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Stochastic Models for time to Recruitment in a Single Grade Manpower System with Correlated Wastages due to Exits and Different Modified Renewal Processes for Breaking Decisions and Involuntary Exits

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Abstract:

Consider any single graded marketing organization in which depletion of manpower takes place due to decisions, exit of personnel etc. These depletion causes three independent different sources of loss of manpower, which is classified into voluntary exit, involuntary exit and frequent breaks taken by the existing workers in the organization. It is assumed that the loss of manpower due to voluntary exit are correlated. By assuming that the inter-involuntary exit times, inter-breaking decision times forms different modified renewal processes, mean and variance of time to recruitment are determined for two different cases of inter-voluntary exit times using a univariate CUM policy of recruitment. The results are numerically illustrated by assuming specific distributions and relevant conclusions are presented.

Keywords: Single grade manpower system, Correlated loss of manpower, Inter-voluntary exit times, Inter-involuntary exit times, Inter-breaking decision times, Different renewal process, Modified Renewal process, Univariate CUM Policy. **AMS Subject Classification (2010):** Primary: 90B70, Secondary: 60H30, 60K05.

INTRODUCTION:

Attrition, which leads to depletion of manpower, is a common phenomenon in any marketing organization. Whenever this organization announces decisions regarding sales target, revision of wages, incentives and perquisites, these decision causes exit (voluntary, involuntary) of personnel from the organization. Another way of depletion may also be due to the existing personnel in the organization when they takes break. Thus the three sources of depletion are due to voluntary exit decisions (e.g. quitting the job, voluntary transfer etc.), involuntary exit decisions (e.g. dismissal, periodic transfer etc.) and breaking decisions (e.g. health illness, celebrations etc.). Hence it would not be realistic by combining the depletion produced by voluntary, involuntary exit decisions and breaking decisions to form a single source of depletion. The loss of manpower due to these decisions will adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to the cost of recruitment. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. A univariate CUM policy of recruitment [10] based on, the replacement policy associated with the shock model approach in reliability theory is stated as follows: *Recruitment is made whenever the cumulative loss of man power exceeds the breakdown threshold.*

Several models of manpower system have been proposed and studied by many authors [4], [5] and [6] extensively in the past. More specifically [10] have initiated the study on finding the expected time to recruitment for a single grade manpower system using shock model approach in reliability theory by considering that attrition is generated by policy decisions. [7] have derived the variance of time to recruitment when the breakdown threshold has two components. Then [1] have studied the problem of time to recruitment with correlated inter decision times and [8] have studied the same problem when inter decision times forms geometric process. Then [11] have derived the variance of time to recruitment for correlated wastages. Later [2] have studied the problem of time to recruitment, when the depletion of manpower is classified into exit of personnel from the organisation and frequent breaks taken by the existing workers in the organisation by considering correlated wastages due to exits. Depletion of manpower due to exits can be involuntary and voluntary exit. It is not so realistic to assume this exit of personnel as a single source of depletion. Hence recently [3] have studied the problem of time to recruitment by classifying the exit of personnel into voluntary and involuntary exit and the breaking decision forms modified renewal process in the sense that before there is any loss in manpower due to breaks, every breaking decision (decisions such as postponing breaking decisions due to any issues in the organisation) is associated with the probability $0<p<1$ to produce loss of manpower. After the first occurrence of loss due to breaks p changes to 1. Considering the assumption of modified renewal process only to the breaking decisions will not be sufficient. Since involuntary exit decisions (decisions such as firing a personnel during workload period, transferring the persons during the project period etc.) has been associated to a probability $0<q'<1$, before the occurrence of loss of manpower. After the first occurrence of loss of manpower probability q equals to 1. Hence in this paper it is assumed inter-involuntary exit decisions and inter breaking decisions forms a modified renewal process. The problem of time to recruitment in this paper is constructed according as the inter-voluntary exit times are in the following cases: (Case-I) as a sequence of exchangeable and constantly correlated exponential random variables and (Case-II) as a geometric process. Mean and variance of time to recruitment are obtained using an univariate CUM policy of recruitment by assuming specific distribution for the loss of manpower and thresholds. The results are numerically illustrated and specific conclusions are made.

