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Operational Readiness of a P.S. Redundant System with Three Types of Failure and Waiting Concept

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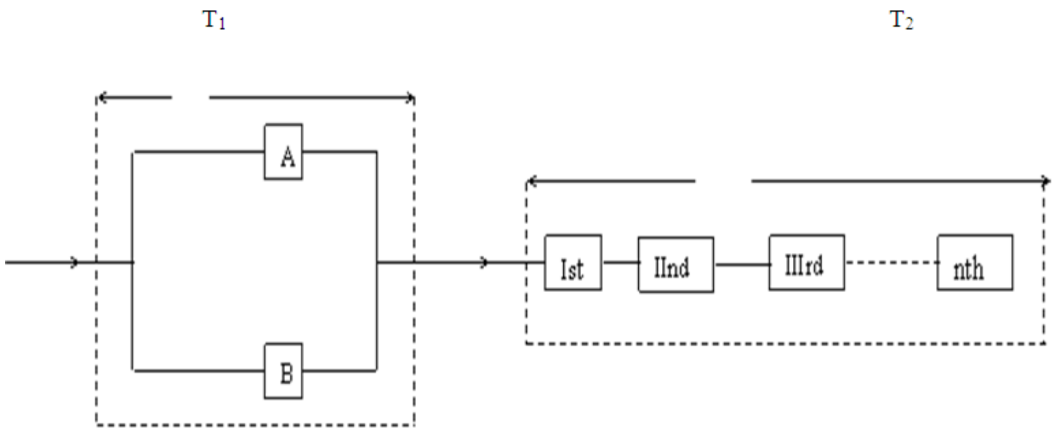
Abstract

This paper deals with a multi-component System (Parallel and Series) which has two Subsystem T_1 and T_2 connected in series. Subsystem T_1 is made up two identical units in Parallel, while T_2 is made up n-identical units in series. Subsystem T_1 fails when both of its units fails and T_2 fails when any one of its unit fail. System goes to complete break down if either both T_1 andT_2 and any one of T_1 and T_2 fails. While on failure of 1-T_1-: one out of two, it goes to reduced efficiency state. Whole system may fail completely from any operable state due to networking failure also. There are three type of failure partial failure (minor), catastrophic (major) and networking failure. All type of failure are assumed to be exponential while repair are distributed generally accept Networking failure which is taken constant. In this paper Author's used S.V technique and L.T of various state probabilities have been evaluated. At last expressions for Cost function, Reliability and Variance of time to failure are obtained by inversion process. Conclusion and result of this paper and few graphical illustrations are also given in the end so as to explain practical utility of the problem.

Keywords: Variances of time to failure, exponential and general distribution, mean time to system failure.

System Configuration



Introduction

An unreliable system has higher probability of failure and when any item of the associated equipment is damaged or destroyed due to some or other causes, consequently the unreliability of the system may results in to the wastage of cost, time of personal as well as national security. Thus a high degree of reliability is required in various practical system i.e. many equipment of DRDO, Nuclear system, Power plants, Fuel system , Self operating system, Space system. Introducing parallel redundancy and using waiting service facility, is found very useful to achieve the requirement of high degree of reliability' In 1979 B.S. Dhillon [2];1985 Murty A.S.R, Verma A.K [3] and 1995 Pandey D, Jacob Mendes [4],et all try various reliability technique to achieve a high degree of reliability for system under consideration. But this does not seem to be sufficient and more is required in this direction.

In view of the above authors have tried to fill the gap by considering a higher reliable & more complex system P.S. redundant system with three types of failure and waiting concept. Mathematical modeling of this model solve by S.V technique and L.T process.

Notations

λ_m / λ'_m	Constant failure rate of any one unit of T_1 in normal efficiency state / failure rate of T_1 degraded state.
λ_M	Constant failure rate of any one unit of subsystem T_2
A	Constant waiting rate for series from $S_2 \to S_3, S_4 \to S_5, S_6 \to S_7$
$\Phi_1(x)/\Phi_2(y)$	General repair rates of single unit / two unit at a time with elapsed repair time x, y in failed state
$P_0(t)$	The probability that , the system is in operable state at time t.
$P_1(x,t)/P_2(t)$	The probability at time t, that the system is under repair in

