Connected strong domination related parameters in an Intuitionistic fuzzy graph

Ponnappan C.Y\(^1\), Muthuraj.R\(^2\), Surulinathan .P\(^3\).

\(^1\)Department of Mathematics, Government Arts College Paramakudi, Tamilnadu, India. Email : Pons_mdu1969@yahoo.com

\(^2\)P.G and Research Department of Mathematics, H.H.The Rajah’s College, Pudukkottai–622001, Email : rnr1973@gmail.com

\(^3\)Department of Mathematics, Lathamathavan Engineering College, Kidaripatti, Alogarkovil, Madurai. Email : Surulinathan_P@yahoo.co.in.

Abstract : In this paper, the concept of Intuitionistic fuzzy inverse connected strong domination number, Intuitionistic fuzzy inverse disconnected strong domination number, Intuitionistic fuzzy pair connected strong domination number are introduced and its properties are discussed. Furthermore this new domination parameter is compared with other known domination parameters.

Keywords: Connected strong domination number, disconnected strong domination number, inverse connected strong domination number, inverse disconnected strong domination number, pair connected strong domination number.

AMS Classification: 05C72, 05C75

1. Introduction:

The study of domination set in graphs was begun by Ore and Berge. The connected domination number was first introduced by E.Sampathkumar and H.B.Walikar [1] Rosenfield [2] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness. A Somasundaram and S.Somasundaram [3] discussed domination in fuzzy graphs. K.T. Atanassov [4] initiated the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. R.Parvathi and G.Thamizhendhi [5] was introduced dominating set and domination number in IFGS. In this paper, we discuss inverse connected domination number, inverse disconnected domination number and pair connected domination number of an intuitionistic fuzzy graphs and obtain the relationship with other known parameters of an IFG G.
II. Preliminaries

Definition: 2.1
An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$ where

(i) $V = \{V_1, V_2, \ldots, V_n\}$ such that $\sigma_1 : V \rightarrow [0, 1]$ and $\sigma_2 : V \rightarrow [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \sigma_1 (v_i) + \sigma_2 (v_i) \leq 1$, for every $v_i \in V$

(ii) $E \subseteq V \times V$ where

$\mu_1 : V \times V \rightarrow [0, 1]$ and $\mu_2 : V \times V \rightarrow [0, 1]$ are such that

$\mu_1 (v_i, v_j) \leq \min \{ \sigma_1 (v_i), \sigma_1 (v_j) \}$

$\mu_2 (v_i, v_j) \geq \max \{ \sigma_2 (v_i), \sigma_2 (v_j) \}$

and $0 \leq \mu_1 (v_i, v_j) + \mu_2 (v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Note: when $\mu_{1ij} = \mu_{2ij} = 0$ for some $i$ and $j$ then there is no edge between $v_i$ and $v_j$ otherwise there exists an edge between $v_i$ and $v_j$.

Definition: 2.2
An IFG $H = (V', E')$ is said to be an IF subgraph (IFSG) of $G = (V, E)$ if $V' \subseteq E$ and $E' \subseteq E$. That is $\sigma_{li}^1 \leq \sigma_{li}^1$ ; $\sigma_{2i}^1 \geq \sigma_{2i}^1$ and $\mu_{li}^1 \leq \mu_{li}^1$ ; $\mu_{2j}^1 \geq \mu_{2j}^1$, for every $i = 1, 2, \ldots, n$

Definition: 2.3
The intuitionistic fuzzy subgraph $H = (V', E')$ is said to be a spanning fuzzy subgraph of an IFG $G = (V, E)$ if $\sigma_1' (u) = \sigma_1 (u)$ and $\sigma_2' (u) = \sigma_2 (u)$ for all $u \in V'$ and $\mu_1' (u, v) \leq \mu_1 (u, v)$ and $\mu_2' (u, v) \geq \mu_2 (u, v)$ for all $u, v \in V$.

Definition: 2.4
Let $G = (V, E)$ be an IFG. Then the vertex cardinality of $G$ is defined by

$P = |V| = \sum_{v_i \in V} \frac{1}{2} (1 + \sigma_1 (v_i) - \sigma_2 (v_i))$ for all $v_i \in V$

Definition: 2.5
Let $G = (V, E)$ be an IFG. Then the edge cardinality of $E$ is defined by

$$q = |E| = \sum_{v_i, v_j \in E} \frac{1}{2} \left( 1 + \mu_1(v_i, v_j) - \mu_2(v_i, v_j) \right)$$

for all $(v_i, v_j) \in E$.