MODEL DESCRIPTION:

Consider an organization with single grade in which exit of personnel takes place either by voluntary and involuntary exit of personnel from the organization or due to breaks taken by the existing workers in the organization. Let X_{I_q}, q = 1,2,3,… be a sequence of random variables denoting the loss of manpower due to the qth involuntary exit of personnel from the organization with Laplace transform $\bar{f}_{X_I}(.)$ with parameter $\alpha_1, \alpha_1 > 0$. Let X_{V_r}, r = 1,2,3,… be a sequence of exponential random variables denoting the loss of manpower due to the rth voluntary exit of personnel from the organization with Laplace transform $\bar{f}_{X_V}(.)$ with parameter $\alpha_2, \alpha_2 > 0$, correlation ρ' and the relation $v' = \alpha_2(1-\rho')$. Let Y_l, l = 1,2,3,… be a sequence of exponential random variables denoting the loss of manpower due to the l^{th} break taken by the existing personnel from the organization with Laplace transform $\bar{g}_{Y_l}(.)$ with parameter γ, $\gamma > 0$. The loss of manpower are assumed to be linear and cumulative. Let $\widetilde{X_{I_q}}$ be the cumulative loss of manpower in the first q involuntary exits of personnel from the organization. Let $\widetilde{X_{V_r}}$ be the cumulative loss of manpower in the first r voluntary exits of personnel from the organization. Let $\widetilde{Y_l}$ be the cumulative loss of manpower in the first l breaks. Let U_{I_q} ; q = 1,2,3… be independent and identically distributed exponential random variables representing the time between q-1th and qth involuntary exit of personnel from the organization with mean $\frac{1}{\lambda_{11}}, \lambda_{11} > 0$ and forms a modified renewal process with parameter λ_{12}, ($\lambda_{12} = q'\lambda_{11}$), $\lambda_{12} > 0$ and 0<q'< 1 in the sense that, before there is any loss in manpower, every involuntary exit decision has a fixed probability q’ of causing loss in manpower in the organization. After the first occurrence of loss in manpower, q’ changes to 1. Let U_{V_r} ; r = 1,2,3,… be independent exponential random variables representing the time between r-1th and rth voluntary exit of personnel from the organization with mean $\frac{1}{\lambda_2}, \lambda_2 > 0$. Let V_l; l = 1,2,3,… be independent and identically distributed exponential random variables representing the time between l-1th and l^{th} break with mean $\frac{1}{\beta_1}, \beta_1 > 0$ and forms modified renewal process with parameter β_2, ($\beta_2 = p\beta_1$), $\beta_2 > 0$ and 0<$p < 1$ in the sense that, before there is any loss in manpower, every breaking decision has a fixed probability p of causing loss in manpower in the organization. After the first occurrence of loss in manpower, p changes to 1. Let $N_{11}(t)$ be the number of involuntary exit decisions in (0,t], $N_{12}(t)$ be the number of voluntary exit decisions in (0,t] and $N_2(t)$ be the number of breaking decisions in (0,t]. Let Z_{11} be the exponential threshold for the cumulative loss of manpower due to involuntary exits of personnel from the organization with parameter $\theta_{11} > 0$, Z_{12} be the exponential threshold for the cumulative loss of manpower due to voluntary exits of personnel from the organization with parameter $\theta_{12} > 0$ and Z_2 be the exponential threshold for the cumulative loss of manpower due to breaks with parameter $\theta_2 > 0$. Let $Z_{11}+Z_{12}+Z_2$ be the breakdown threshold for the organization. Let W be the time to recruitment for the organization with the distribution function L(.), density function $l(.)$ and the Laplace transform $\bar{l}(.)$. It is assumed that the loss of man hours due to involuntary exits, voluntary exits and breaks, inter-involuntary and voluntary exit times, inter-breaking decision times and breakdown threshold are stochastically independent.

