzy graph G.



p = 3; q = 3;

d(a) = 0.8; d(b) = 2.1; d(c) = 1.3; d(d) = 0.8; d(e) = 2.3; d(f) = 1; d(g) =0.9;

$\Delta = 2.3$; $\frac{p}{2(\Delta+1)} = 0.45$; $\gamma_{ctp}(G) = 1.6$; $2q - p+1 = 4$

Therefore, $\frac{p}{2(\Delta+1)} \leq \gamma_{ctp}(G) \leq 2q - p+1$.

4.12 Theorem

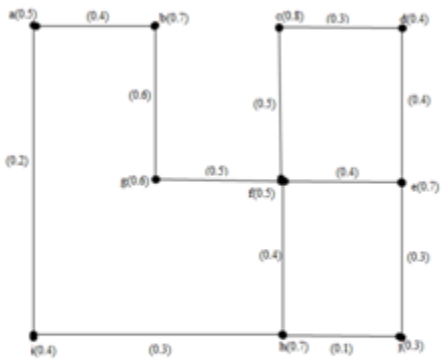
If H is a connected spanning sub graph of a fuzzy graph G, then $\gamma_{ctp}(G) \leq \gamma_{ctp}(H)$.

Proof

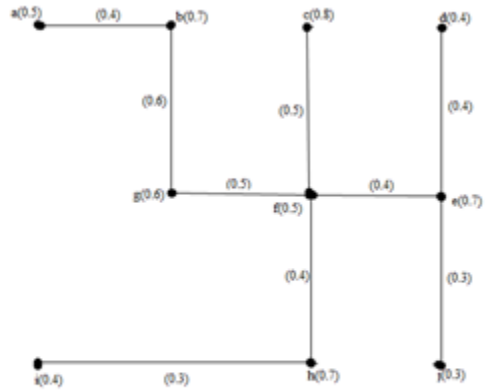
Since every connected total perfect dominating set of H is also a connected total perfect dominating set of G.

4.13 Example

Consider the fuzzy graph G.



Fuzzy Graph G



Fuzzy Spanning subgraph H

Connected total perfect dominating set of G = {b, g, f, e, h}

Connected total perfect domination number $\gamma_{ctp}(G) = 3.2$

Connected total perfect dominating set of H = {b, g, f, e, h}

Connected total perfect domination number $\gamma_{ctp}(H) = 3.2$

$\gamma_{ctp}(G) \leq \gamma_{ctp}(H)$.

4.14 Theorem

For any connected fuzzy graph of order p, $\gamma_{ctp}(G) \leq p - \Delta(G)+1$.

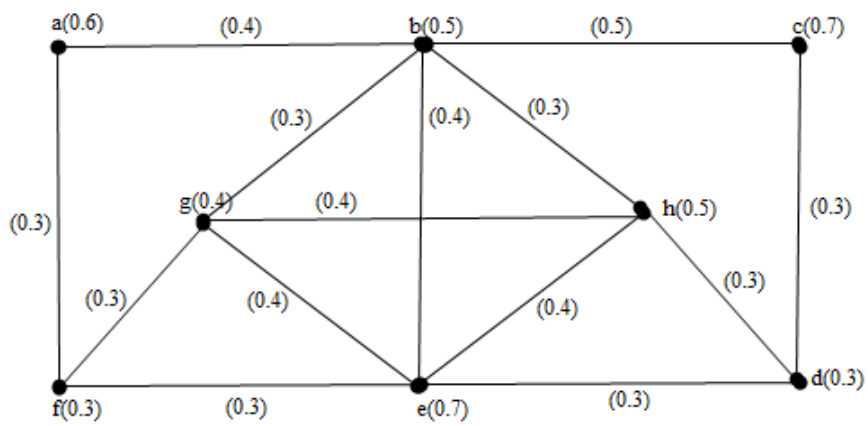
Proof

Let deg(v) = $\Delta(G)$. Then a spanning tree P_{ct} of G can be formed in which v is dominates to each of its neighbours. So, P_{ct} has a vertex v of degree Δ and hence has at least Δ end vertices. We know that $\gamma_{ctp} \leq$ p –cardinlaity of membership values of end vertices in G . Since

$\Delta \leq$cardinality of membership values of end vertices in G , $\gamma_{ctp} \leq p - \Delta(G)+1$.

4.15 Example

Consider the fuzzy graph G,



Connected total perfect dominating set = {b, e}

Connected total perfect domination number $\gamma_{ctp} = 1.2$

$p - \Delta(G)+1 = 4 - 2+1 = 3.$

Therefore, $\gamma_{ctp} \leq p - \Delta(G)+1$.

4.16 Theorem

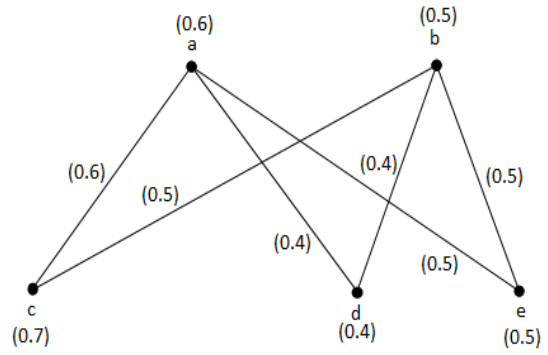
A connected total perfect domination number $\gamma_{fctp} = \min(\sigma(u))+ \min(\sigma(v))$ for all $u \in V_1$ and $v \in V_2$ for any complete bipartite fuzzy graph G.

Proof

In a complete bipartite fuzzy graph, all edges are strong arcs. Each vertex in V_1 is dominate with all vertices in V_2. Hence in a complete bipartite fuzzy graph, the connected total perfect dominating set are V_1 and V_2 and any set containing exactly two vertices, one in V_1 and other in V_2. Among this, if V_1contains only one vertex, say u and V_2 contains only one vertex, say v which are the minimum membership values then connected total perfect dominating set is equal to {u, v} are the vertices of V_1 and V_2. Therefore, connected total perfect domination number $\gamma_{fctp} = \min(\sigma(u))+ \min(\sigma(v))$ for all $u \in V_1$ and $v \in V_2$. Hence proved.

4.17 Example

Consider the complete bipartite fuzzy graph G.



Connected total perfect domination number γ_{ctp} = min(σ(u))+ min(σ(v)) = 0.5 +0.4 = 0.9.

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