**Definition: 2.6**

Let $G = (V, E)$ be an IFG. Then the cardinality of $G$ is defined to be $|G| = |V| + |E| = p + q$.

**Definition: 2.7**

The number of vertices is called the order of an IFG and is denoted by $O(G)$. The number of edge is called size of IFG and is denoted by $S(G)$.

**Definition: 2.8**

The vertices $v_i$ and $v_j$ are said to the neighbors in IFG either one of the following conditions hold:

(i) $\mu_1(v_i, v_j) > 0$, $\mu_2(v_i, v_j) > 0$

(ii) $\mu_1(v_i, v_j) = 0$, $\mu_2(v_i, v_j) > 0$

(iii) $\mu_1(v_i, v_j) > 0$, $\mu_2(v_i, v_j) = 0$, for $v_i, v_j \in V$

**Definition: 2.9**

A path in an IFG is a sequence of distinct vertices $v_1, v_2, \ldots v_n$ such that either one of the following conditions is satisfied.

(i) $\mu_1(v_i, v_j) > 0$, $\mu_2(v_i, v_j) > 0$ for some $i$ and $j$

(ii) $\mu_1(v_i, v_j) = 0$, $\mu_2(v_i, v_j) > 0$ for some $i$ and $j$

(iii) $\mu_1(v_i, v_j) > 0$, $\mu_2(v_i, v_j) = 0$ for some $i$ and $j$

**Note:** The length of the path $P = v_1, v_2, \ldots v_{n+1}$ ($n > 0$) is $n$.

**Definition: 2.10**

Two vertices that are joined by a path is called connected.

**Definition: 2.11**

An edge $e=(x,y)$ of an IFG $G = (V, E)$ is called an effective edge if $\mu_1(x, y) = \min \{\sigma_1(x), \sigma_1(y)\}$ and $\mu_2(x, y) = \max \{\sigma_2(x), \sigma_2(y)\}$.

**Definition: 2.12**
An IFG $G = (V, E)$ is said to be complete IFG if $\mu_{ij} = \min \{\sigma_{1i}, \sigma_{1j}\}$ and $\mu_{2ij} = \max \{\sigma_{2i}, \sigma_{2j}\}$ for every $v_i, v_j \in V$.

**Definition: 2.13**

The complement of an IFG, $G = (V, E)$ is an IFG, $\overline{G} = (\overline{V}, \overline{E})$, where

(i) $\overline{V} = V$

(ii) $\overline{\sigma}_{1i} = \sigma_{1i}$ and $\overline{\sigma}_{2i} = \sigma_{2i}$, for all $i = 1, 2, \ldots n$

(iii) $\overline{\mu}_{ij} = \min \{\sigma_{1i}, \sigma_{1j}\} - \mu_{ij}$

(iv) $\overline{\mu}_{2ij} = \max \{\sigma_{2i}, \sigma_{2j}\} - \mu_{2ij}$ for all $i = 1, 2, \ldots n$

**Definition: 2.14**

An IFG, $G = (V, E)$ is said to bipartite the vertex set $V$ can be partitioned into two non empty sets $V_1$ and $V_2$ such that

(i) $\mu_1 (v_i, v_j) = 0$ and $\mu_2 (v_i, v_j) = 0$ if $v_i, v_j \in V_1$ (or) $v_i, v_j \in V_2$

(ii) $\mu_1 (v_i, v_j) > 0$ and $\mu_2 (v_i, v_j) > 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some $i$ and $j$.

$\mu_1 (v_i, v_j) = 0$ and $\mu_2 (v_i, v_j) > 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some $i$ and $j$.

$\mu_1 (v_i, v_j) > 0$ and $\mu_2 (v_i, v_j) = 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some $i$ and $j$.

**Definition: 2.15**

A bipartite IFG, $G = (V, E)$ is said to be complete if 

$\mu_1 (v_i, v_j) = \min \{\sigma_{1i}(v_i), \sigma_{1j}(v_j)\}$

$\mu_2 (v_i, v_j) = \max \{\sigma_{2i}(v_i), \sigma_{2j}(v_j)\}$

for all $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{(\sigma_1, \sigma_2, \mu_1, \mu_2)}$

**Definition: 2.16**

A vertex $u \in V$ of an IFG $G = (V, E)$ is said to be an isolated vertex if $\mu_1 (u, v) = 0$ and $\mu_2 (u, v) = 0$, for all $v \in V$. That is $N(u) = \phi$. Thus an isolated vertex does not dominate any other vertex in $G$.