ANALYTICAL RESULTS:

Let the event {W > t} be the time to recruitment which occurs beyond the time t, and

$\{\widetilde{X_{I_{N_{11}(t)}}} + \widetilde{X_{V_{N_{12}(t)}}} + \widetilde{Y_{N_2(t)}} < Z_{11} + Z_{12} + Z_2\}$ represent the event that the cumulative loss of man hours due to the three types of decisions does not cross the breakdown threshold upto the time t. It is proved that the occurrence of these two events are equal. Hence

$$\{W > t\} \Leftrightarrow \{\widetilde{X_{I_{N_{11}(t)}}} + \widetilde{X_{V_{N_{12}(t)}}} + \widetilde{Y_{N_2(t)}} < Z_{11} + Z_{12} + Z_2\}$$

Now conditioning upon $N_{11}(t), N_{12}(t)$ and $N_2(t)$ by using the result of renewal theory [9], the distribution function and density function for time to recruitment are derived. Hence the r^{th} moment for the time to recruitment is determined by taking the r^{th} derivative with respect to s for the Laplace transform of the time to recruitment at s = 0.

THEOREM-1

Let $Z_i, i = 1,2,3, ...k$ be a sequence of exchangeable and constantly correlated exponential random variables with the correlation ρ, $\rho \in [-1,1]$, mean u and the relation v = u(1- ρ). If the probability density function of Zi; i = 1,2,3,…k is $\frac{1}{u} e^{-\left(\frac{t}{u}\right)}, u > 0, 0 < t < \infty$ and the k-fold convolution of the distribution of these random variables is $$Z_K(t)=\frac{1-\rho}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i\left(1-\sum_{j=0}^{k+i-1}e^{-\left[\frac{t}{u}\right]}\frac{\int_0^{\frac{t}{u}}\left(e^{-z}z^{k+i-1}\right)dz}{(k+i-1)!}\right)$$

then the Laplace Stieltje's transform for $Z_K(t)$ is $\overline{Z_K}(\mathrm{s})=\dfrac{(1-\rho)(vs+1)^{1-k}}{(1-\rho)(vs+1)+k\rho vs}$

Proof: By taking the Laplace transform and using the relation of Laplace transform and the Laplace Stieltje's transform,

$$\overline{Z_K}(\mathrm{s})=\frac{s(1-\rho)}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i\left(\frac{1}{s}-\sum_{j=0}^{k+i-1}\frac{(k+i-j-1)!}{v^{k+i-j-1}\left(s+\left(\frac{1}{v}\right)^{k+i-j}\right.}\,\,(k+i-j-1)!\right)$$

Simplifying the above expression, then the Laplace Stieltje's transform for $Z_k(t)$ is reduced

to $$\overline{Z_K}(\mathrm{s})=\frac{(1-\rho)}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i(1+vs)^{-(k+i)}$$

Doing further simplifications the Laplace Stieltje's transform is proved. Henceforth the moments are derived by taking the r^{th} derivative with respect to s for the Laplace Stieltje's transform of $Z_k(t)$ at s=0.

Case-I: In case-I inter-voluntary exit times are assumed to form a sequence of exchangeable and constantly correlated exponential random variables with mean λ_2, correlation ρ with the relation v = λ_2 (1- ρ)

Using the Theorem-1 taking derivative with respect to s for the Laplace transform of W at s=0 gives the mean time to recruitment for the present case.

E(W) = C1 D1 – C2D2 + C3D3 (1)

where

$$
\begin{aligned}
D_1 = {} & (1-\mathrm{y}_1)\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_2}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_3}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_4}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_5}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_6}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_7}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_8}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_9}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{11})
\end{aligned}
$$

$$
\begin{aligned}
D_2 = {} & (1-\mathrm{y}_{11})\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{12}}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{13}}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{14}}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{15}}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{16}}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{17}}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{18}}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{19}}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{12})
\end{aligned}
$$

$$
\begin{aligned}
D_3 = {} & (1-\mathrm{y}_{21})\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{22}}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_2) \\
& +\frac{y_{23}}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{24}}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_2) \\
& +\frac{y_{25}}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{26}}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{22}) \\
& +\frac{y_{27}}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{28}}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_2) \\
& +\frac{y_{29}}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_2)
\end{aligned}
$$

Now differentiating twice the Laplace transform of W with respect to s and at s = 0, the second moment of W is determined. From these results the variance of time to recruitment for the present case is determined.