	degraded states / failed states due to failure of one- unit of T_1/ both units of T_1 and the elapsed repair time lies in the interval $(x, x+\Delta)$ /failure of both-T_1 and waiting for expire at the t .
$P_3(y,t)/P_5(y,t)/P_7(x,t)$	The probability at time t when the system is under repair from state and the elapsed repair time lies in the interval $(y, y+\Delta)$, or $(x, x+\Delta)$
$P_4(t)/P_6(t)$	The probability at time t that the system is in failed state due to failure of any T_2- unit.
$P_8(t)$	The probability at time t when the system is in failed state due to networking failure.
δ	Constant networking failure rate
μ	Immediate constant repair rate for networking failure
$S_i(x) = i(x)\cdot e^{-\int_0^x i(x)dx}$	(By Davis formula)
$F_i(x) = \dfrac{1-\overline{S}_i(x)}{x}$	

Mathematical formulation of this model

By probability and continuity argument the difference- differential equations for stochastic process which is continuous in time discrete in space are as

$$\left[\frac{d}{dt} + 2\lambda_m + n\lambda_M + \delta\right]P_0(t) = \int_0^\infty P_1(x,t)\Phi_1(x)dx + \int_0^\infty P_3(y,t)\Phi_2(y)dy$$

$$+\int_0^\infty P_5(y,t)\Phi_2(y)dy + \int_0^\infty P_7(x,t)\Phi_1(x)dx \;\; + \mu\cdot P_8(t) \tag{1}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \lambda_m' + n\lambda_M + \Phi(x) + \delta\right]P_1(x,t) = 0 \tag{2}$$

$$\left[\frac{d}{dt} + a\right]P_2(t) = \lambda_m'\cdot P_1(t) \tag{3}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \Phi_2(y)\right]P_3(y,t) = 0 \tag{4}$$

$$\left[\frac{d}{dt} + a\right]P_4(t) = n\lambda_M P_1(t) \tag{5}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial t} + \Phi_2(y)\right]P_5(y,t) = 0 \tag{6}$$

$$\left[\frac{d}{dt}+a\right]\mathrm{P}_6(t)= n\lambda_M \mathrm{P}_0(t) \tag{7}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\Phi_1(x)\right]\mathrm{P}_7(x,t) = 0 \tag{8}$$

$$\left[\frac{d}{dt}+\mu\right]\mathrm{P}_8(t) = \delta\left\{\mathrm{P}_0(t)+\int_0^\infty \mathrm{P}_1(x,t)dx\right\} \tag{9}$$

Boundary Condition

$$\mathrm{P}_1(0,t) = 2\lambda_m \mathrm{P}_0(t) \tag{10}$$

$$\mathrm{P}_3(0,t) = a\mathrm{P}_2(t) \tag{11}$$

$$\mathrm{P}_5(0,t) = a\mathrm{P}_4(t) \tag{12}$$

$$\mathrm{P}_7(0,t) = a\cdot \mathrm{P}_6(t) \tag{13}$$

Initial conditions

$$\mathrm{P}_k(0) = \begin{cases}1 & \text{if } K=0\\ 0 & \text{if } K=1,\text{------}8\end{cases} \ldots. \tag{14}$$

Solution of the model

Taking Laplace transform of equation (1) to (13) and using (14) we get

$$\overline{\mathrm{P}}_0(s) = \frac{1}{A(s)} \tag{15}$$

where

$$A(s)=s+2\lambda_m+n\lambda_M+\delta-2\lambda_m\cdot\overline{S_{\Phi_1}}(s+\lambda'_m+n\lambda_M+\delta)$$
$$-\frac{2a\lambda'_m\lambda_m}{(s+a)}F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta)\overline{S}_{\Phi_2}-2a\cdot n\lambda_M\cdot\lambda_m F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta)\overline{S}_{\Phi_2}(s)$$
$$-\frac{n\lambda_M\cdot a}{(s+a)}\overline{S}_{\Phi_1}(s)-\frac{\delta}{(s+\mu)}\left[1-2\lambda_m\cdot F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta)\right]$$

$$\overline{\mathrm{P}}_1(s) = \frac{2\lambda_m}{\mathrm{A}(s)}F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta) \tag{16}$$

$$\overline{\mathrm{P}}_2(s) = \frac{2\lambda'_m\lambda_m}{\mathrm{A}(s)(s+a)}F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta) \tag{17}$$