**Definition: 2.17**

Let $G = (V, E)$ be an IFG on $V$. Let $u, v \in V$, we say that $u$ dominate $v$ in $G$ if there exits an effective edge between them.
Definition: 2.18

Let $G = (V, E)$ be an intuitionistic fuzzy graph $G$ on the vertex set $V$. Let $x, y \in V$, we say that $x$ dominates $y$ in $G$ if $\mu_1(x, y) = \min \{\sigma_1(x), \sigma_1(y)\}$ and $\mu_2(x, y) = \max \{\sigma_2(x), \sigma_2(y)\}$. A subset $D$ of $V$ is called a dominating set in IFG $G$ if for every $v \in V - D$, there exists $u \in D$ such that $u$ dominates $v$. A dominating set $D$ of an IFG is said to be minimal dominating set if no proper subset of $D$ is a dominating set.

Definition: 2.19

Let $G = (V, E)$ be a IFG without isolated vertices. A subset $D_{cs}$ of $V$ is said to be an intuitionistic fuzzy connected strong domination set if both induced subgraphs $<D_{cs}>$ and $<V-D_{cs}>$ are connected. The intuitionistic fuzzy connected strong domination number $\gamma_{cs}(G)$ is the minimum intuitionistic fuzzy cardinality taken over all connected strong dominating sets of $G$.

III. Intuitionistic Fuzzy inverse Connected Strong Domination Number

Definition: 3.1

Let $D_{cs}$ be the minimum connected strong dominating set of intuitionistic fuzzy graph $G = (V,E)$. If $V-D_{cs}$ contains a connected strong dominating set $D'_{cs}$ then $D'_{cs}$ is called the inverse connected strong dominating set with respect to $D_{cs}$. The inverse connected strong domination number $\gamma'_{cs}(G)$ is the minimum intuitionistic fuzzy cardinality taken over all minimal inverse connected strong dominating sets of $G$.

Example: 3.2
Fig.3.1

\[ D_{cs}(G) = \{v_2\}, \quad V-D_{cs}(G) = \{v_1,v_3,v_4,v_5\} \quad D_{cs}^{-1}(G) = \{v_4,v_5\} \]

**Remark: 3.3**

Every graph without isolated vertices contains an inverse dominating set with respect to a minimum dominating set. Here the intuitionistic fuzzy graph whose underlying crisp graph has no isolated vertices.

**Theorem: 3.4**

If \( G = (V, E) \) is an intuitionistic fuzzy graph then \( \gamma'_{cs}(G) < p \).

**Proof:**

By definition 3.1, \( \gamma'_{cs}(G) < p \) is obvious.

**Observation: 3.5**

1. \( \gamma_{cs}(K_{\sigma_1,\sigma_2}) = |v_1|, \) \( v_1 \) is the vertex having minimum intuitionistic fuzzy cardinality.

   Then

   \( \gamma'_{cs}(K_{\sigma_1,\sigma_2}) = |v_j|, \) \( v_j \) is the vertex having second minimum intuitionistic fuzzy cardinality.

2. Inverse connected strong dominating sets do not exist for Path, Cycle, Star, Double star, Sub division of star and Corona.

3. If \( \gamma_{cs}(W_{n+1}) = |v_1| \), then \( \gamma'_{cs}(W_{n+1}) = p - \max\{|u|+|v|+|v_1|\}\), where \( u \) and \( v \) are adjacent vertices and \( v_1 \) is the center of the wheel.

4. If \( \gamma_{cs}(k_{\sigma_1\sigma_2,\mu\mu_2}) = \min\{|v_1|\} + \min\{|v_j|\} \) where \( v_i \in V_1 \) and \( v_j \in V_2 \), then

   \( \gamma'_{cs}(k_{\sigma_1\sigma_2,\mu\mu_2}) = \min\{|v_1|\} + \min\{|v_2|\} \) where \( v_i \in V_1 \) and \( v_i \in V_2 \) are the second minimum intuitionistic fuzzy cardinalities.