Theorem-2: Let Z_k be a sequence of non-negative random variables and *a* be a positive constant. If a normalized stochastic process $\{V_k\}_{k=1}^{\infty}$, where $V_k = a^{n-1}Z_k$, k=1,2,3,… is a geometric process with a parameter *a,* then the Laplace transform for V_k is $\overline{V_K}(\mathrm{s}) = \prod_{r=1}^{k}\overline{f}\left(\frac{s}{a^{r-1}}\right)$

Proof: The distribution function $V_k(t) = F(a^{k-1}t)$; k = 1,2,3,… is determined from the definition of geometric process and by differentiating the distribution function with respect to t, the density function of $v_k(t) = a^{k-1}f(a^{k-1}t)$; k = 1, 2, 3, … is derived.

Now using the property, *Laplace transform of the convolution of random variables is the product of their Laplace transforms,* the Laplace transform for $v_k(t)$ is derived. Hence the theorem is proved.

Corollary: If a = 1, then the sequence of random variables Vk, k = 1, 2, 3; … forms an ordinary renewal process.

Remark: If a > 1, $\{V_k\}_{k=1}^{\infty}$ is stochastically decreasing and when 0 < a < 1, $\{V_k\}_{k=1}^{\infty}$ forms a stochastically increasing sequence.

Case-II: In this case inter-voluntary exit times are assumed to form a geometric process with a parameter *a* > 0.

Using Theorem-2, the mean time to recruitment for Case-III is determined by taking first derivative with respect to s for the Laplace transform of W at s=0.

$$E(W) = C1\ D4 - C2D5 + C3D6 \tag{2}$$

$$\begin{aligned}
D_4 = {} & (1-y_1)\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_2}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_3}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_4}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_5}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_6}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_7}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{11})+\frac{y_8}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{11}) \\
& +\frac{y_9}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{11})
\end{aligned}$$

$$\begin{aligned}
D_5 = {} & (1-y_{11})\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{12}}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{13}}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{14}}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{15}}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{16}}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{17}}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{12})+\frac{y_{18}}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{12}) \\
& +\frac{y_{19}}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_{12})
\end{aligned}$$

$$\begin{aligned}D_6 &= (1-\mathrm{y}_{21})\sum_{r=1}^{\infty}(\overline{f'_{U_{V_r}}}(0)-\overline{f'_{U_{V_{r+1}}}}(0))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{22}}{\mu_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_1))-(\overline{f_{U_{V_r}}}(\mu_1)))\overline{f_{X_{V_r}}}(\theta_2)\\
&+\frac{y_{23}}{\mu_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\mu_2))^r-(\overline{f_{U_{V_r}}}(\mu_2)))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{24}}{\tau_1}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_1))-(\overline{f_{U_{V_r}}}(\tau_1)))\overline{f_{X_{V_r}}}(\theta_2)\\
&+\frac{y_{25}}{\tau_2}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}(\tau_2))-(\overline{f_{U_{V_r}}}(\tau_2)))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{26}}{(\tau_1+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_1))))\overline{f_{X_{V_r}}}(\theta_{22})\\
&+\frac{y_{27}}{(\tau_2+\mu_1)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_1)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_1))))\overline{f_{X_{V_r}}}(\theta_2)+\frac{y_{28}}{(\tau_1+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_1+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_1+\mu_2))))\overline{f_{X_{V_r}}}(\theta_2)\\
&+\frac{y_{29}}{(\tau_2+\mu_2)}\sum_{r=1}^{\infty}((\overline{f_{U_{V_r}}}((\tau_2+\mu_2)))-(\overline{f_{U_{V_r}}}((\tau_2+\mu_2))))\overline{f_{X_{V_r}}}(\theta_2)\end{aligned}$$

Taking second derivative for the Laplace transform of W with respect to s and at s = 0, the second moment for W is derived. From these two results the variance of time to recruitment is determined for the present case.

Note: Geometric process assumed for the sequence of random variables in the inter-voluntary exit times reduced to an ordinary renewal process when a=1. Then the results for an ordinary renewal process can be reduced from Eqn. (2).