$$\overline{\mathrm{P}}_3(s) = \frac{2a\lambda'_m\lambda_m}{\mathrm{A}(s)(s+a)}F_{\Phi_1}(s+\lambda'_m+n\lambda_M+\delta) \tag{18}$$

$$\overline{P}_4(s) = \frac{2\lambda_m \cdot n\lambda_M}{A(s)(s+a)} F_{\Phi_1}(s+\lambda'_m + n\lambda_M + \delta) \tag{19}$$

$$\overline{P}_5(s) = \frac{2a\lambda_m \cdot n\lambda_M}{A(s)(s+a)} F_{\Phi_1}(s+\lambda'_m + n\lambda_M + \delta) \cdot F_{\Phi_1}(s) \tag{20}$$

$$\overline{P}_6(s) = \frac{n\lambda_M}{A(s)(s+a)} \tag{21}$$

$$\overline{P}_7(s) = \frac{a \cdot n\lambda_M}{A(s)(s+a)} F_{\Phi_1}(s) \tag{22}$$

$$\overline{P}_8(s) = \frac{\delta}{(s+\mu)} \frac{1}{A(s)} \left\{1 + 2\lambda_m \cdot F_{\Phi_1}(s+\lambda'_m + n\lambda_M + \delta)\right\} \tag{23}$$

When all repairs follow the exponential time distribution

Setting $\overline{S}_{\Phi_i} = \dfrac{\Phi_i}{s+\Phi_i}, i = 1,2$ *,we get*

$\overline{P}_0(s) = \dfrac{1}{B(s)}$ *,where*

$$\begin{aligned} B(s) = {} & 2\lambda_m + n\lambda_M + \delta - 2\lambda_m \cdot \frac{\Phi_1}{(s+\lambda'_m + n\lambda_M + \delta + \Phi_1)} \\ & - \frac{2a\lambda'_m \lambda_m}{(s+a)} \frac{\Phi_1}{(s+\lambda'_m + n\lambda_M + \delta + \Phi_1)} \frac{\Phi_2}{(s+\Phi_2)} \\ & - 2a\lambda_m\lambda'_m \cdot \frac{\Phi_1}{(s+\lambda'_m + n\lambda_M + \Phi_1 + \delta)} \frac{\Phi_2}{(s+\Phi_2)} - \frac{n\lambda_M \cdot a}{(s+a)} \frac{\Phi_1}{(s+\Phi_1)} \\ & + \frac{\delta}{(s+\mu)} \left\{1 + 2\lambda_m \cdot \frac{\Phi_1}{(s+\lambda'_m + n\lambda_M + \delta + \Phi_1)}\right\} \end{aligned} \tag{24}$$

$$\overline{P}_1(s) = \frac{2\lambda_m}{B(s)} \frac{1}{(s+\lambda'_m + n\lambda_M + \Phi_1 + \delta)} \tag{25}$$

$$\overline{P}_2(s) = \frac{2\lambda'_m \lambda_m}{B(s)(s+a)} \frac{1}{(s+\lambda'_m + n\lambda_M + \Phi_1 + \delta)} \tag{26}$$

$$\overline{P}_3(s) = \frac{2a\lambda'_m \lambda_m}{B(s)(s+a)} \frac{1}{(s+\lambda'_m + n\lambda_M + \Phi_1 + \delta)} \frac{1}{(s+\Phi_2)} \tag{27}$$

$$\overline{P}_4(s) = \frac{2a\lambda_m \cdot n\lambda_M}{B(s)\cdot(s+a)} \frac{1}{(s+\lambda'_m + n\lambda_M + \Phi_1 + \delta)} \tag{28}$$

$$\overline{P}_5(s) = \frac{2a\lambda_m \cdot n\lambda_M}{B(s)(s+a)} \frac{1}{(s+\lambda'_m + n\lambda_M + \delta)} \frac{1}{(s+\Phi_2)} \tag{29}$$

$$\overline{P}_6(s) = \frac{n\lambda_M}{B(s)\cdot(s+a)} \tag{30}$$

$$\overline{P}_7(s) = \frac{a\cdot n\lambda_M}{B(s)(s+a)} \frac{1}{(s+\Phi_1)} \tag{31}$$

$$\overline{P}_8(s) = \frac{\delta}{B(s)(s+\mu)} \left\{1 + \frac{2\lambda_m}{(s+\lambda_m + n\lambda_M + \Phi_1 + \delta)}\right\} \tag{32}$$