5. \( \gamma'_{cs}(nFan) = p - \{ \min\{s,t\} + |v_1| \} \), where \( v_1 \) is the vertex adjacent to all other vertices further \( v_1 \) dominates all other vertices and \( s = \sum_{i=2}^{n-1} |V_i|, \) \( t = \sum_{i=3}^{n} |V_i| \).
6. For Petersen graph, $\gamma_{cs}(G) + \gamma'_cs(G) = p$.

**Theorem: 3.6**

For any intuitionistic fuzzy graphs $G = (V, E)$ with at least one inverse connected strong dominating set, $\gamma'_cs(G) \geq \gamma_{cs}(G)$.

**Proof:**

Let $G = (V, E)$ be an intuitionistic fuzzy graph without isolated vertices. Then $G$ has one or more connected strong dominating sets, $\gamma_{cs}(G)$ is the minimum fuzzy cardinality of the connected strong dominating set $D_{cs}$ of $G$. $V - D_{cs}$ contains another connected strong dominating set $D'_{cs}$ with fuzzy cardinality $\gamma'_cs(G)$. Clearly $\gamma'_cs(G) \geq \gamma_{cs}(G)$.

**Theorem: 3.7**

Let $G = (V, E)$ be a complete intuitionistic fuzzy graph $k_{\sigma_1,\sigma_2}$. Then

$\gamma'_cs(G) \leq \gamma'_cs(G - v), \, v \in k_{\sigma_1,\sigma_2}$.

**Proof:**

Let $G = (V, E)$ be the complete intuitionistic fuzzy graph $k_{\sigma_1,\sigma_2}$. By definition of complete intuitionistic fuzzy graph, all edges are effective. Therefore each vertex dominates the remaining vertices. By theorem complete intuitionistic fuzzy graph $K_{\sigma_1,\sigma_2}$ then $\gamma_{cs}(K_{\sigma_1,\sigma_2}) = |v| = \frac{1}{2} \{1 + \sigma_1(v) - \sigma_2(v)\}, \, v$ is the vertex having minimum intuitionistic fuzzy cardinality. Clearly,

$\gamma_{cs}(k_{\sigma_1,\sigma_2}) = |v_i|, \, v_i$ is the vertex of minimum intuitionistic fuzzy cardinality. Let $D'_{cs}$ be the inverse connected strong dominating set of $G$. Since $\gamma_{cs}(k_{\sigma_1,\sigma_2}) = \sigma(v_i), \, v_i$ is the vertex having minimum intuitionistic fuzzy cardinality, then the inverse connected strong domination number $\gamma'_cs(k_{\sigma_1,\sigma_2}) = |v_j|$, where $v_j$ is the vertex having second minimum intuitionistic fuzzy cardinality. If any intuitionistic fuzzy vertex is removed from the vertex set, then the intuitionistic fuzzy inverse connected strong domination number will be greater than or equal to $\gamma'_cs(k_{\sigma_1,\sigma_2})$.

Hence $\gamma'_cs(k_{\sigma_1,\sigma_2}) \leq \gamma'_cs(k_{\sigma_1,\sigma_2} - v), \, v \in k_{\sigma_1,\sigma_2}$.
Theorem: 3.8

For any intuitionistic fuzzy graphs $G = (V,E)$ with atleast one isolated vertex

$$\gamma'_{cs}(G) = 0.$$ 

Proof:

Let $D_{cs}$ be a $\gamma_{cs}$-set of intuitionistic fuzzy graph $G$ and $u \in D_{cs}$ be an isolated vertex. Then $\mu_1(\{u,v\}) < \sigma_1(u) \land \sigma_2(v)$, $\mu_2(\{u,v\}) > \sigma_1(u) \lor \sigma_2(v)$ for all $v \in V - D_{cs}$, thus $\gamma'_{cs}(G) = 0$.

Theorem: 3.9

For any intuitionistic fuzzy graph $G = (V, E)$ with $\gamma'_{cs}$-set, $\gamma_{cs}(G) + \gamma'_{cs}(G) \leq p$. Further equality holds if $V - D_{cs}$ is independent and contains inverse dominating set $D'_{cs}$ with respect to $D_{cs}$.