The expressions for the notations used in the equations (1) and (2) are given below.

$$A_1=\frac{\alpha_1}{\alpha_1+\theta_{11}},\ A_2=\frac{\alpha_2}{\alpha_2+\theta_{11}},\ A_3=\frac{\gamma}{\gamma+\theta_{11}},\ A_4=\frac{\alpha_1}{\alpha_1+\theta_{12}},\ A_5=\frac{\alpha_2}{\alpha_2+\theta_{12}},\ A_6=\frac{\gamma}{\gamma+\theta_{12}},$$

$$A_7=\frac{\alpha_1}{\alpha_1+\theta_2},\ A_8=\frac{\alpha_2}{\alpha_2+\theta_2},\ A_9=\frac{\gamma}{\gamma+\theta_2};\mathrm{h}_1=\beta_1\beta_2;\mathrm{h}_2=\beta_2;\mathrm{s}_1=\lambda_{11}\lambda_{12};\mathrm{s}_2=\lambda_{12};$$

$$\mathrm{h}_3=\beta_1+\beta_2-\beta_1A_3;\mathrm{h}_4=\beta_1\beta_2-\beta_1\beta_2A_3;\mathrm{s}_3=\lambda_{11}+\lambda_{12}-\lambda_{11}A_1;\mathrm{s}_4=\lambda_{11}\lambda_{12}-\lambda_{11}\lambda_{12}A_1;$$

$$\tau_1=\frac{-s_3-\sqrt{s_3^{\,2}-4s_4}}{2};\tau_2=\frac{-s_3+\sqrt{s_3^{\,2}-4s_4}}{2};\mu_1=\frac{-h_3-\sqrt{h_3^{\,2}-4h_4}}{2};\tau_2=\frac{-h_3+\sqrt{h_3^{\,2}-4h_4}}{2};$$

$$\mathrm{y}_1=\frac{\tau_1\tau_2(1-A_3)h_1+\mu_1\mu_2(1-\mathrm{A}_1)s_1-(1-A_1)(1-A_3)\mathrm{s}_1h_1)}{\tau_1\tau_2\mu_1\mu_2};\mathrm{y}_2=\frac{(1-A_3)(1-A_1)\mathrm{s}_1(h_2\mu_1-h_1)-\tau_1\tau_2(1-A_3)(h_2\mu_1-h_1)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_1};$$

$$\mathrm{y}_3=\frac{(1-A_3)(1-A_1)\mathrm{s}_1(h_1-h_2\mu_2)-\tau_1\tau_2(1-A_3)(h_1-h_2\mu_2)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_2};\mathrm{y}_4=\frac{(1-A_1)(\mathrm{s}_2\tau_1-s_1)(1-A_3)h_1-\mu_2\mu_1(1-A_1)(\mathrm{s}_2\tau_1-s_1)}{(\tau_2-\tau_1)\tau_1\mu_2\mu_1};$$

$$\mathrm{y}_5=\frac{(1-A_1)(s_1-\mathrm{s}_2\tau_2)(1-A_3)h_1-\mu_2\mu_1(1-A_1)(s_1-\mathrm{s}_2\tau_2)}{(\tau_2-\tau_1)\tau_2\mu_2\mu_1};\mathrm{y}_6=\frac{(1-A_3)(1-A_1)(h_2\mu_1-h_1)(\mathrm{s}_2\tau_1-s_1)}{(\tau_2-\tau_1)(\mu_2-\mu_1)\mu_1\tau_1};$$

$$\mathrm{y}_7=\frac{(1-A_3)(1-A_1)(h_2\mu_1-h_1)(s_1-\mathrm{s}_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_1\tau_2};\mathrm{y}_8=\frac{(1-A_3)(1-A_1)(h_1-h_2\mu_2)(\mathrm{s}_2\tau_1-s_1)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_1};$$

$$y_9=\frac{(1-A_3)(1-A_1)(h_1-h_2\mu_2)(s_1-\mathrm{s}_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_2};\mathrm{y}_{11}=\frac{\tau_1\tau_2(1-A_6)h_1+\mu_1\mu_2(1-\mathrm{A}_4)s_1-(1-A_4)(1-A_6)\mathrm{s}_1h_1)}{\tau_1\tau_2\mu_1\mu_2};$$