It can be easily verified that

$$\sum_{i=1}^{8} P_i(s) = \frac{1}{s}$$

Evaluation of Up State Probability

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_1(s)$$

$$\overline{P}_{up}(s) = \frac{1}{(s+2\lambda_m + n\lambda_M + \delta)} \left[1 + \frac{2\lambda_m}{(s+\lambda'_m + n\lambda_M + \delta)}\right] \text{,by inversion process} \tag{33}$$

$$P_{up}(t) = \frac{1}{(2\lambda_m - \lambda'_m)} \left[2\lambda_m \cdot \exp\{-((\lambda'_m + n\lambda_M + \delta)t)\} - \lambda'_m \cdot \exp\{-(2\lambda_m + \lambda'_m + \delta)t\}\right] \tag{34}$$

To obtain expression for reliability, setting all repair rates zero in equation (33). We get

$$\overline{R}(s) = \frac{1}{(s+2\lambda_m + n\lambda_M + \delta)} \text{,by inversion process we get}$$

$$R(t) = \exp\{-(2\lambda_m + n\lambda_M + \delta)t\} \tag{35}$$

$$H(t) = \frac{C_1}{(2\lambda_m - \lambda'_m)} \left[\frac{2\lambda_m(1-\exp\{-(\lambda'_m + n\lambda_M + \delta)\})}{(\lambda'_m + n\lambda_M + \delta)} - \lambda' m\left(\frac{1-\exp\{-(2\lambda_m + \lambda' m + \delta)\}}{(2\lambda_m + \lambda'_m + \delta)}\right)\right] - C_2 t \tag{36}$$

V.T.T.F of the System

$$\sigma^2 = -2 \lim_{s\to 0} \frac{dR(s)}{ds} - (M.T.S.F)^2 \Rightarrow \sigma^2 = \frac{2}{\{2\lambda_m + n\lambda_M + \delta\}^2} - \frac{1}{(2\lambda_m + n\lambda_M + \delta)^2} \tag{37}$$

Mean Time to System Failure

$$M.T.S.F = \lim_{S\to 0} R(s) \Rightarrow M.T.S.F = \frac{1}{2\lambda_m + n\lambda_M + \delta} \tag{38}$$

Numerical computations

Reliability Analysis: Taking and varying t from 0 to 20,in equation (35),we obtain

Table 1.0

T	R(t) for n=1	R(t) for n=2	R (t) for n=j
0	1	1	1
1	.9598291299	.9408232398	0.922193691
2	.9212719587	.8851436850	.8504412045
3	.8842636626	.8327681557	.7842715138
4	.8487420219	.7834876343	.7232502424
5	.8146473164	.7371233744	.6669768109
6	.7819222490	.6935028012	.6150818073
7	.7505117288	.6524635522	.5672245624
8	.7203630197	.6138528730	.5230909131
9	.6914254105	.5775270488	.4823911401
10	.6636502500	.5433508691	.4448580662
11	.6369908422	.5111971250	.4102453023
12	.6114023658	.4809461353	.3783256297
13	.5868418008	.4524853012	.3488950900
14	.5632678551	.4257086870	.3217437042
15	.5406408953	.4005166261	.2967100143
16	.5189228802	.3768153497	.2736241034
17	.4980690982	.3545166381	.2523344220
18	.4706909820	.3335374921	.2327012121
19	.4588646466	.3137998239	.2145955898
20	.4404336545	.2952301669	.1978986991