Proof:

Let $D_{cs}$ be a $\gamma_{cs}$-set of $G$. If $D'_{cs}$ is an inverse connected strong dominating set of $G$ with respect to $D_{cs}$ then $D'_{cs} \subseteq V - D_{cs}$. Therefore $|D'_{cs}| \leq |V - D_{cs}|$. Thus $\gamma'_{cs}(G) \leq p - \gamma_{cs}(G)$. Since $V - D_{cs}$ is independent and contains an inverse connected strong dominating set $D'_{cs}$ with respect to $D_{cs}$. Therefore $V - D_{cs}$ itself is an inverse dominating set of the fuzzy graph $G$.

Remark: 3.10

Equality holds for $P_n$. That is $\gamma_{cs}(P_n) + \gamma'_{cs}(P_n) = p$.

Theorem: 3.11

For any intuitionistic fuzzy graph $G = (V, E)$, $\gamma'_{cs}(G) \leq \Gamma(G)$

Proof:

Let $D_{cs}$ be a $\gamma_{cs}$-set of $G$. $V - D_{cs}$ is independent and contains an inverse connected strong dominating set $D'_{cs}$ with respect to $D_{cs}$. Therefore $V - D_{cs}$ itself is an inverse dominating set of the intuitionistic fuzzy graph $G$. Clearly $\gamma'_{cs}(G) = \Gamma(G)$. Suppose $V - D_{cs}$ contains atleast two connected strong dominating set, then the minimum intuitionistic fuzzy cardinality of a connected strong dominating set in $V - D_{cs}$ is $\gamma'_{cs}(G)$. Therefore $\gamma'_{cs}(G) \leq \Gamma(G)$.
Theorem: 3.12

For any intuitionistic fuzzy graphs $G = (V, E)$, $\gamma'_{cs} (G) \leq p - \delta_N$.

Proof:

Let $G$ be an intuitionistic fuzzy graph, let $v \in V$ with $d_N(v) = \delta_N$. Suppose $D'_{cs}$ is an inverse connected strong dominating set of $G$ and $D'_{cs} \subseteq V - N(v)$.

Then $\gamma'_{cs} (G) = |V - N(v)| = P - |N(v)| \leq p - \delta_N$.

Theorem: 3.13

For any intuitionistic fuzzy graphs $G = (V, E)$, $\gamma'_{cs} (G) < p$.

Proof:

Let $G = (V, E)$ be an intuitionistic fuzzy graph without isolated vertices having connected strong dominating set. Then $\gamma_{cs} (G) > 0$. By theorem 3.9, $\gamma_{cs} (G) + \gamma'_{cs} (G) \leq p$. Clearly $\gamma'_{cs} (G) < p$.

Theorem: 3.14

For any intuitionistic fuzzy graphs $G = (V, E)$, $\gamma_{cs} (G) + \gamma'_{cs} (\overline{G}) < 2p$, where $\gamma'_{cs} (\overline{G})$ is the inverse connected strong domination number of $\overline{G}$.

Proof:

Let $G$ be a fuzzy graph without isolated vertices. Since by theorem 3.4, $\gamma'_{cs} (G) < p$ therefore $\gamma_{cs} (G) + \gamma'_{cs} (\overline{G}) \neq 2p$. Thus $\gamma_{cs} (G) + \gamma'_{cs} (\overline{G}) < 2p$.

Theorem: 3.15

For any intuitionistic fuzzy graphs $G = (V, E)$, $\gamma'_{cs} (G) \leq \beta (G)$. 
IV. Intuitionistic fuzzy inverse disconnected strong domination number

Definition: 4.1

Let \( D_{ds} \) be the minimum disconnected strong dominating set of an intuitionistic fuzzy graph \( G = (V, E) \). If \( V - D_{ds} \) contains a disconnected strong dominating set \( D'_{ds} \) then \( D'_{ds} \) is called the inverse disconnected strong dominating set with respect to \( D_{ds} \). The inverse disconnected strong domination number \( \gamma'_{ds}(G) \) is the minimum intuitionistic fuzzy cardinality taken over all minimal inverse disconnected strong dominating sets of \( G \).

Observation: 4.2

1. Disconnected strong dominating sets do not exist for intuitionistic fuzzy complete graph and nFan.
2. Let \( G = (V,E) \) be an intuitionistic fuzzy complete bipartite graph \( k_{\sigma_1\sigma_2,\mu_1\mu_2} \),
   
   Obviously \( \gamma_{ds}(G) = \min \left\{ \sum_{i=1}^{n} |V_i|, \sum_{j=1}^{n} |V_j| \right\} \). Therefore \( \gamma_{ds}(G) + \gamma'_{ds}(G) = p \).