$$\mathrm{y}_{12}=\frac{(1-A_6)(1-A_4)\mathrm{s}_1(h_2\mu_1-h_1)-\tau_1\tau_2(1-A_6)(h_2\mu_1-h_1)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_1};\mathrm{y}_{13}=\frac{(1-A_6)(1-A_4)\mathrm{s}_1(h_1-h_2\mu_2)-\tau_1\tau_2(1-A_6)(h_1-h_2\mu_2)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_2};$$

$$\mathrm{y}_{14}=\frac{(1-A_4)(\mathrm{s}_2\tau_1-s_1)(1-A_6)h_1-\mu_2\mu_1(1-A_4)(\mathrm{s}_2\tau_1-s_1)}{(\tau_2-\tau_1)\tau_1\mu_2\mu_1};\mathrm{y}_{15}=\frac{(1-A_4)(s_1-\mathrm{s}_2\tau_2)(1-A_6)h_1-\mu_2\mu_1(1-A_4)(s_1-\mathrm{s}_2\tau_2)}{(\tau_2-\tau_1)\tau_2\mu_2\mu_1};$$

$$\mathrm{y}_{16}=\frac{(1-A_6)(1-A_4)(h_2\mu_1-h_1)(\mathrm{s}_2\tau_1-s_1)}{(\tau_2-\tau_1)(\mu_2-\mu_1)\mu_1\tau_1};\mathrm{y}_{17}=\frac{(1-A_6)(1-A_4)(h_2\mu_1-h_1)(s_1-\mathrm{s}_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_1\tau_2};$$

$$\mathrm{y}_{18}=\frac{(1-A_6)(1-A_4)(h_1-h_2\mu_2)(\mathrm{s}_2\tau_1-s_1)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_1};y_{19}=\frac{(1-A_6)(1-A_4)(h_1-h_2\mu_2)(s_1-\mathrm{s}_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_2};$$

$$y_{21}=\frac{\tau_1\tau_2(1-A_9)h_1+\mu_1\mu_2(1-A_7)s_1-(1-A_7)(1-A_9)s_1h_1}{\tau_1\tau_2\mu_1\mu_2};\ y_{22}=\frac{(1-A_9)(1-A_7)s_1(h_2\mu_1-h_1)-\tau_1\tau_2(1-A_9)(h_2\mu_1-h_1)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_1};$$

$$y_{23}=\frac{(1-A_9)(1-A_7)s_1(h_1-h_2\mu_2)-\tau_1\tau_2(1-A_9)(h_1-h_2\mu_2)}{\tau_1\tau_2(\mu_2-\mu_1)\mu_2};\ y_{24}=\frac{(1-A_7)(s_2\tau_1-s_1)(1-A_9)h_1-\mu_2\mu_1(1-A_7)(s_2\tau_1-s_1)}{(\tau_2-\tau_1)\tau_1\mu_2\mu_1};$$

$$y_{25}=\frac{(1-A_7)(s_1-s_2\tau_2)(1-A_9)h_1-\mu_2\mu_1(1-A_7)(s_1-s_2\tau_2)}{(\tau_2-\tau_1)\tau_2\mu_2\mu_1};\ y_{26}=\frac{(1-A_9)(1-A_7)(h_2\mu_1-h_1)(s_2\tau_1-s_1)}{(\tau_2-\tau_1)(\mu_2-\mu_1)\mu_1\tau_1};$$

$$y_{27}=\frac{(1-A_9)(1-A_7)(h_2\mu_1-h_1)(s_1-s_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_1\tau_2};\ y_{28}=\frac{(1-A_9)(1-A_7)(h_1-h_2\mu_2)(s_2\tau_1-s_1)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_1};$$

$$y_{29}=\frac{(1-A_9)(1-A_7)(h_1-h_2\mu_2)(s_1-s_2\tau_2)}{(\mu_2-\mu_1)(\tau_2-\tau_1)\mu_2\tau_2};C_1=\frac{\theta_{12}\theta_2}{(\theta_{12}-\theta_{11})(\theta_2-\theta_{11})};C_2=\frac{\theta_{11}\theta_2}{(\theta_{12}-\theta_{11})(\theta_2-\theta_{12})};C_3=\frac{\theta_{12}\theta_{11}}{(\theta_2-\theta_{11})(\theta_2-\theta_{12})};$$

$$\theta_{11}\neq\theta_{12}\neq\theta_2, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9\neq 1, \mu_1\neq\mu_2, \beta_1\neq\beta_2, \lambda_{11}\neq\lambda_{12}, \tau_1\neq\tau_2$$