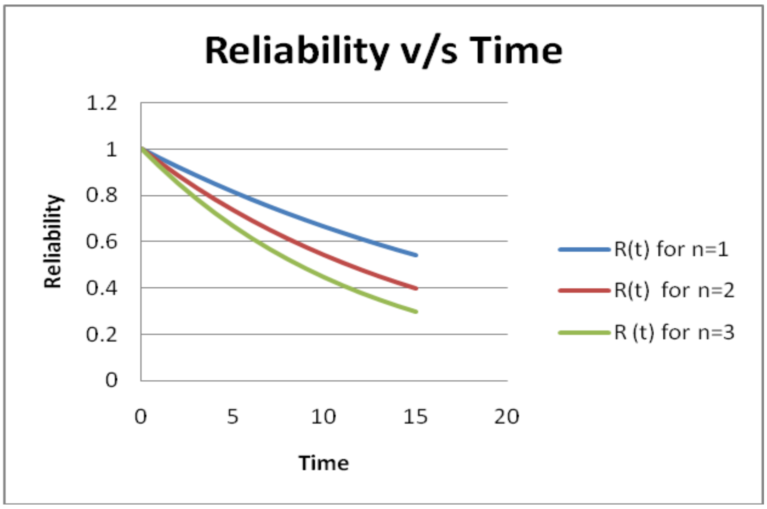


Figure 1.0

Cost Function Analysis

$Taking \lambda_m = 0.001, \lambda'_m = 0.003, \lambda_M = 0.002, \delta = 0.001, C_1 = 100,\ for C_2 = 50,60,70,$ in equation (36),we get

(a) For, n=1

Table 1.2 (a)

T	*H(t) C_2=50*	*H(t) C_2=60*	*H(t) C_2=70*
0	*0*	*0*	*0*
1	48.46589330	38.46589330	28.46589330
2	93.92617164	73.92617167	53.92617167
3	136.4725797	106.4725797	76.47257970
4	176.1940617	136.1940616	96.19406170
5	213.1768468	163.1768468	113.1768468
6	247.5045323	187.5045323	127.5045323
7	279.2581635	209.2581635	149.2581635
8	308.5163120	228.5163120	148.5163120
9	335.3551509	245.3551509	155.3551509
10	359.8485283	259.8485283	159.8485283

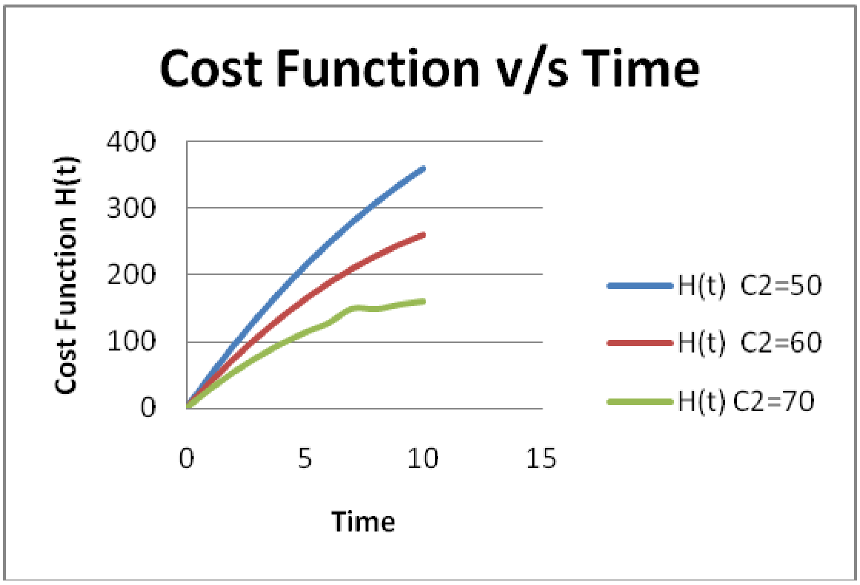


Figure 1.2 (a)

(b) For, n=2

Table 1.2 (b)

T	H(t), C_2=50	H(t) , C_2=60	H(t), C_2=70
0	0	0	0
1	46.51971250	36.51971245	26.51971245
2	86.35009632	66.35009632	46.35009632
3	119.8814524	89.88145242	59.88145242
4	147.4829141	107.4829141	67.48291410
5	169.5035533	119.5035533	69.50355333
6	186.2734306	126.2734306	58.10459095
7	198.1045904	128.1045904	45.29200634
8	205.1144769	118.1147690	58.10459045
9	208.1144769	106.8354761	28.11447640

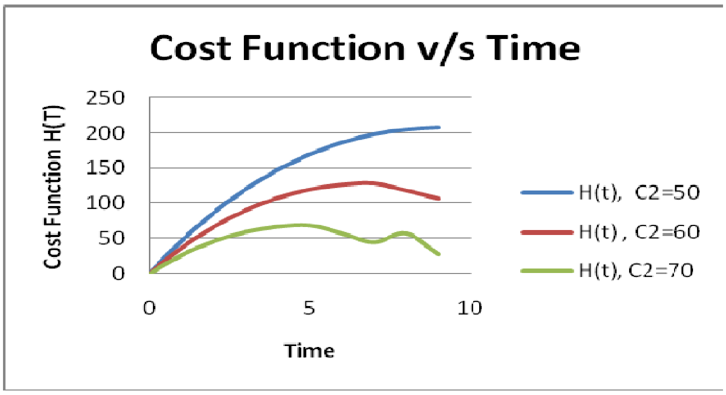


Figure 1.2 (b)