Theorem: 4.3

For any intuitionistic fuzzy path \( P_n \), \( \gamma'_{cs}(P_n) = \Gamma(P_n) \).

Proof:

The intuitionistic fuzzy path \( P_n \) contains only two dominating sets.

Clearly \( \gamma'_{cs}(P_n) = \Gamma(P_n) \).

V. Intuitionistic fuzzy pair connected strong domination number

Definition: 5.1

Let \( G = (V, E) \) be an intuitionistic fuzzy graph, the dominating set \( D_{pcs} \) is called pair connected strong dominating set with respect to \( D_{cs} \) if \( <D_{cs}> \) and \( <D'_{cs}> \) are connected. The pair connected strong dominating number \( \gamma_{pcs} \) is the minimum intuitionistic fuzzy cardinality taken over all minimal connected strong dominating set of \( G \).

Theorem: 5.2

If \( G = (V, E) \) is an intuitionistic fuzzy graph then \( \gamma(G) \leq \gamma_{pcs}(G) \leq \gamma'_{cs}(G) \).

Proof:
Let $G = (V,E)$ is an intuitionistic fuzzy graph. Every connected strong dominating set is a dominating set. Therefore $\gamma(G) \leq \gamma_{cs}(G)$. By definition 5.1, $\gamma_{pcs}(G) = \gamma_{cs}(G)$. By theorem 3.5, $\gamma_{cs}(G) \leq \gamma'_{cs}(G)$. Clearly $\gamma(G) \leq \gamma_{pcs}(G) \leq \gamma'_{cs}(G)$.

**Theorem: 5.3**

If $G = (V, E)$ is an intuitionistic fuzzy graph then $\gamma_{pcs}(G) + \gamma_{pcs}(\overline{G}) < p$.

**Proof:**

Let $G = (V, E)$ be an intuitionistic fuzzy graph, By definition 2.19 and 5.1, $\gamma_{pcs}(G) = \gamma_{cs}(G)$ and by an earlier result $\gamma_{cs}(G) < p$. Suppose $G$ is an intuitionistic fuzzy graph in which all the edges are effective then the intuitionistic fuzzy compliment of $G$ is totally disconnected. Therefore $\gamma_{pcs}(G) = 0$. Otherwise $\gamma_{pcs}(G) < p$. Clearly $\gamma_{pcs}(G) + \gamma_{pcs}(\overline{G}) < p$.

**Theorem: 5.4**

If $G = (V, E)$ is a complete intuitionistic fuzzy graph $K_{\sigma_1, \sigma_2}$ then $\gamma_{pcs}(G) + \gamma_{pcs}(\overline{G}) = |u|$, where $u$ and $v$ are the intuitionistic fuzzy vertices having first two minimum intuitionistic fuzzy cardinality.

**Proof:**

Let $G = (V, E)$ be a complete intuitionistic fuzzy graph $K_{\sigma_1, \sigma_2}$. By the definition of complete intuitionistic fuzzy graph, each vertex is adjacent with one another also all edges are effective. Moreover each vertex is dominated by any other vertex. Let $D_{cs}$ be the connected strong dominating set of $G$ and $D'_{cs}$ is the inverse connected strong dominating set of $G$ with respect to $D_{cs}$. $\gamma_{pcs}(G)$ is the pair connected strong domination number of $G$. By definition 2.19 and 3.1, $D_{cs} = \{u / \text{where } u \text{ is the intuitionistic fuzzy vertex having minimum cardinality}\}$ and $D'_{cs} \subseteq V - D_{cs}$, $D'_{cs} = \{v / \text{where } v \text{ is the intuitionistic fuzzy vertex having second minimum cardinality}\}$. $\gamma_{pcs}(G)$ is the pair connected dominating number of $G$ which is equal to $\gamma_{cs}(G)$. Obviously $\gamma_{pcs}(\overline{G}) = 0$, since $\overline{G}$ is totally disconnected.

Clearly $\gamma_{pcs}(G) + \gamma_{pcs}(\overline{G}) = |u|$.

**Acknowledgement**

Thanks are due to the referees for their valuable comments and suggestions.
References