NUMERICAL ILLUSTRATION:

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically using MATLAB. The performance measures are calculated for all the three cases.

Case-I: Effect of ρ , ρ′ for the mean and variance of time to recruitment is studied by fixing the value of the parameters q=0.9; p = 0.9; $\theta11$ = 0.001; $\theta12$ = 0.02, $\theta2$ = 0.03, α2=0.7, α1=0.9, γ=0.08, β1=0.9, $\lambda11$=0.8, $\lambda2$=0.1.

ρ′	ρ	E(W)	V(W)
0.1	0.4	5.5501	$3.0692\text{x}10^5$
0.2	0.4	5.5493	$3.0686\text{x}10^5$
0.3	0.4	5.5478	$3.0677\text{x}10^5$
-0.3	0.4	5.5477	$3.0676\text{x}10^5$
-0.2	0.4	5.5492	$3.0686\text{x}10^5$
-0.1	0.4	5.5501	$3.0692\text{x}10^5$
0.1	0.5	5.5427	$3.0642\text{x}10^5$
0.1	0.6	5.5331	$3.0579\text{x}10^5$
0.1	0.7	5.5208	$3.0498\text{x}10^5$
0.1	-0.7	5.5331	$3.0584\text{x}10^5$
0.1	-0.6	5.5406	$3.0632\text{x}10^5$
0.1	-0.5	5.5469	$3.0673\text{x}10^5$

Case-II: Effect of a , ρ′ for the mean and variance of time to recruitment is studied by fixing the value of the parameters q=0.9; p = 0.6; $\theta11$ = 0.009; $\theta12$ = 0.05, $\theta2$ = 0.07, α2=0.7, α1=0.9, γ=0.08, β1=0.9, $\lambda11$=0.8, $\lambda2$=0.2.

ρ'	a	E(W)	V(W)
0.3	0.91	29.5859	1.5672×10^3
0.3	0.93	33.9673	2.3087×10^3
0.3	0.95	37.1501	3.0947×10^3
0.3	0.97	39.4819	3.9453×10^3
0.3	1.01	42.4584	5.8540×10^3
0.3	1.03	43.3807	6.9880×10^3
0.3	1.05	44.0473	8.1913×10^3
0.3	1.07	44.5198	9.5017×10^3
0.31	0.9	26.7893	1.2017×10^3
0.33	0.9	26.7888	1.2010×10^3
0.37	0.9	26.7878	1.1997×10^3
-0.37	0.9	26.7864	1.1995×10^3
-0.33	0.9	26.7878	1.2009×10^3
-0.31	0.9	26.7885	1.2016×10^3

CONCLUSION:

In Case-(I), if the negative value of correlation (loss due to voluntary exit) increases by fixing the other parameters the mean time to recruitment and variance of time to recruitment increases. When the positive value of correlation (loss due to voluntary exit) increases by fixing the other parameters the mean time to recruitment variance of time to recruitment decreases. If the negative value of correlation (inter-voluntary exit times) increases by fixing the other parameters the mean time to recruitment and variance of time to recruitment increases. When the positive value of correlation (inter-voluntary exit times) increases by fixing the other parameters the mean time to recruitment variance of time to recruitment decreases.

In Case-(II), for (a<1, a>1) increasing the parameter for geometric process by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment increases. If the negative value of correlation (loss due to voluntary exit) increases by fixing the other parameters the mean time to recruitment and variance of time to recruitment increases. When the positive value of correlation (loss due to voluntary exit) increases by fixing the other parameters the mean time to recruitment variance of time to recruitment decreases.

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