(c) For, n=3

Table 1.2 (c)

T	H.1(t), C_2=50	H(t) , C_2=60	H(t), C_2=70
0	0	0	0
1	44.59919930	34.59919930	24.59919930
2	78.97169741	58.97169741	38.97169741
3	103.9326676	73.93266759	43.93266675
4	120.2379251	80.23792507	54.23979250
5	128.5881019	78.58810188	60.58810188
6	129.6325321	69.63253213	48.63253213
7	123.9728680	53.97286799	40.97286799
8	112.1664470	32.16644766	34.14644766
9	94.72941189	28.72929411	30.72929411
10	72.13964768	20.13964768	19.13964768

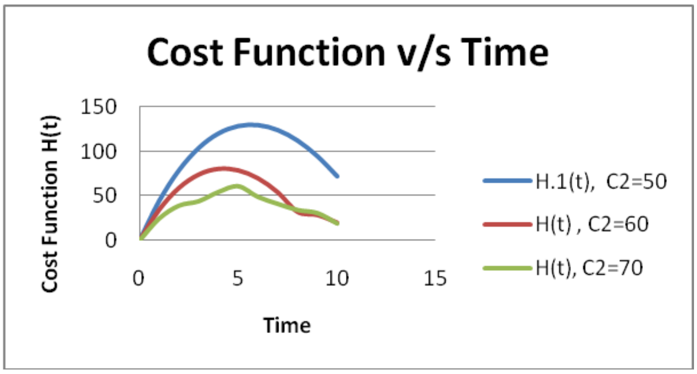


Figure 1.2 (c)

Variance of time to failure: Taking $\lambda_m = 0.001, \delta = 0.001, \lambda_M = 0.002$ in equation (37) for n=1,2,3 we get

Table 1.30

S.No	Variance of time to failure		
	n=1	n=2	n=3
1	31.2500	62.5000	75.555550
2	23.8095	38.4615	45.714285
3	19.2308	27.7778	34.239032
4	16.1290	21.7391	29.502555
5	13.8889	17.8571	24.675182
6	12.1951	15.1515	18.023752
7	10.8696	13.1579	16.765832
8	9.80392	11.6279	14.675182
9	8.92857	10.4167	12.062875
10	9.61538	9.43396	9.353827

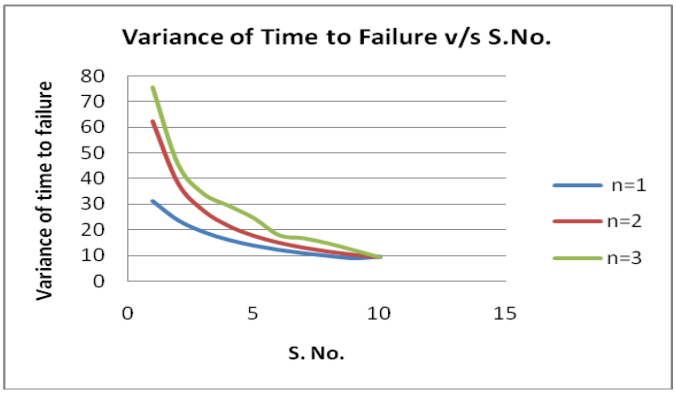


Figure 1.30

Conclusion of This paper / Result

- From the Table 1.0 and Graph 1.0 authors conclude that reliability of this P-S system decreases as well as time increase i.e. as time t increase reliability of the system decreases.
- By the inspection, Table 1.2 (a), 1.2(b) & 1.2(c) and graph fig 1.2(a), fig1.2 (b),& fig-1.2(c) .Expected profit H(t) decrease rapidly as C2 increase and there is no harm by using n identical unit except initial burden.
- By the Inspection Table 1.30& graph , fig 1.30 if P-S system consist by more than two identical unit than variance of time to failure is large but for less than two units it is small, while for a long internal it being constant.

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